Kinematics and Dynamics of Machines	<ul> <li>There are various methods of performing kinematic analysis of mechanisms, including <i>graphical, analytical,</i> and <i>numerical.</i> The choice of a method depends on the problem at hand and on available computational means.</li> <li>One way to analyze kinematics of mechanisms is relative velocities and accelerations method.</li> </ul>
3. Kinematic Analysis of Mechanisms	
(Relative Velocity Method)	<ul> <li>Another sufficient method for simple and many compound mechanisms is <i>loop-closure equation</i> method which is presented here in vector notation.</li> </ul>
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Introduction

## **Relative Velocity**

 From preliminary dynamics we know that the rate of change of a general vector Q with respect to a fixed frame OXYZ and with respect to a frame Oxyz rotating with an angular velocity Ω is

$$(\dot{\mathbf{Q}})_{OXYZ} = (\dot{\mathbf{Q}})_{OXYZ} + \mathbf{\Omega} \times \mathbf{Q}$$

OR

 $\frac{DQ}{Dt} = \frac{dQ}{dt} + \omega \times Q$ 



- If frame QXYZ is fixed and frame Oxyz rotates with angular velocity , then, Postilon vector for the particle P is the same in both frames but the rate of change depends on the choice of frame.
- The absolute velocity of the particle P is

$$\vec{v}_P = \left(\dot{\vec{r}}\right)_{OXYZ} = \vec{\Omega} \times \vec{r} + \left(\dot{r}\right)_{OXYZ}$$

• Absolute acceleration for the particle P is

 $\vec{a}_{P} = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) + 2\vec{\Omega} \times \left(\dot{\vec{r}}\right)_{Oxyz} + \left(\ddot{\vec{r}}\right)_{Oxyz}$ 

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The most general motion of a rigid body in space is equivalent, to the sum of a translation and a rotation. Considering two particles *A* and *B* of the body:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

where  $\mathbf{v}_{B/A}$  is the velocity of *B* relative to a frame *AX*'*Y*'*Z*' attached to *A* and of fixed orientation. Denoting by  $\mathbf{r}_{B/A}$  the position vector of *B* relative to *A*, we write

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{\omega} \times \mathbf{r}_{B/A}$$

where  $\boldsymbol{\omega}$  is the angular velocity of the body at the instant considered. The acceleration of *B* is, by similar reasoning

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

or

 $\mathbf{a}_B = \mathbf{a}_A + \mathbf{\alpha} \times \mathbf{r}_{B/A} + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}_{B/A})$ 

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**Example :** The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine : 1. velocity of piston, 2. angular velocity of connecting rod, 3. velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4. velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.



**Example :** In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle  $BAD = 60^{\circ}$ .



**Example :** In Fig. 7.9, the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm; AB = 44 mm; BC 49 mm; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.



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## Vector Algebra and Analysis

A vector **r** has two components in the (x,y) plane:  $r_x$  and  $r_y$ . Note that the bold font identifies vectors. The following two notations for a vector in a component form will be used:



The result of the cross-product of two vectors is a vector perpendicular to the plane in which the original two vectors are lying. Thus, if the two vectors are lying in the (x,y) plane, then their product will have a *z*-direction.

$$\mathbf{u} = \mathbf{a}^T \times \mathbf{b} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & 0 \\ b_x & b_y & 0 \end{bmatrix}$$
$$\mathbf{u} = (a_y 0 - b_y 0) \mathbf{i} - (a_x 0 - b_x 0) \mathbf{j} + (a_x b_y - b_x a_y) \mathbf{k} = (a_x b_y - b_x a_y) \mathbf{k}$$
$$|\mathbf{u}| = u = (a_x b_y - b_x a_y)$$

 $\mathbf{u} = ab \ (\cos \alpha \ \sin \beta - \cos \beta \ \sin \alpha) = ab \ \sin(\beta - \alpha)$ 

 An addition/subtraction of two (or more) vectors is a vector whose elements are found by addition/subtraction of the corresponding *x*- and *y*components of the original vectors. If **a** = (*ax*, *ay*)<sup>T</sup> and **b** = (*bx*, *by*)<sup>T</sup>, then

$$\mathbf{a} + \mathbf{b} = (a_x + b_x, a_y + b_y)^T$$

 The scalar (or dot) product of two vectors is a scalar, which is found by multiplication of the corresponding *x*- and *y*-components of two vectors and then the summation of results. For the two vectors **a** and **b**, their scalar product is

$$\mathbf{d} = \mathbf{a}^{T}\mathbf{b} = (a_{x}, a_{y})\begin{bmatrix} b_{x} \\ b_{y} \end{bmatrix} = a_{x}b_{x} + a_{y}b_{y}$$

• If vectors **a** and **b** are given in the form  $\mathbf{a} = a (\cos \alpha, \sin \alpha)^T \mathbf{b} =$ , the  $b (\cos \beta, \sin \beta)^T$  scalar product takes the form

$$\mathbf{d} = \mathbf{a}^T \mathbf{b} = ab(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = ab\cos(\alpha - \beta)$$

 $\mathbf{a} = a(t) [\cos \alpha(t), \sin \alpha(t)]^T$ 

$$\frac{d\mathbf{a}(t)}{dt} = \frac{da(t)}{dt} \left[\cos\alpha(t), \sin\alpha(t)\right]^{T} + a(\mathbf{t}) \left[-\sin\alpha(t), \cos\alpha(t)\right]^{T} \frac{d\alpha(t)}{dt}$$

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Kinematic Analysis Of The Slider-crank Mechanism Using Loop-closure Method



• In order to perform a velocity analysis, we can derivate position equation with respect to time:

$$\sum_{t=1}^{N} \dot{r}_i(t) [\cos \theta_i(t), \sin \theta_i(t)]^T + r_i(t) [-\sin \theta_i(t), \cos \theta_i(t)]^T \omega_i(t) = 0$$

• And for acceleration analysis:

$$\sum_{i=1}^{N} \ddot{r_i} [\cos \theta_i, \sin \theta_i]^T + 2\dot{r_i} [-\sin \theta_i, \cos \theta_i]^T \omega_i$$
$$-r_i [\cos \theta_i, \sin \theta_i]^T \omega_i^2 + r_i [-\sin \theta_i, \cos \theta_i]^T \alpha_i = 0$$

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Example 1: In below figure an inverted slider-crank mechanism is shown.

- a. Write a loop-closure equation for this mechanism.
- b. If the input is the crank angle, what are the unknowns?
- c. Solve the equation for the unknowns.
- d. Express the position of point *P* in terms of the input angle.



- **Example 2:** In below figure a slider-crank mechanism is shown.
- a. Write a loop-closure equation for this mechanism.
- b. If the input is the crank angle, what are the unknowns?
- c. Solve the equation for the unknowns.
- d. Express the position of point *P* in terms of the input angle.

