CHAPTER 5

Kinematics of Rigid Bodies

Kinematics of Rigid Bodies

❑ Applications

A battering ram is an example of curvilinear translation – the ram stays horizontal as it swings through its motion.

Kinematics of Rigid Bodies

❑ Applications

How can we determine the velocity of the tip of a turbine blade?

Kinematics of Rigid Bodies

❑ Applications

Planetary gear systems are used to get high reduction ratios with minimum weight and space. How can we design the correct gear ratios?

Kinematics of Rigid Bodies

☐ Applications

Biomedical engineers must determine the velocities and accelerations of the leg in order to design prostheses.

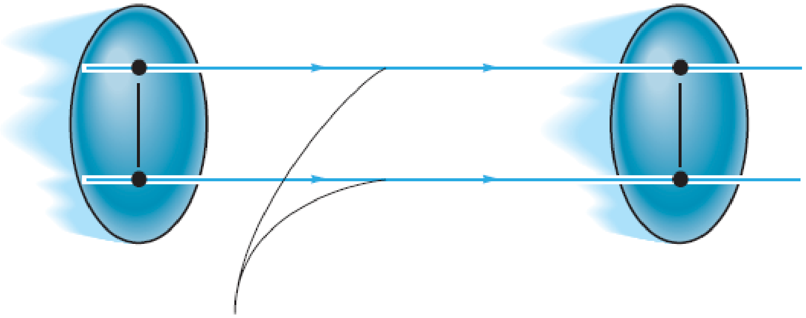
Kinematics of Rigid Bodies

❑ Introduction

- Kinematics of rigid bodies: **relations** between **time** and the **positions**, **velocities**, and **accelerations** of the **particles forming a rigid body**.
- **Classification of rigid body motions:**

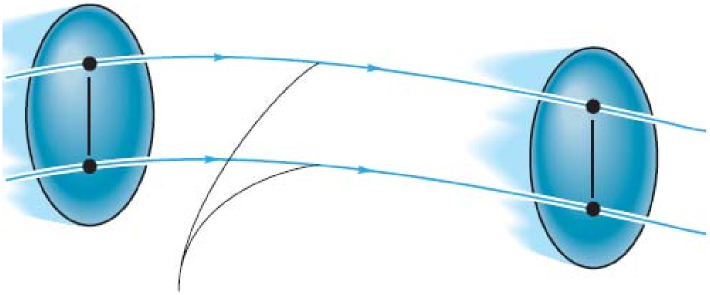
Translation

Rectilinear Translation



Path of rectilinear translation

Curvilinear Translation



Path of curvilinear translation

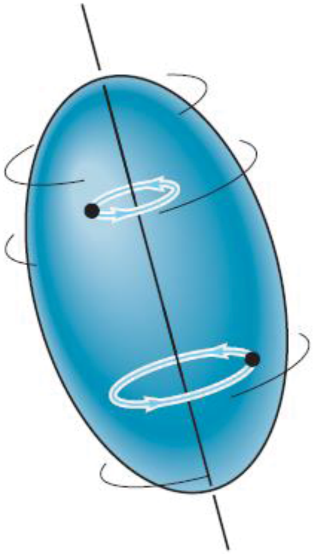
Kinematics of Rigid Bodies

☐ Introduction

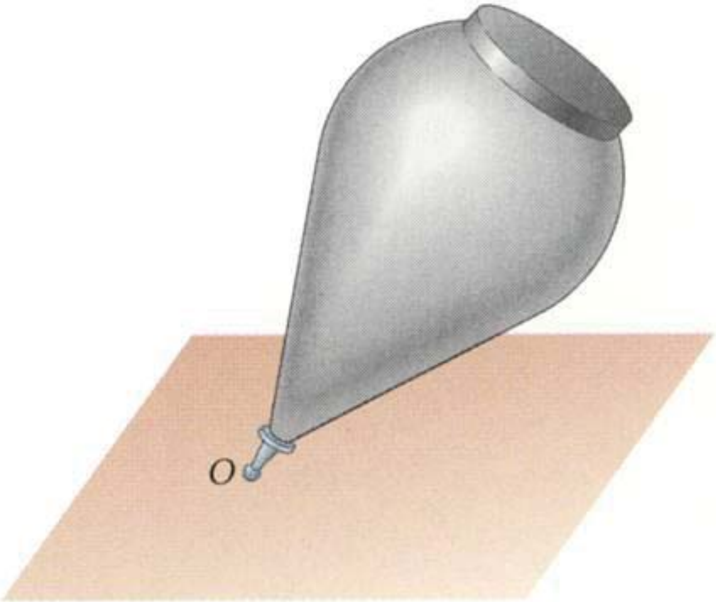
- Kinematics of rigid bodies: **relations** between **time** and the **positions**, **velocities**, and **accelerations** of the particles **forming a rigid body**.

- **Classification of rigid body motions:**

Rotation about a fixed axis ***Motion about a fixed point***



Rotation about a fixed axis

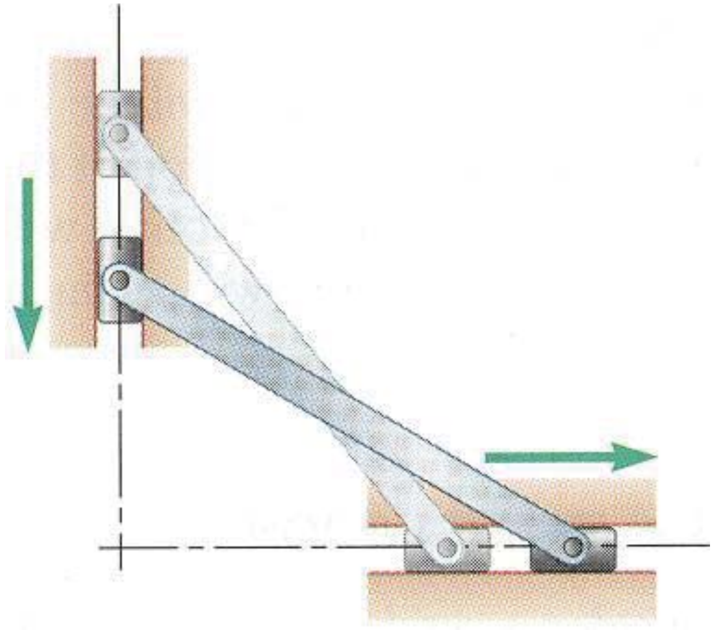


Kinematics of Rigid Bodies

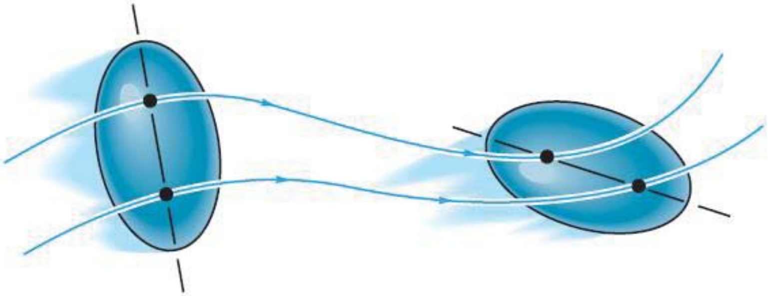
❑ Introduction

- Kinematics of rigid bodies: **relations** between **time** and the **positions**, **velocities**, and **accelerations** of the **particles forming a rigid body**.
- **Classification of rigid body motions:**

General plane motion



General motion



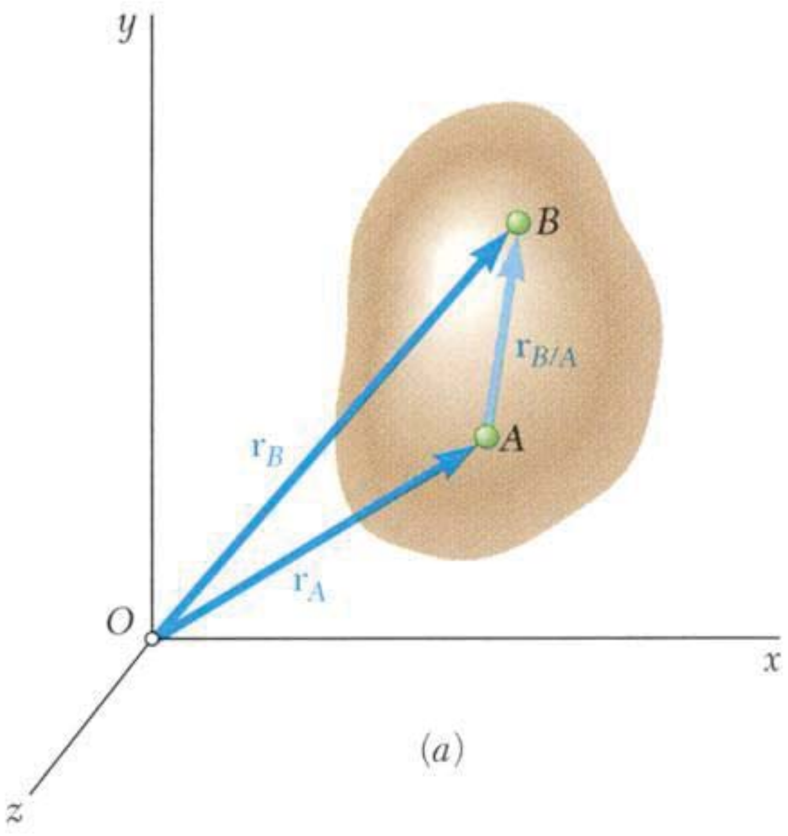
Kinematics of Rigid Bodies

❑ Translation

- Consider rigid body in translation:
  - Direction of any straight line inside the body is constant,
  - All particles forming the body move in parallel lines.

- For any two particles in the body,

$$\boxed{\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}}$$



Kinematics of Rigid Bodies

❑ Translation

$$\boxed{\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}}$$

- Differentiating with respect to time,

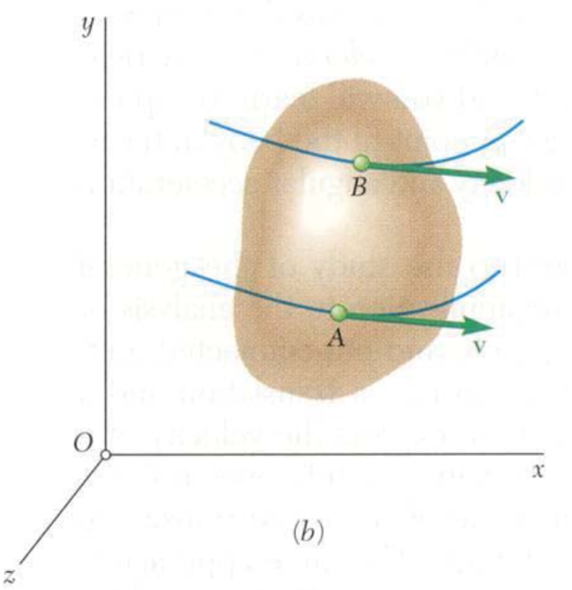
$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \cancelto{0}{\dot{\vec{r}}_{B/A}} = \dot{\vec{r}}_A \qquad \Rightarrow \boxed{\vec{v}_B = \vec{v}_A}$$

All particles have the same velocity.

