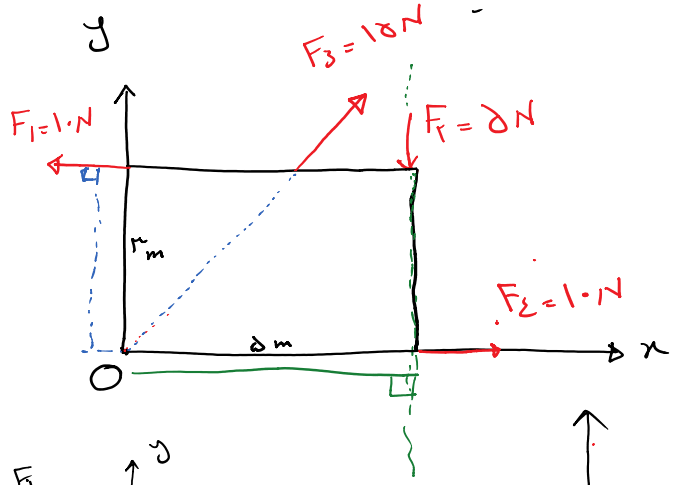
Meeting_004

Wednesday, October 14, 2020 2:28 PM

مثال ۱: گشتاور نیروها حول نقطه O را به صورت اسکالری محاسبه کنید.

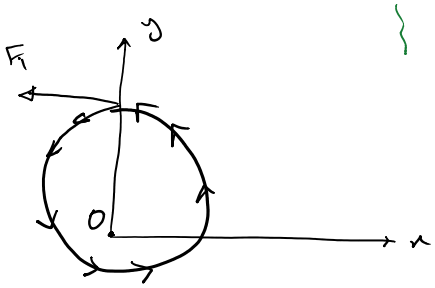


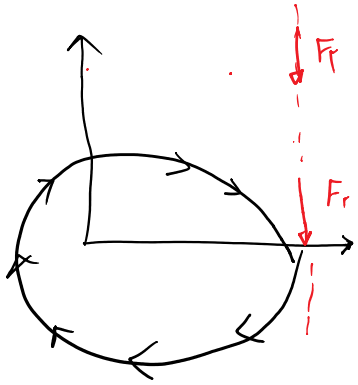
$M_1 = F_1 d_1 = 10 \times 2 = 20\ N.m$ (+)

$M_2 = F_2 \times d_2 = 5 \times 5 = 25\ N.m$ (−)

$M_3 = F_3 . d_3 = 15 \times 0 = 0$

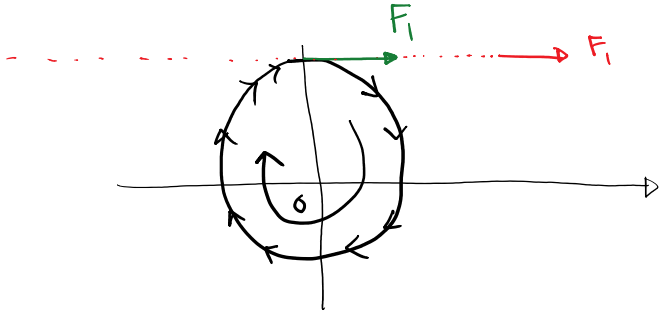
$M_4 = F_4 d_4 = 0$



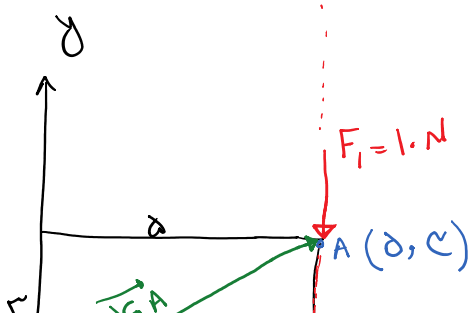


$\vec{M} = M_1 + M_2 + M_3 + M_4$

$= +20 - 25 + 0 + 0 = +5$ (+)

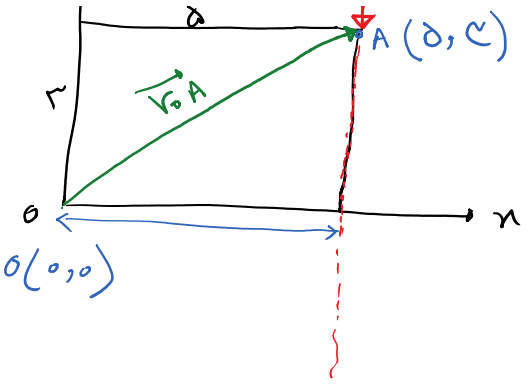


مثال: گشتاور نیرو حول به صورت برداری حول نقطه O را محاسبه کنید



$\vec{F_1} = -10\,\vec{j}$

$\vec{r}_{OA} = (5-0)\vec{i} + (2-0)\vec{j} = 5\vec{i} + 2\vec{j}$



$\vec{r}_{OA} = (2-0)\hat{i} + (2-0)\hat{j} = 2\hat{i} + 2\hat{j}$

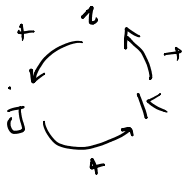
$\vec{M}_O = \vec{r} \times \vec{F} = (2\hat{i} + 2\hat{j}) \times (-1 \cdot \hat{j})$

$= -2 \cdot \hat{k} + (-2 \times 0)$

$= -2 \cdot \hat{k}$

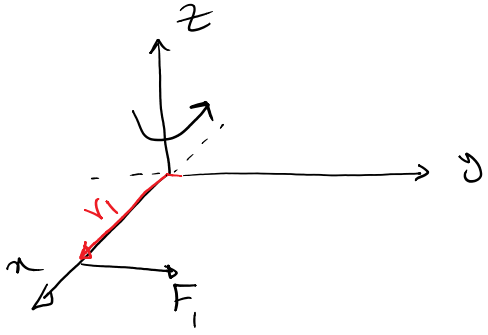
$|M| = 2$

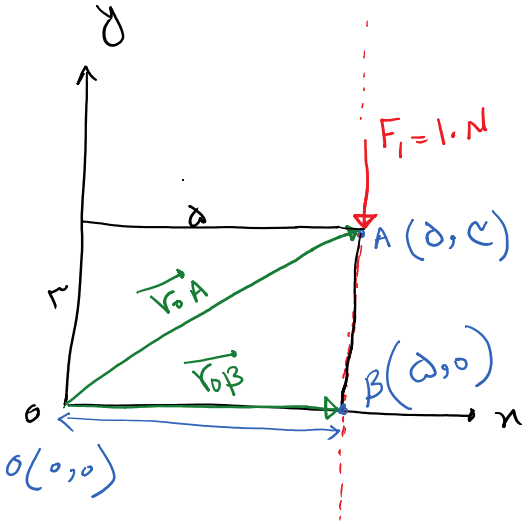
$i \times i = j \times j = k \times k = 0$



$M_O = F \times d = 1 \times 2 = 2$ ↻ $= -2 \cdot k$

$M = r_1 \times F_1$



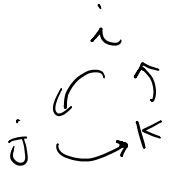


$\vec{F}_1 = -1 \cdot \hat{j}$

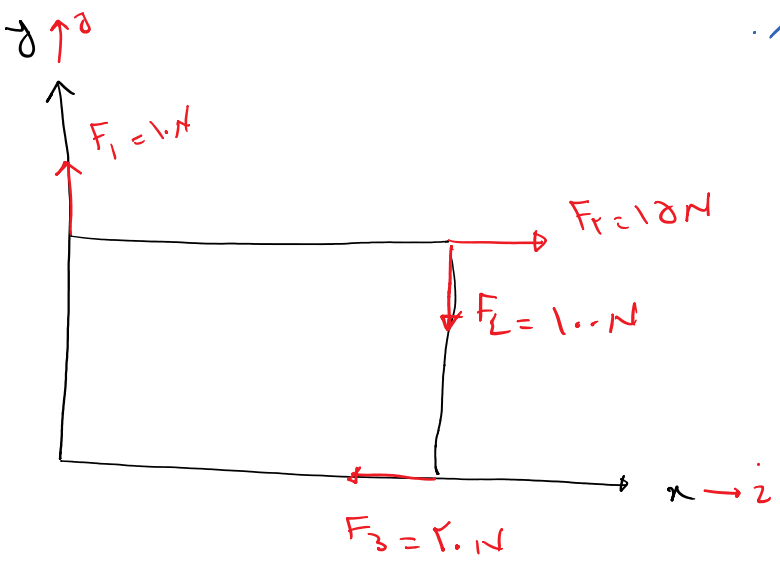
$\vec{r}_{OB} = (2\hat{i}) = 2\hat{i}$

$\vec{M} = \vec{r}_{OA} \times \vec{F}_1 = 2\hat{i} \times (-1 \cdot \hat{j}) = -2 \cdot \hat{k}$

$i \times i = j \times j = k \times k = 0$



در شکل زیر نیروها را بصورت بردار بنویسید.