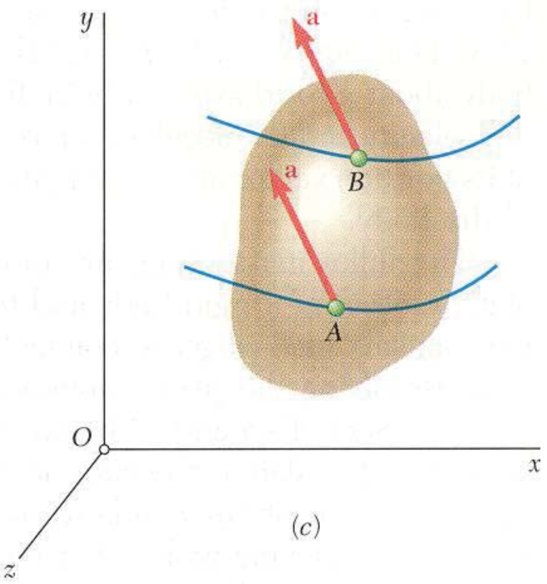
- Differentiating with respect to time again,

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \cancelto{0}{\ddot{\vec{r}}_{B/A}} = \ddot{\vec{r}}_A \qquad \Rightarrow \boxed{\vec{a}_B = \vec{a}_A}$$

All particles have the same acceleration.



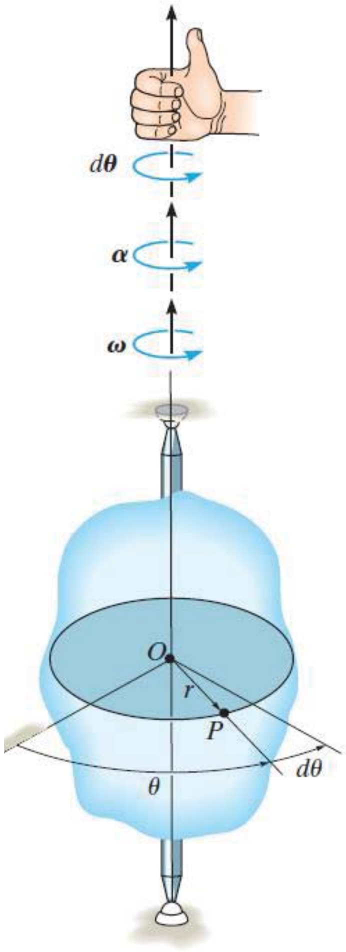
(b)



(c)

Kinematics of Rigid Bodies

❑ Rotation about a Fixed Axis

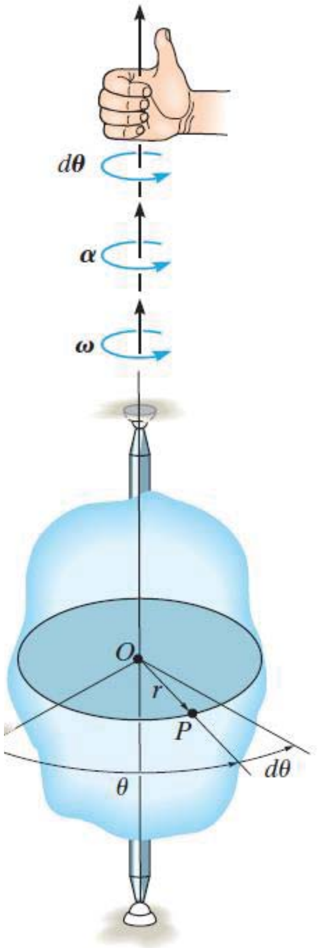


Angular Motion. Since a point is without dimension, it cannot have angular motion. *Only lines or bodies undergo angular motion*. For example, consider the body shown in Fig. 16–4*a* and the angular motion of a radial line *r* located within the shaded plane.

Angular Position. At the instant shown, the *angular position* of *r* is defined by the angle θ, measured from a *fixed* reference line to *r*.

Kinematics of Rigid Bodies

❑ Rotation about a Fixed Axis

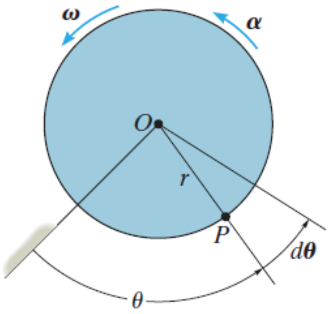


Angular Displacement. The change in the angular position, which can be measured as a differential $d\boldsymbol{\theta}$, is called the *angular displacement*.* This vector has a *magnitude* of $d\theta$, measured in degrees, radians, or revolutions, where 1 rev = 2π rad. Since motion is about a *fixed axis*, the direction of $d\boldsymbol{\theta}$ is *always* along this axis. Specifically, the *direction* is determined by the right-hand rule; that is, the fingers of the right hand are curled with the sense of rotation, so that in this case the thumb, or $d\boldsymbol{\theta}$, points upward, Fig. 16–4*a*. In two dimensions, as shown by the top view of the shaded plane, Fig. 16–4*b*, both θ and $d\theta$ are counterclockwise, and so the thumb points outward from the page.

Angular Velocity. The time rate of change in the angular position is called the *angular velocity* $\boldsymbol{\omega}$ (omega). Since $d\boldsymbol{\theta}$ occurs during an instant of time dt, then,

$(\circlearrowleft +)$

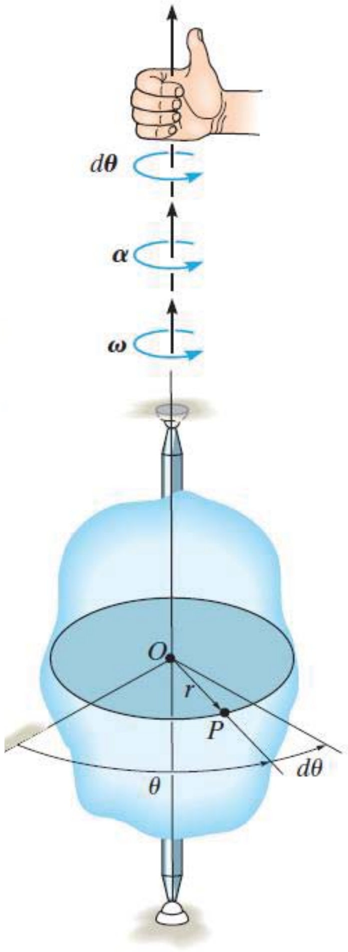
$$\boxed{\omega = \frac{d\theta}{dt}}$$



Kinematics of Rigid Bodies

❑ Rotation about a Fixed Axis

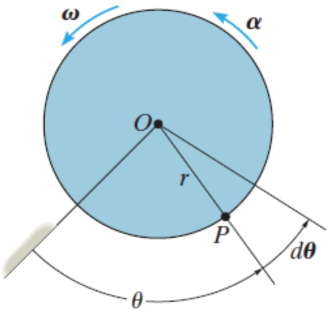
Angular Acceleration. The *angular acceleration* **α** (alpha) measures the time rate of change of the angular velocity. The *magnitude* of this vector is



$(\circlearrowleft +)$ $$\boxed{\alpha = \frac{d\omega}{dt}}$$

$(\circlearrowleft +)$ $$\alpha = \frac{d^2\theta}{dt^2}$$

$(\circlearrowleft +)$ $$\boxed{\alpha\, d\theta = \omega\, d\omega}$$



Kinematics of Rigid Bodies

❑ Equations Defining the Rotation of a Rigid Body About a Fixed Axis

- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

- Recall

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad \Rightarrow \qquad \boxed{\alpha = \frac{d^2\theta}{dt^2} \quad or \quad \alpha = \omega\frac{d\omega}{d\theta}}$$

- *Uniform Rotation*, $\alpha = 0$:

$$\boxed{\theta = \theta_0 + \omega\, t}$$

- *Uniformly Accelerated Rotation*, α = constant:

$$\boxed{\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0) \end{aligned}}$$

Kinematics of Rigid Bodies

Rotation About a Fixed Axis. Velocity

- Consider rotation of rigid body about a fixed axis *AA'*

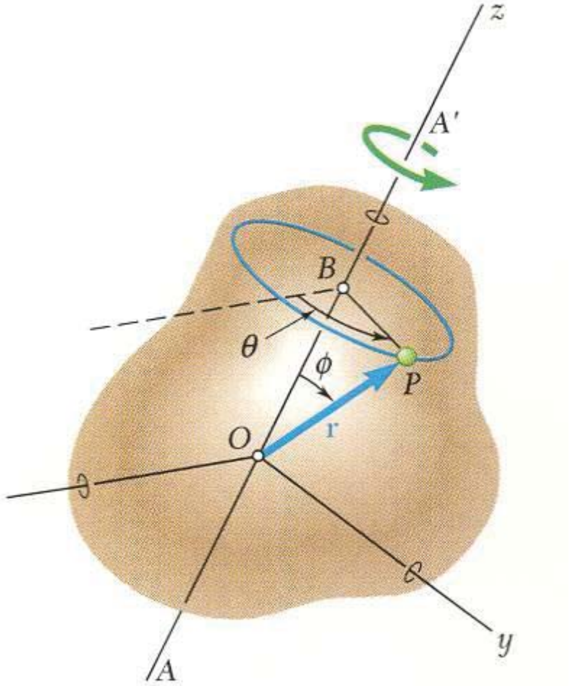
- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle *P* is tangent to the path with magnitude $v = ds/dt$

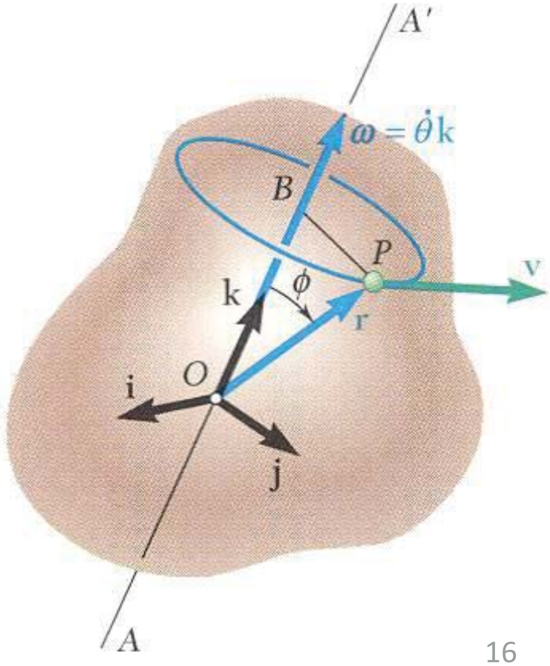
$$\Delta s = (BP)\,\Delta\theta = (r\sin\phi)\,\Delta\theta$$