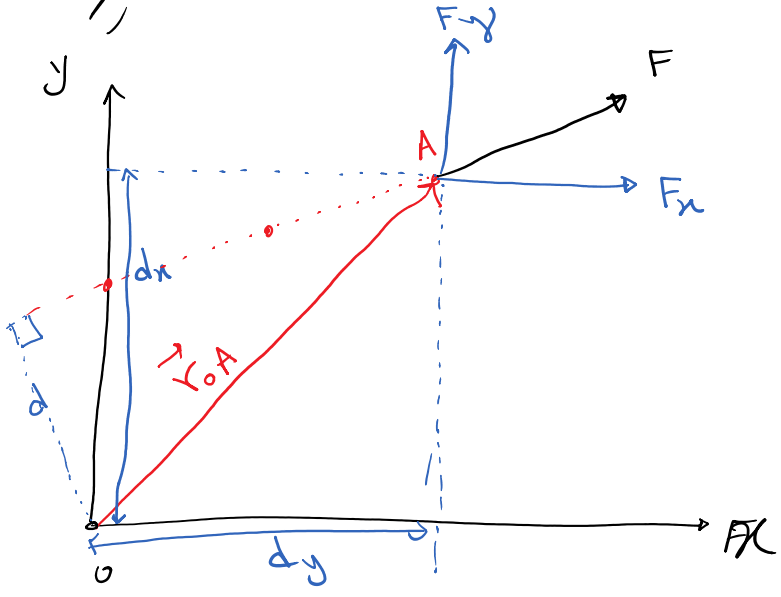
$\vec{F_1} = 10\,j$ $\quad F_2 = 15\,i$

$F_3 = -20\,i$ $\quad F_4 = -100\,j$

نیروهایی که بردارشان محورها یا موازی محورهای مختصات هستند فقط دارای یک مولفه‌ی i یا j یا k هستند.

قضیه وارینون

گشتاور یک نیرو حول یک نقطه برابر است با گشتاور مولفه‌های آن نیرو حول همان نقطه

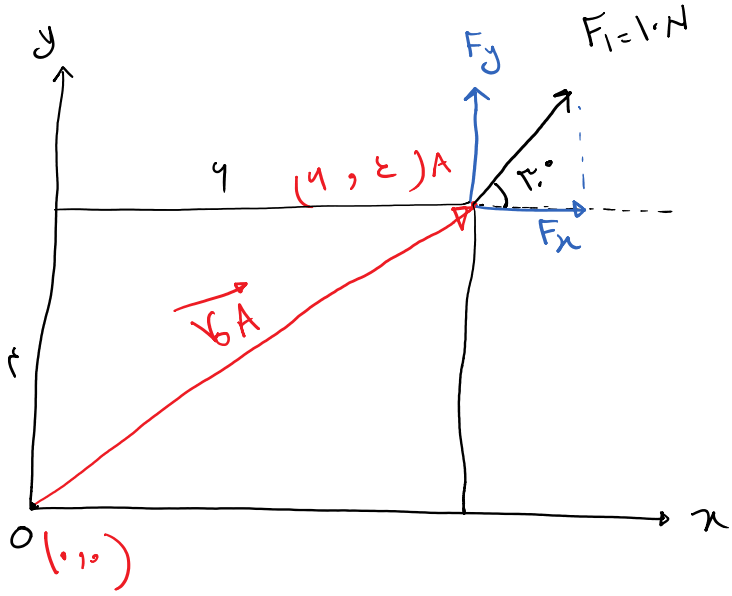


$\vec{M} = \vec{r}_{OA} \times \vec{F}$

$\vec{M} = \vec{r}_{OA} \times (\vec{F_x} + \vec{F_y})$

$= \vec{r}_{OA} \times \vec{F_x} + \vec{r}_{OA} \times \vec{F_y}$

در تکلیف زیر گشتاور نیروی F حول نقطه O را بیابید



$$\vec{F_1} = F_1\cos 30\,i + F\sin 30\,j$$

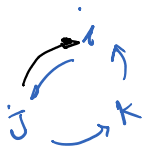
$$\vec{F_1} = 10\times\frac{\sqrt{3}}{2}\,i + 10\times\frac{1}{2}\,j$$