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} \quad \Rightarrow \quad \boxed{v = r\dot{\theta}\sin\phi}$$

- The same result is obtained from

$$\boxed{\begin{aligned} &\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega}\times\vec{r} \\ &\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = angular\ velocity \end{aligned}}$$



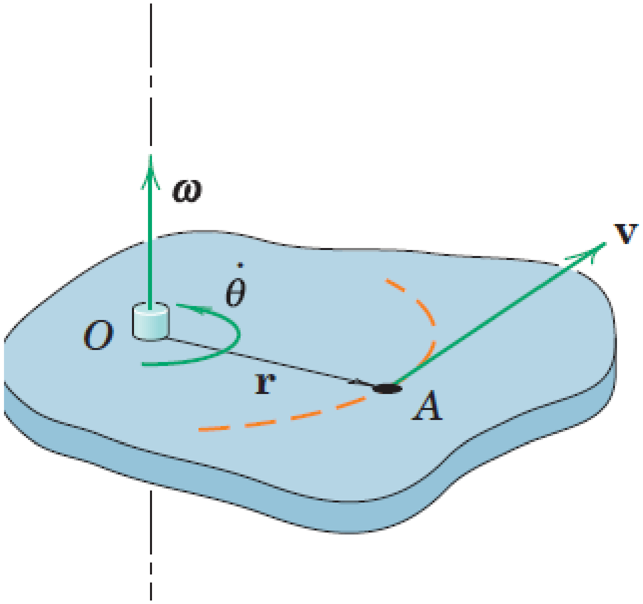


Kinematics of Rigid Bodies

❑ Rotation About a Fixed Axis. Velocity

$$\mathbf{v} = \dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}$$

The order of the vectors to be crossed must be retained. The reverse order gives $\mathbf{r} \times \boldsymbol{\omega} = -\mathbf{v}$.



Kinematics of Rigid Bodies

☐ Concept Quiz

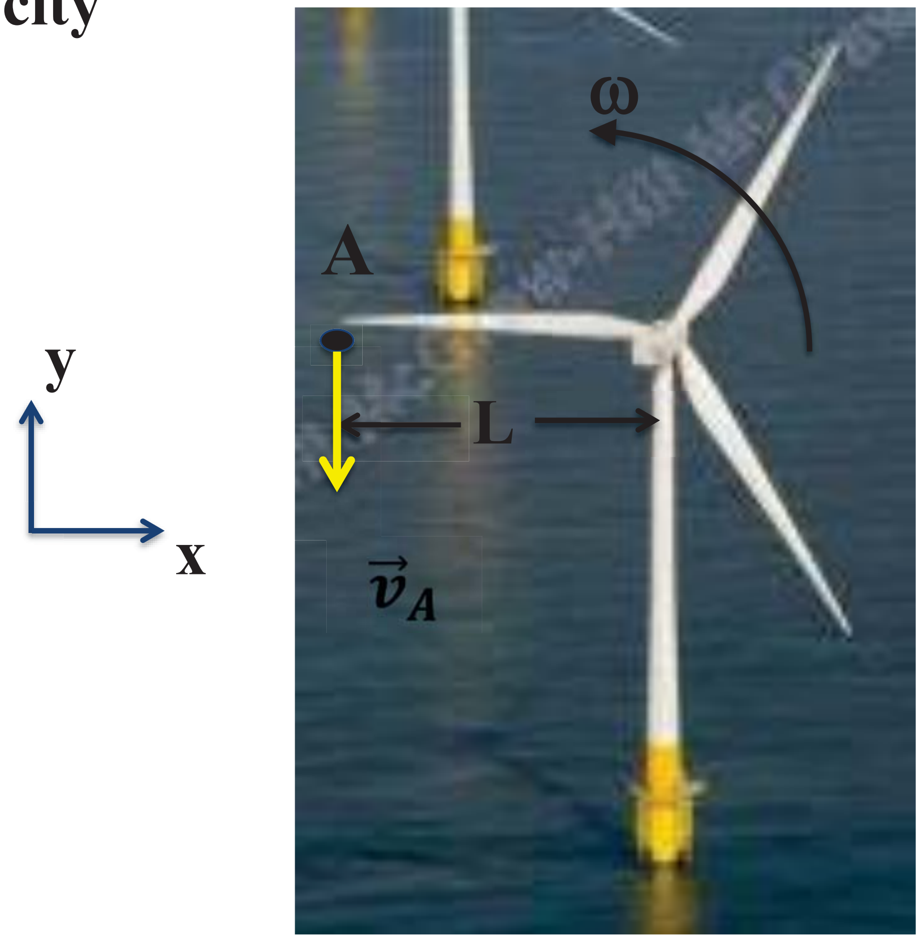
What is the direction of the velocity of point A on the turbine blade?

- **a)** →
- **b)** ←
- **c)** ↑
- [✓ **d)** ↓]

$$\vec{v}_A = \vec{\omega} \times \vec{r}$$

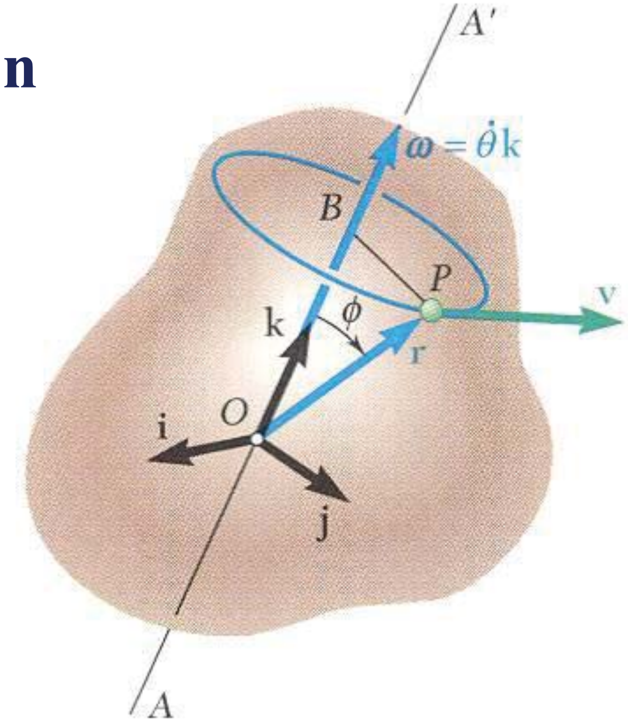
$$\vec{v}_A = \omega \vec{k} \times (-L)\vec{i}$$

$$\vec{v}_A = -L\omega \vec{j}$$



Kinematics of Rigid Bodies

Rotation About a Fixed Axis. Acceleration



- Differentiating to determine the acceleration,

$$\vec{\mathrm{a}} = \frac{d\vec{\mathrm{v}}}{dt} = \frac{\mathrm{d}}{dt}(\vec{\omega}\times\vec{r}) = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$$\Rightarrow \boxed{\vec{\mathrm{a}} = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\vec{v}}$$

$$\boxed{\begin{aligned}\frac{d\vec{\omega}}{dt} &= \vec{a} = angular\ acceleration \\ &= a\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}\end{aligned}}$$

- Acceleration of P is combination of two vectors,

$$\boxed{\vec{\mathrm{a}} = \vec{a}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})}$$

$\vec{a}\times\vec{r}$: tangential acceleration component

$\vec{\omega}\times(\vec{\omega}\times\vec{r})$: radial acceleration component

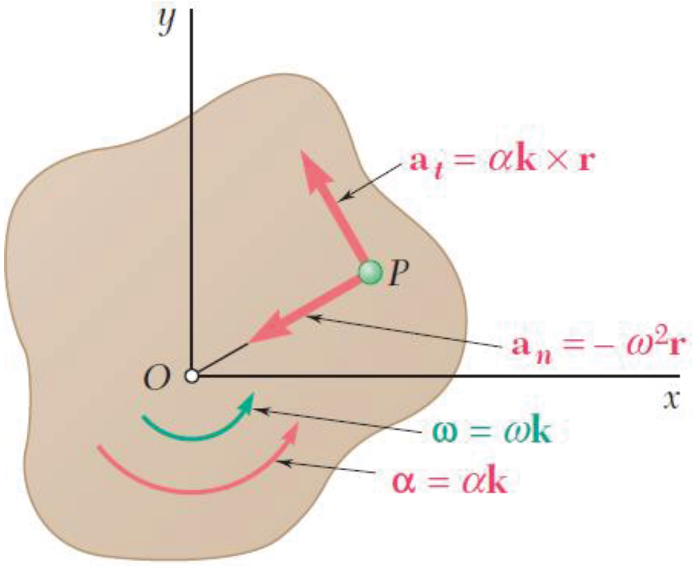
Kinematics of Rigid Bodies

❑ Rotation About a Fixed Axis. Representative Slab

- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.

- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) \quad \Rightarrow \quad \boxed{\vec{a} = \alpha\vec{k}\times\vec{r} - \omega^2\vec{r}}$$



- Resolving the acceleration into tangential and normal components,

$$\boxed{\begin{array}{ll} \vec{a}_t = \alpha\vec{k}\times\vec{r} & a_t = r\alpha \\ \vec{a}_n = -\omega^2\vec{r} & a_n = r\omega^2 \end{array}}$$

Kinematics of Rigid Bodies

❑ Concept Quiz

What is the direction of the normal acceleration of point A on the turbine blade?

[✓ a)] →

b) ←

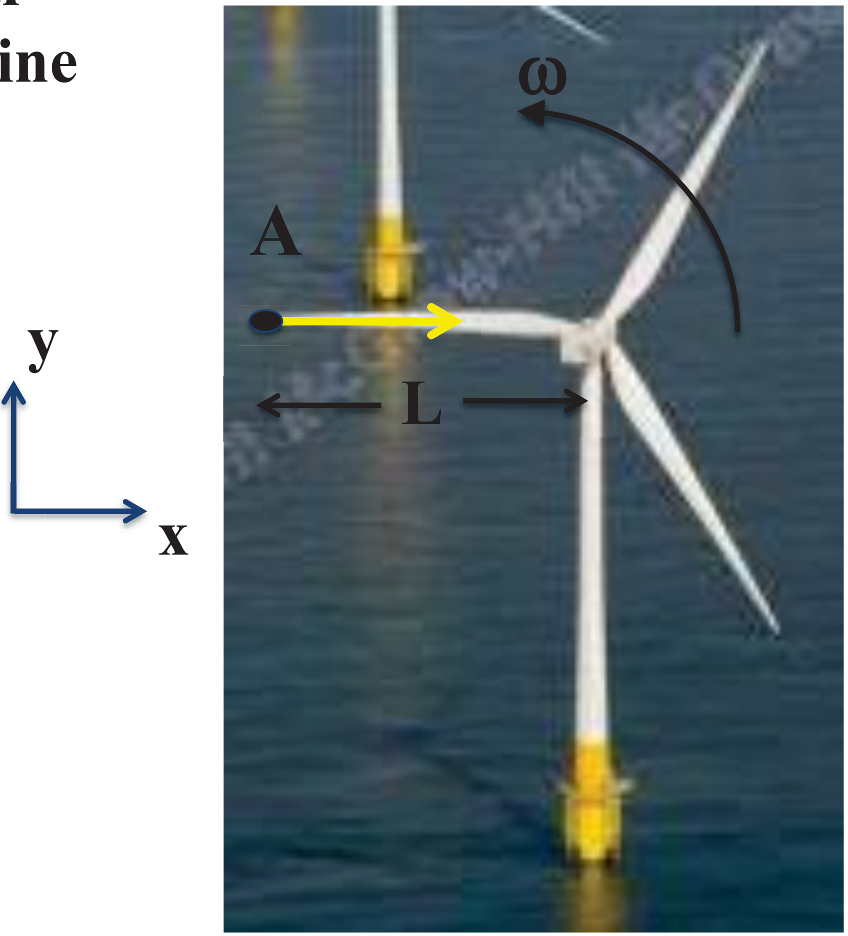
c) ↑

d) ↓

$$\vec{a}_n = -\omega^2 \vec{r}$$

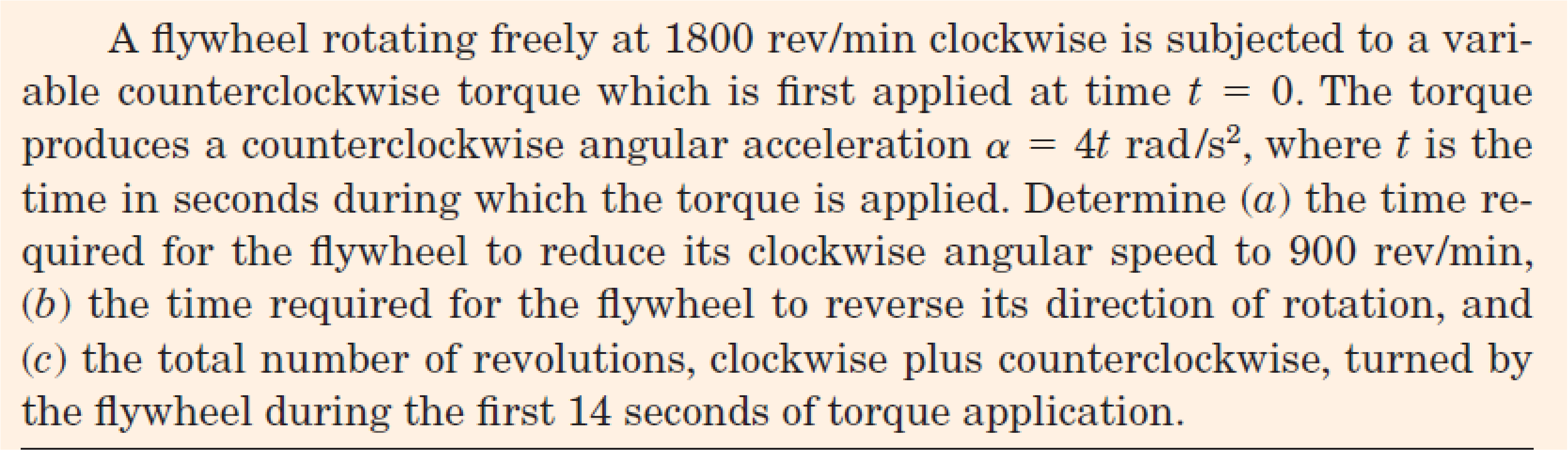
$$\vec{a}_n = -\omega^2 (-L\vec{i}\,)$$

$$\vec{a}_n = L\omega^2\,\vec{i}$$



Kinematics of Rigid Bodies

❑ Sample Problem 01

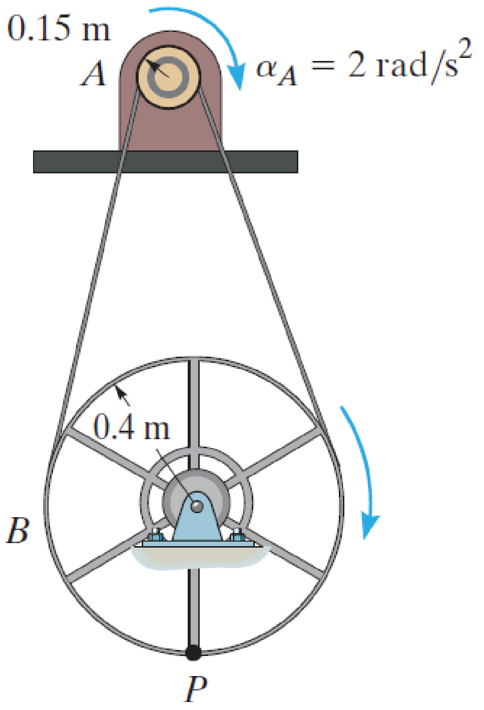


A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. The torque produces a counterclockwise angular acceleration $\alpha = 4t$ rad/s^2, where t is the time in seconds during which the torque is applied. Determine (a) the time required for the flywheel to reduce its clockwise angular speed to 900 rev/min, (b) the time required for the flywheel to reverse its direction of rotation, and (c) the total number of revolutions, clockwise plus counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

Kinematics of Rigid Bodies

❑ Sample Problem 02

The motor shown in the photo is used to turn a wheel and attached blower contained within the housing. The details are shown in Fig. 16–7*a*. If the pulley *A* connected to the motor begins to rotate from rest with a constant angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$, determine the magnitudes of the velocity and acceleration of point *P* on the wheel, after the pulley has turned two revolutions. Assume the transmission belt does not slip on the pulley and wheel.

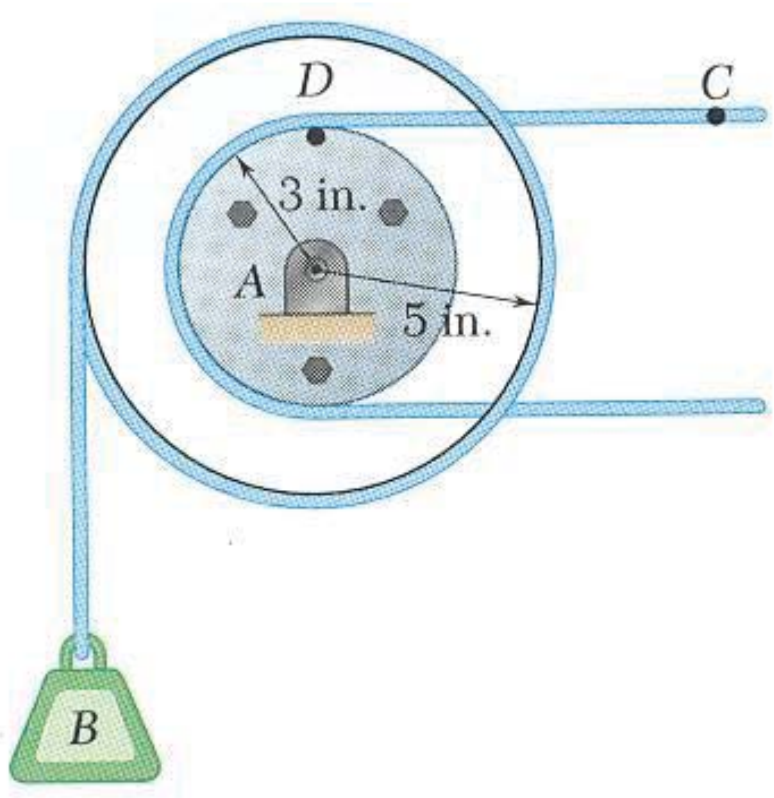


Kinematics of Rigid Bodies

❑ Sample Problem 03

Cable *C* has a constant acceleration of 9 in/s^2 and an initial velocity of 12 in/s, both directed to the right.

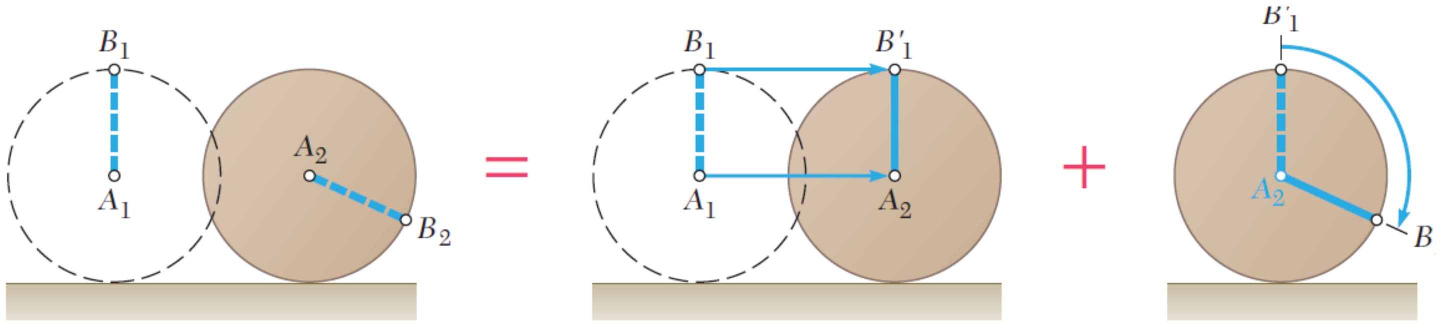
Determine *(a)* the number of revolutions of the pulley in 2 s, *(b)* the velocity and change in position of the load *B* after 2 s, and *(c)* the acceleration of the point *D* on the rim of the inner pulley at *t* = 0.