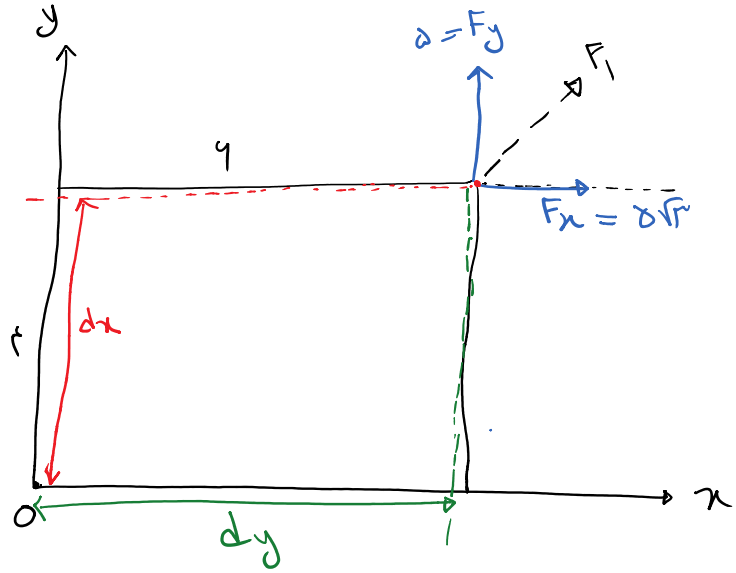
$$= \underbrace{5\sqrt{3}\,i}_{F_x} + \underbrace{5\,j}_{F_y}$$

$$\vec{r}_{OA} = (4i) + 4j = 4i + 4j$$

$$\vec{M}_O = \vec{r}_{OA}\times\vec{F} = (4i+4j)\times(5\sqrt{3}\,i + 5j) = 20\,k - 20\sqrt{3}\,k$$

$$= (20 - 20\sqrt{3})\,k$$





$$M_x = F_x\cdot d_x = 5\sqrt{3}\times 4$$

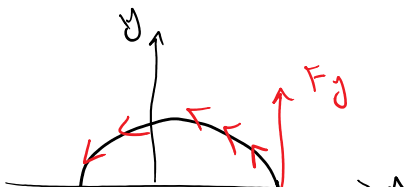
$$= 20\sqrt{3}\ (-)$$

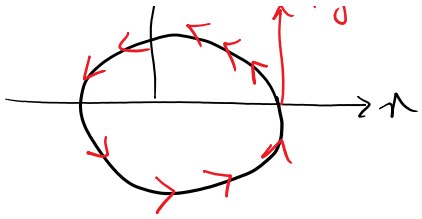
$$M_y = F_x\times d_y = 5\times 4 = 20\ (+)$$

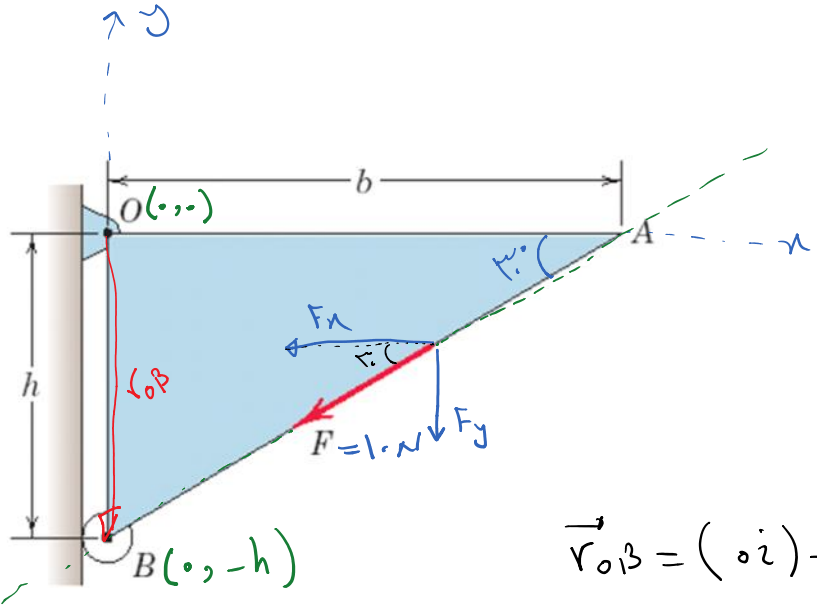
$$M = M_x + M_y$$

$$= -20\sqrt{3} + 20 = 20 - 20\sqrt{3}$$

$$= (20 - 20\sqrt{3})\,k$$







$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

$$= -F\cos 15^\circ\,\vec{i} - F\sin 15^\circ\,\vec{j}$$

$$= -10\times\frac{\sqrt{3}}{2}\vec{i} - 10\times\frac{1}{2}\vec{j}$$

$$= -5\sqrt{3}\,\vec{i} - 5\vec{j}$$

$$\vec{r}_{OB} = (0\vec{i}) + (-h)\vec{j} = -h\vec{j}$$

$$\vec{M}_O = \vec{r}_{OA}\times\vec{F} = (-h\vec{j})\times(-5\sqrt{3}\vec{i} - 5\vec{j}) = \boxed{-5\sqrt{3}h\,\vec{k}}$$