Kinematics of Rigid Bodies

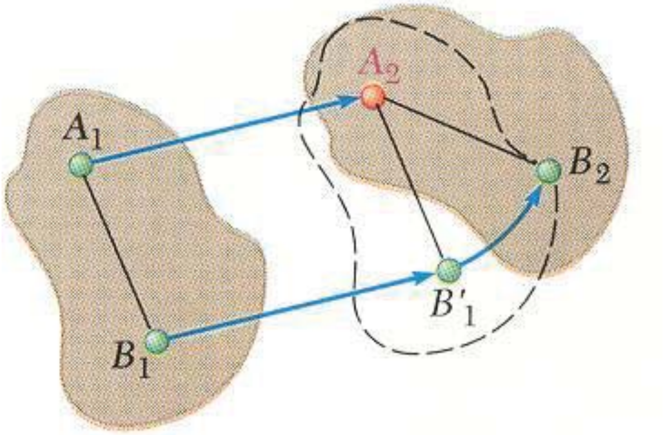
General Plane Motion

- *General plane motion* is ***neither a translation nor a rotation.***
- General plane motion can be considered as ***the sum of a translation and rotation.***



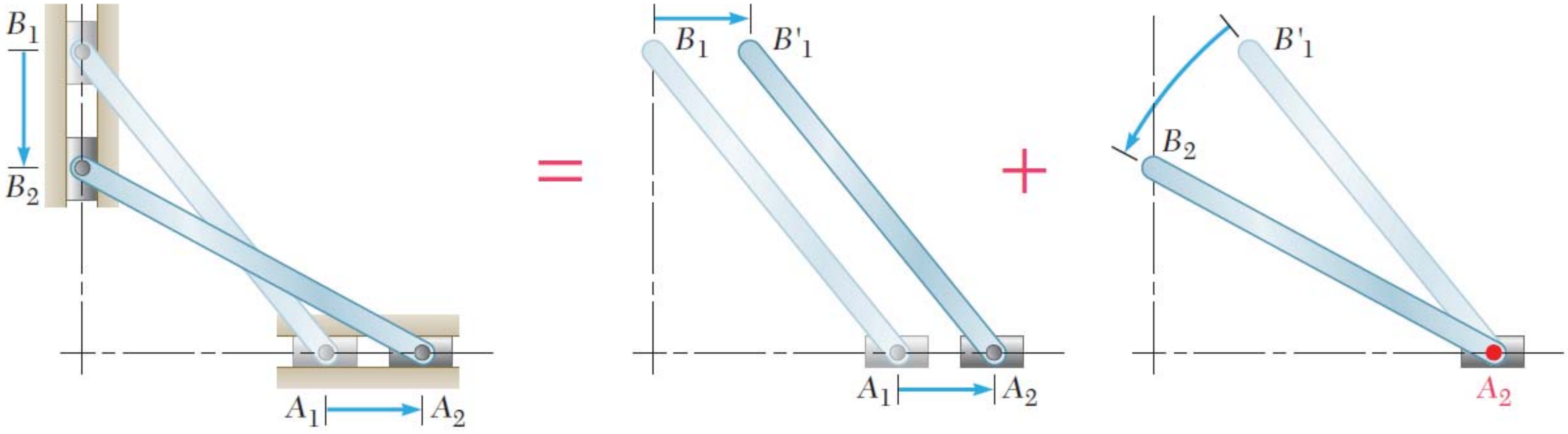
Plane motion = Translation with *A* + Rotation about *A*

- Displacement of particles *A* and *B* to A_2 and B_2 can be divided into two parts:
    - translation to A_2 and B_1'
    - rotation of B_1' about A_2 to B_2

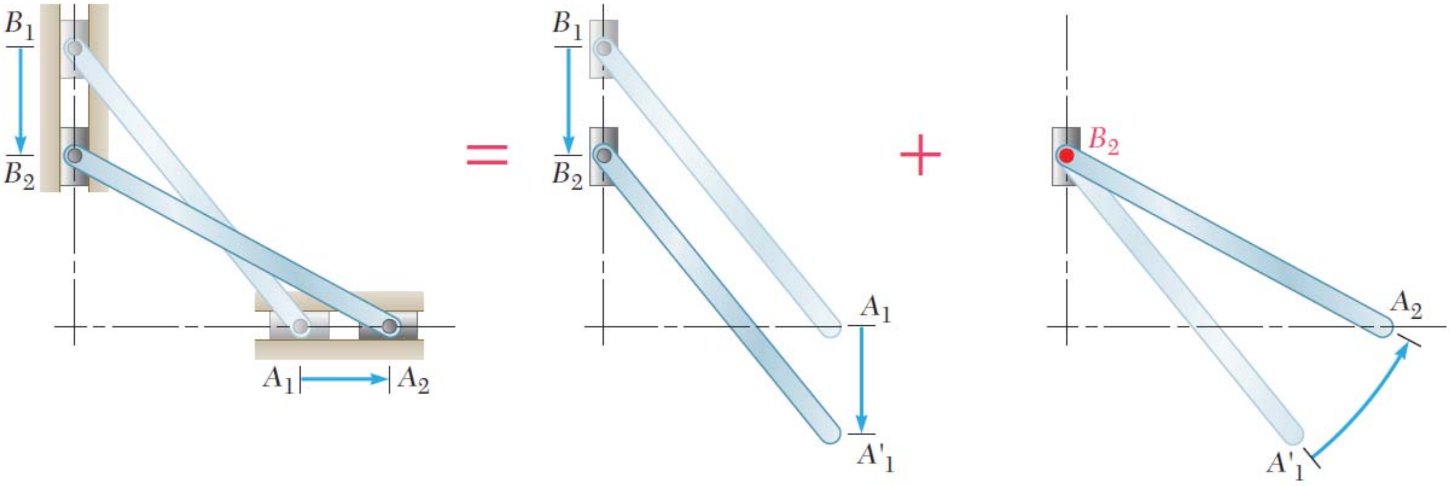


Kinematics of Rigid Bodies

❑ General Plane Motion



Plane motion = Translation with A + Rotation about A

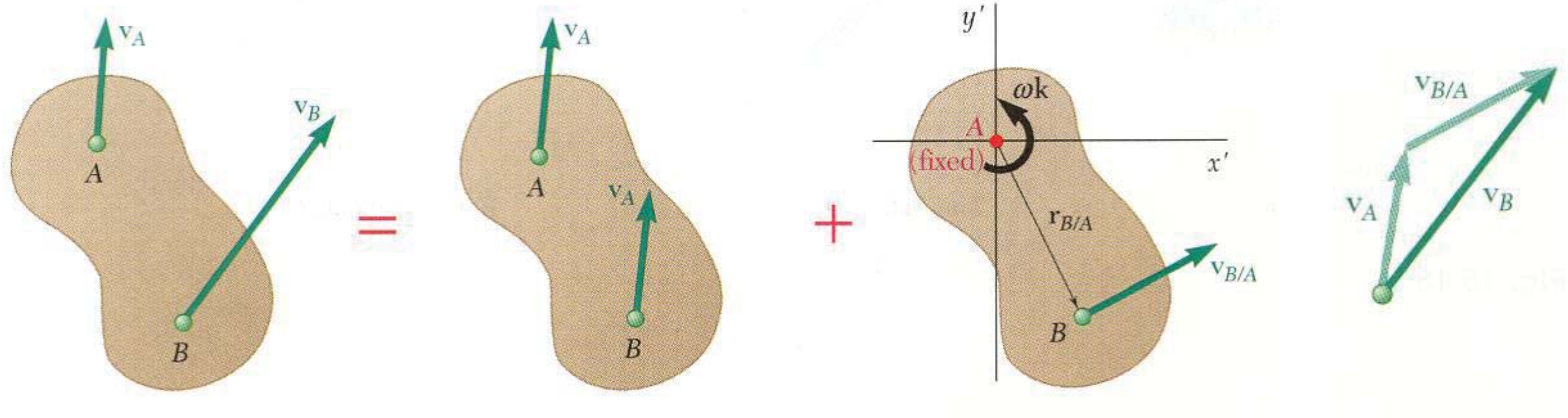


Plane motion = Translation with B + Rotation about B

Kinematics of Rigid Bodies

❑ Absolute and Relative Velocity in Plane Motion

- Any plane motion can be replaced by a translation of an arbitrary reference point *A* and a simultaneous rotation about *A*.



Plane motion = Translation with *A* + Rotation about *A* $v_B = v_A + v_{B/A}$

$$\boxed{\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}}$$

$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{B/A}$$

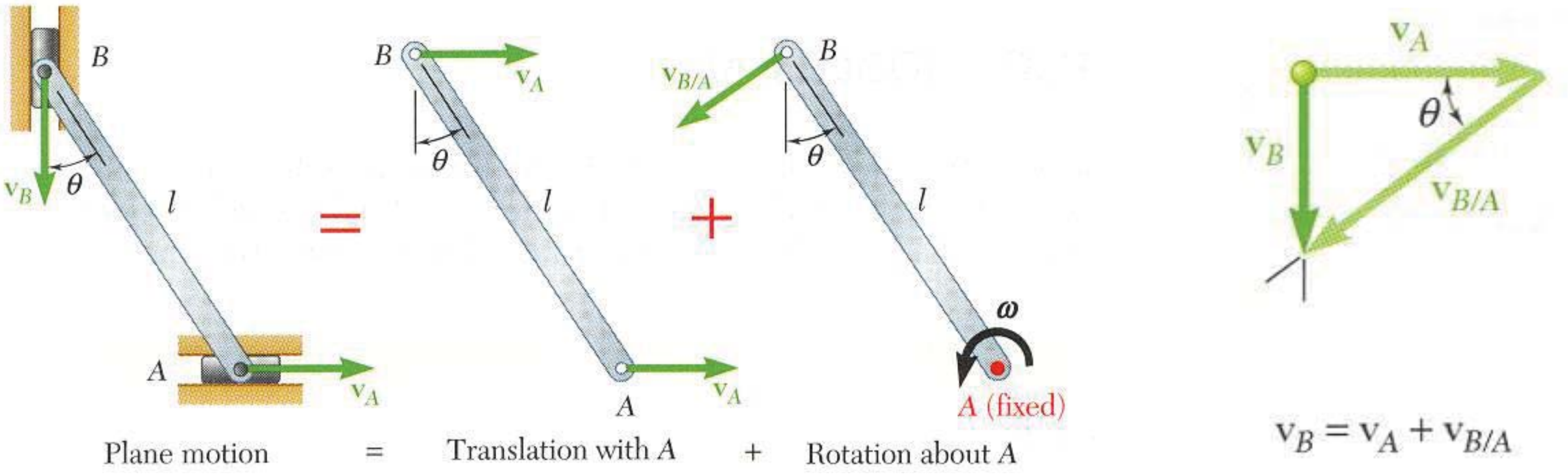
$$v_{B/A} = r\omega$$

$$\Rightarrow \boxed{\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}}$$

Kinematics of Rigid Bodies

❑ Absolute and Relative Velocity in Plane Motion

- Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.



- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

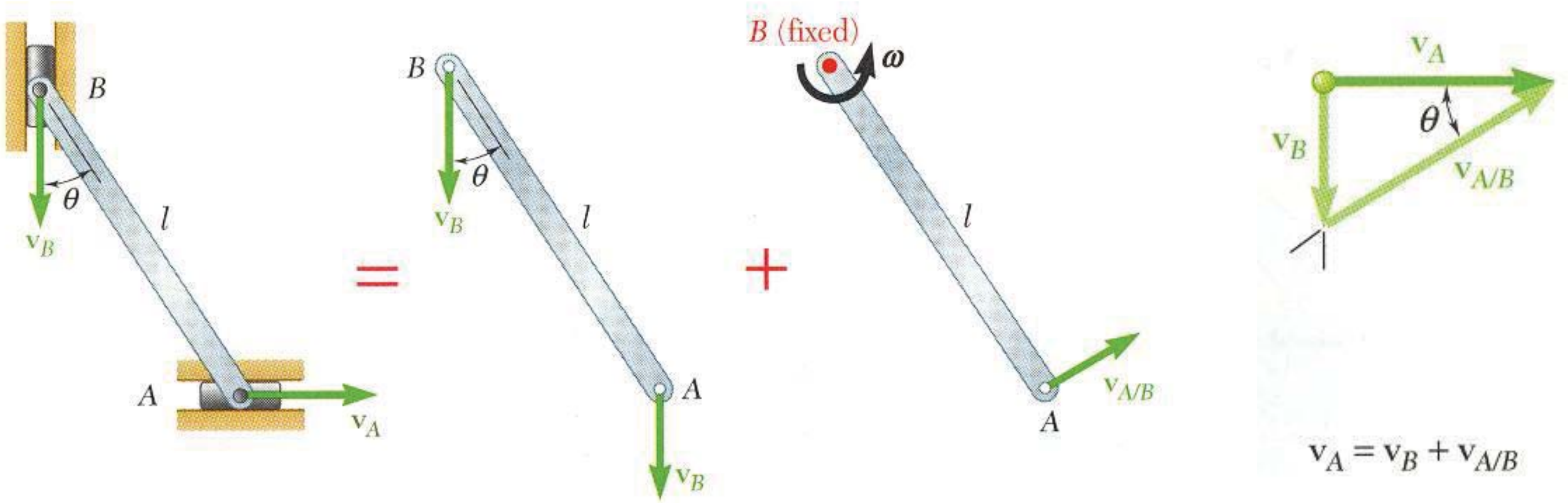
$$\frac{v_B}{v_A} = \tan\theta \quad \Rightarrow \quad \boxed{v_B = v_A \tan\theta}$$

$$\cos\theta = \frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} \quad \Rightarrow \quad \boxed{\omega = \frac{v_A}{l\cos\theta}}$$

Kinematics of Rigid Bodies

❑ Absolute and Relative Velocity in Plane Motion

- Selecting point *B* as the reference point and solving for the velocity v_A of end *A* and the angular velocity ω leads to an equivalent velocity triangle.



Plane motion = Translation with *B* + Rotation about *B*

- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. ***The sense of the relative velocity is dependent on the choice of reference point.***

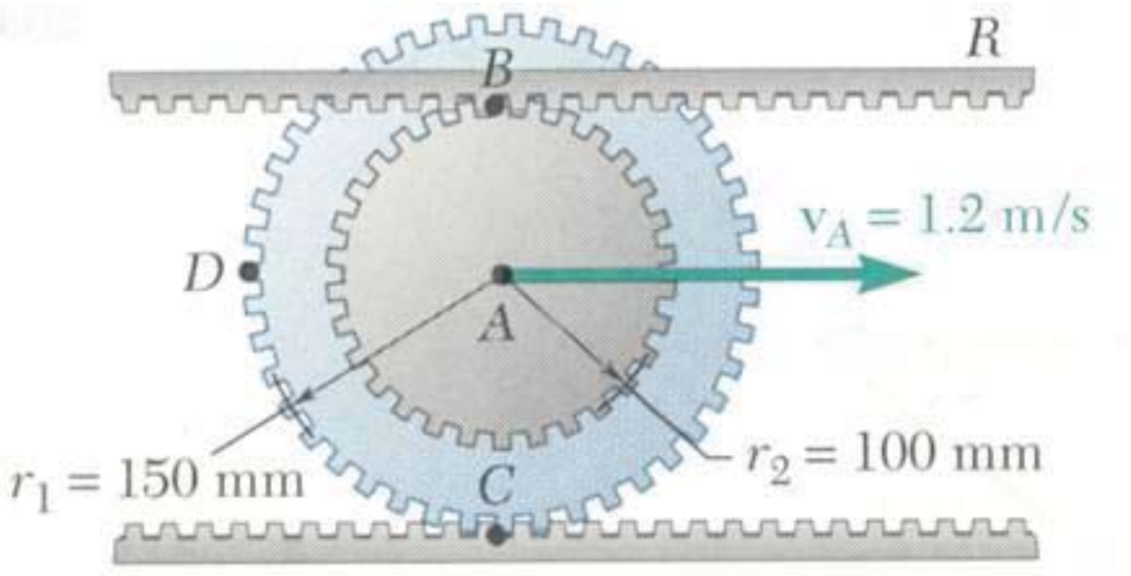
- Angular velocity ω of the rod in its rotation about *B* is the same as its rotation about *A*. ***Angular velocity is not dependent on the choice of reference point.***

Kinematics of Rigid Bodies

❑ Sample Problem 02

The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s.

Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack *R* and point *D* of the gear.

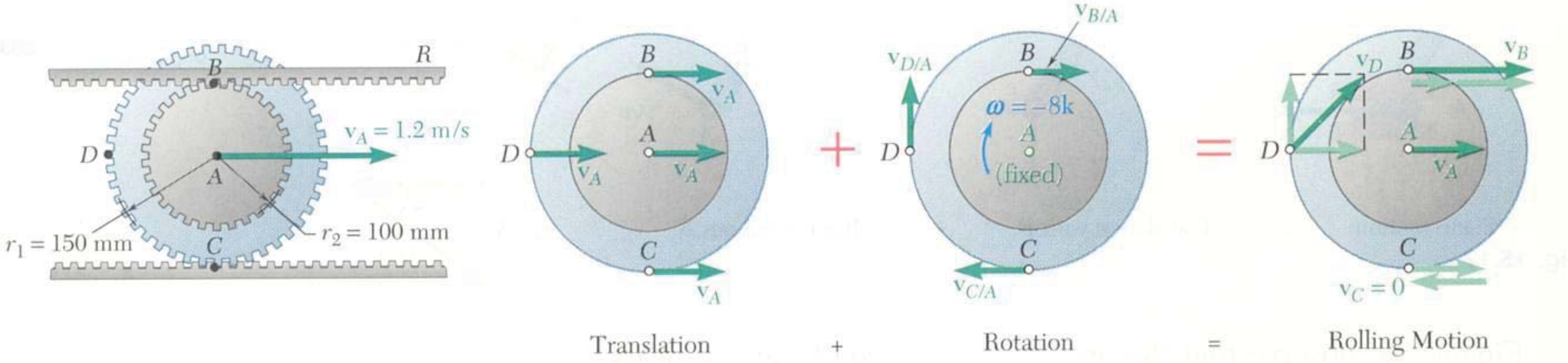


Kinematics of Rigid Bodies

❑ Sample Problem 02

SOLUTION:

- For any point P on the gear, $\boxed{\vec{v}_P = \vec{v}_A + \vec{v}_{P/A} = \vec{v}_A + \omega \vec{k} \times \vec{r}_{P/A}}$

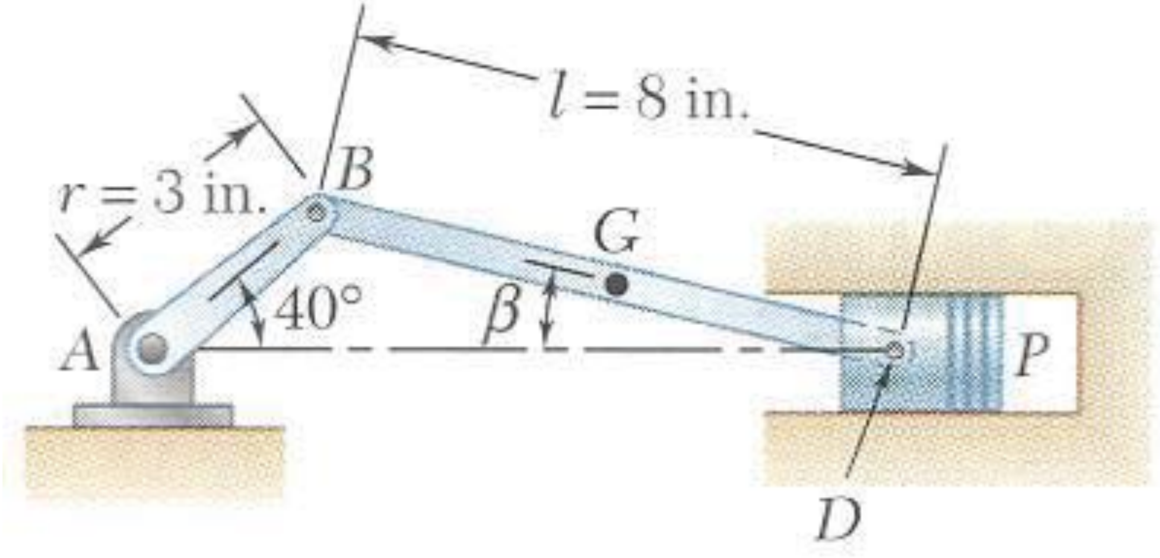


Kinematics of Rigid Bodies

❑ Sample Problem 03

The crank *AB* has a constant clockwise angular velocity of 2000 rpm.

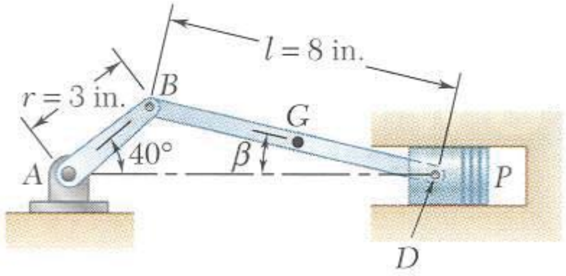
For the crank position indicated, determine *(a)* the angular velocity of the connecting rod *BD*, and *(b)* the velocity of the piston *P*.

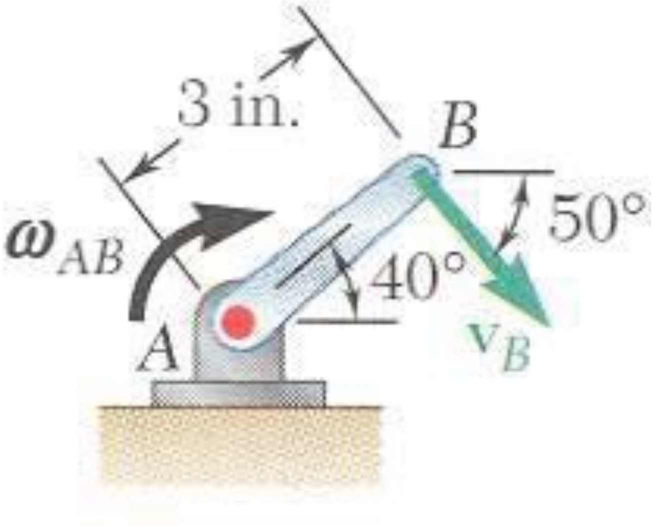


Kinematics of Rigid Bodies

❑ Sample Problem 03

SOLUTION:



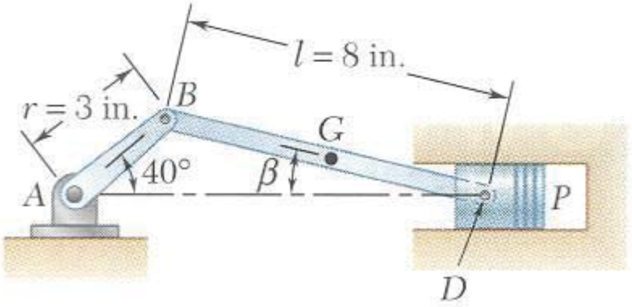


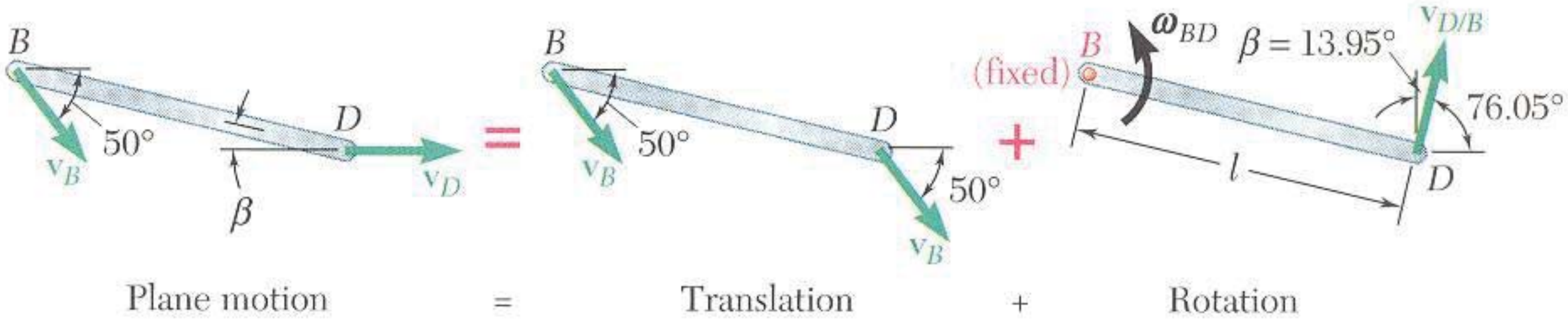
Kinematics of Rigid Bodies

❑ Sample Problem 03

SOLUTION:

- The direction of the absolute velocity $\vec{v}_D$ is ***horizontal***. The direction of the relative velocity $\vec{v}_{D/B}$ is ***perpendicular to BD***. Compute the angle between the horizontal and the connecting rod from the law of sines.

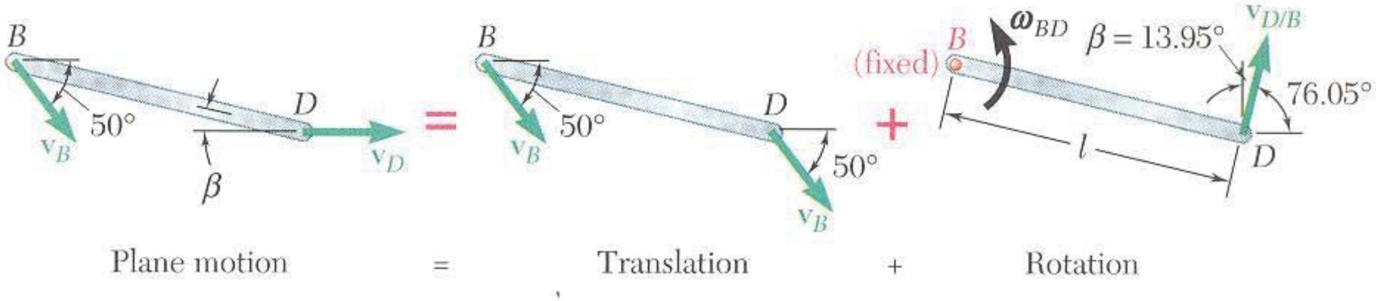




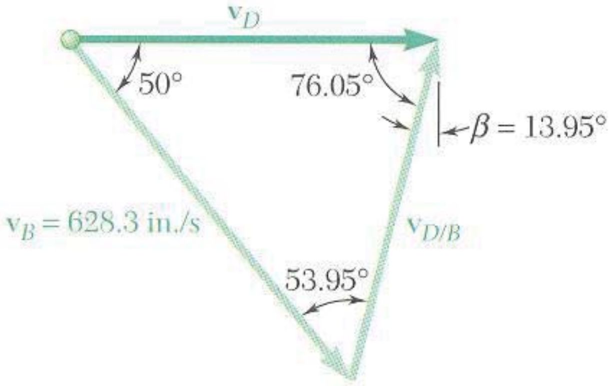
Kinematics of Rigid Bodies

❑ Sample Problem 03

SOLUTION:



- Determine the velocity magnitudes v_D and $v_{D/B}$ from the vector triangle.

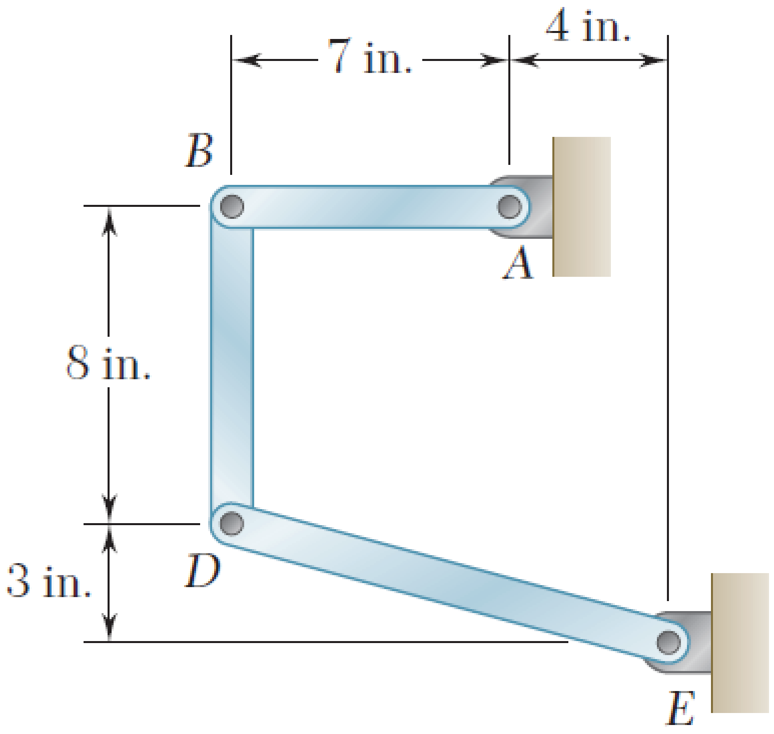


$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

Kinematics of Rigid Bodies

☐ Sample Problem 04

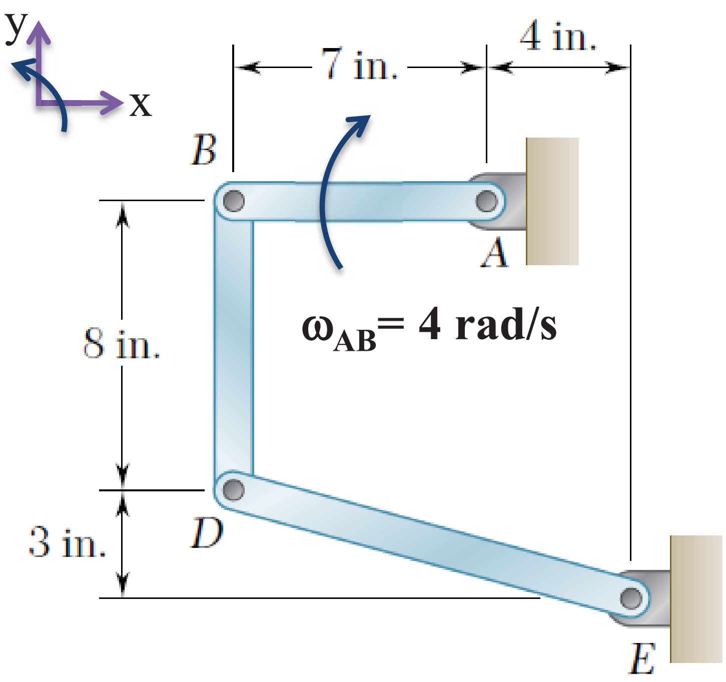
In the position shown, bar *AB* has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars *BD* and *DE*.



Kinematics of Rigid Bodies

❑ Sample Problem 04

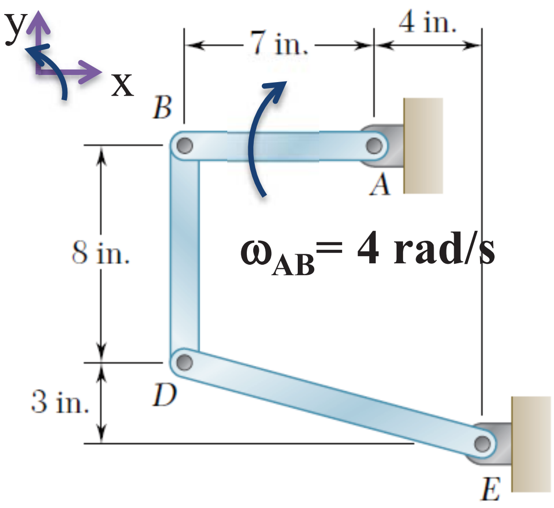
SOLUTION:



Kinematics of Rigid Bodies

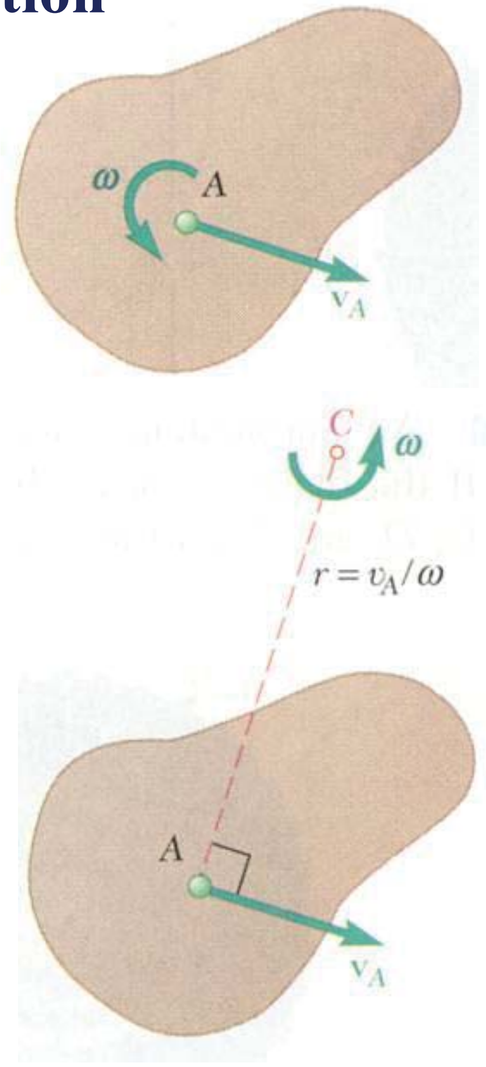
❑ Sample Problem 04

SOLUTION:



Kinematics of Rigid Bodies

❑ Instantaneous Center of Rotation in Plane Motion



- Plane motion of all particles in a slab can always be replaced by the ***translation of an arbitrary point A and a rotation about A*** with an angular velocity that is independent of the choice of *A*.

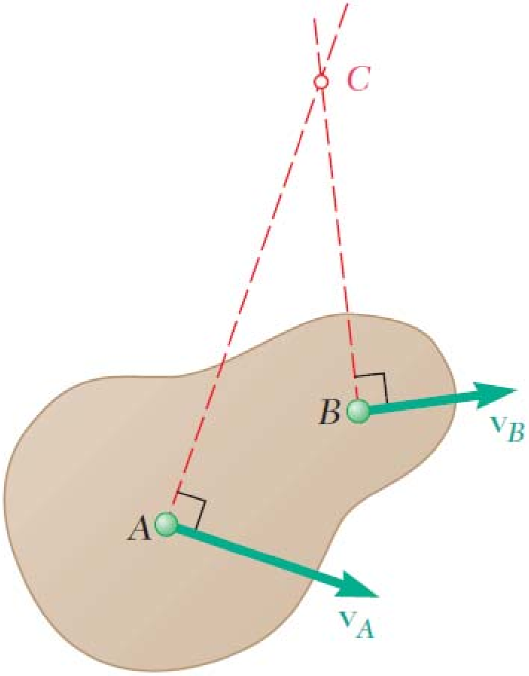
- The same translational and rotational velocities at *A* are obtained by allowing the slab to ***rotate with the same angular velocity about the point C on a perpendicular to the velocity at A.***

- ***The velocity of all other particles in the slab are the same as originally defined*** since the angular velocity and translational velocity at *A* are equivalent.

- As far as the velocities are concerned, the slab seems to rotate about the ***instantaneous center of rotation C.***

Kinematics of Rigid Bodies

☐ Instantaneous Center of Rotation in Plane Motion



- If the velocity at two points A and B are known, the instantaneous center of rotation lies at ***the intersection of the perpendiculars to the velocity vectors through A and B .***

- If the velocity vectors are parallel, ***the instantaneous center of rotation is at infinity and the angular velocity is zero.***

$$\boxed{r = \frac{v_A}{\omega} \to \infty \quad \Rightarrow \quad \omega \to 0}$$

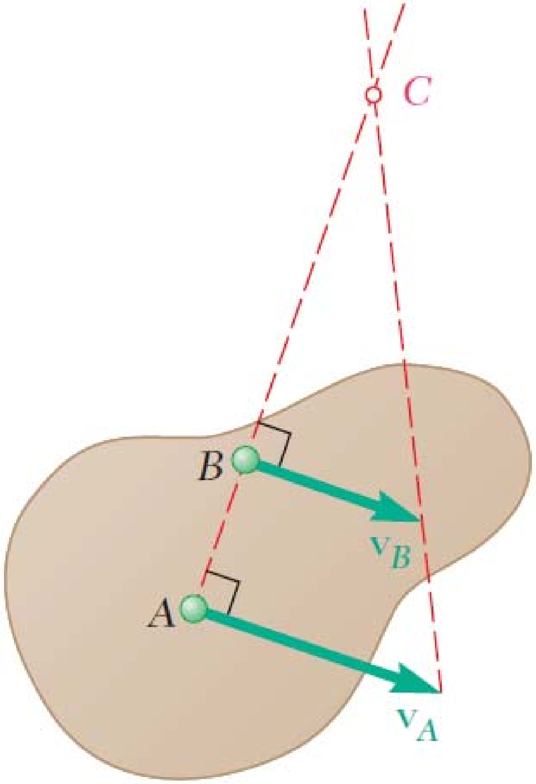
Kinematics of Rigid Bodies

☐ Instantaneous Center of Rotation in Plane Motion

- If the velocity vectors at A and B are perpendicular to the line AB, ***the instantaneous center of rotation lies at the intersection of the line AB with the line joining the extremities of the velocity vectors at A and B.***

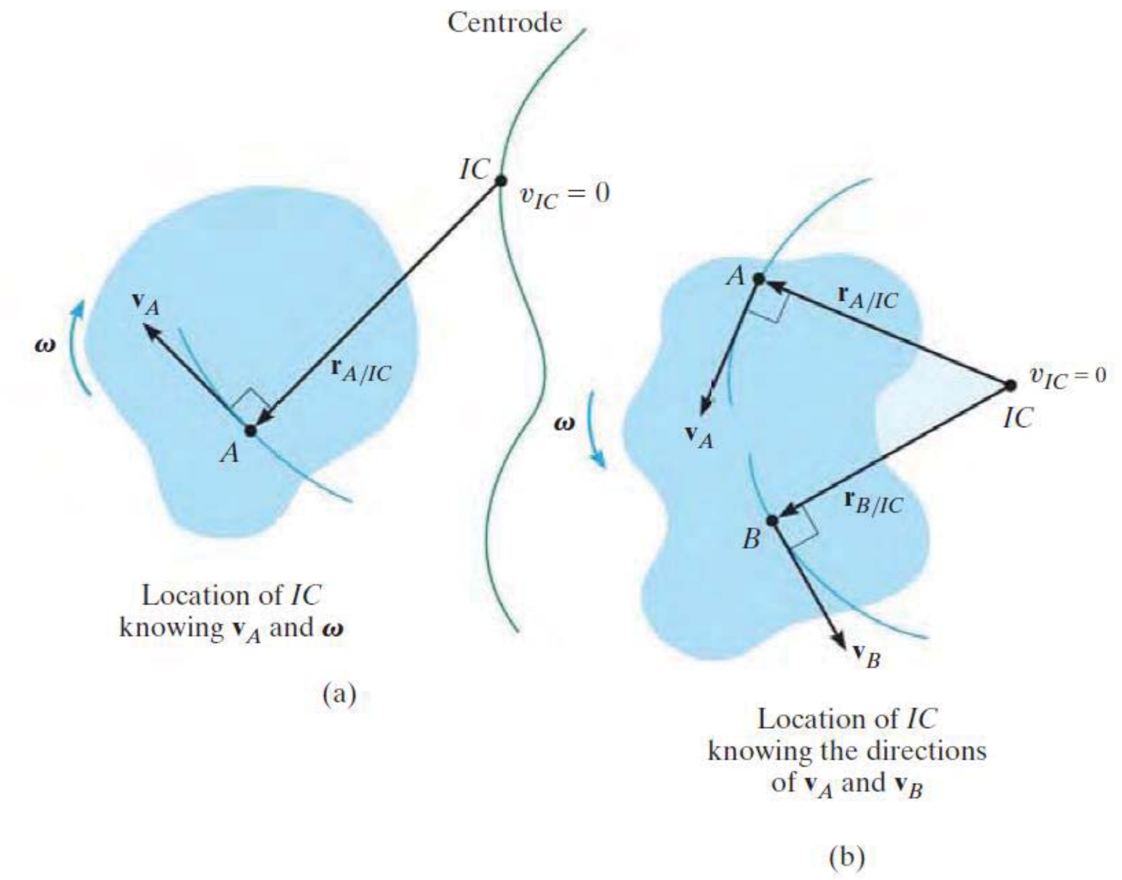
- If the velocity magnitudes are equal, ***the instantaneous center of rotation is at infinity and the angular velocity is zero.***

$$\boxed{r = \frac{v_A}{\omega} \to \infty \quad \Rightarrow \quad \omega \to 0}$$



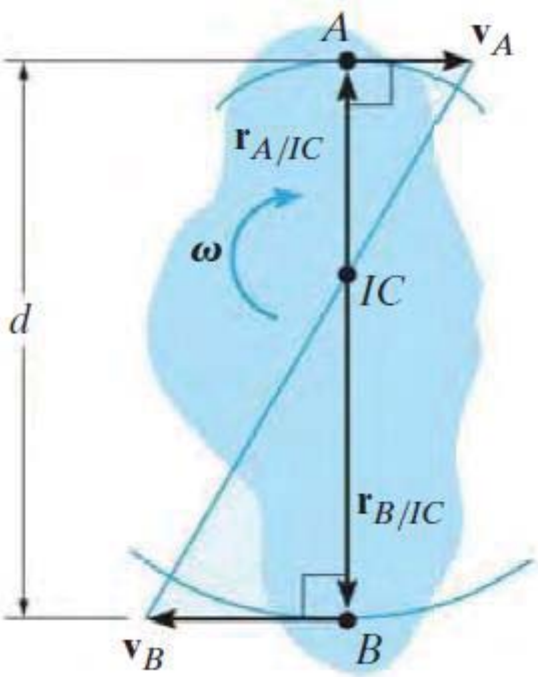
Kinematics of Rigid Bodies

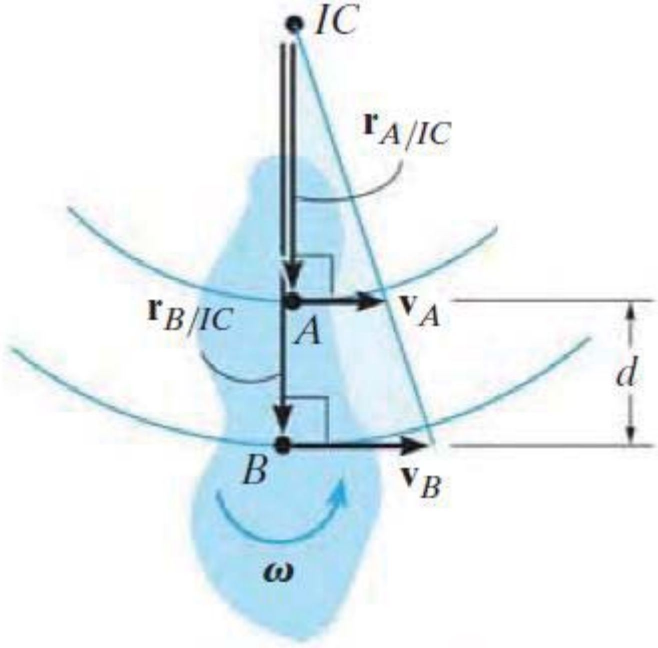
❑ Instantaneous Center of Rotation in Plane Motion



Kinematics of Rigid Bodies

❑ Instantaneous Center of Rotation in Plane Motion



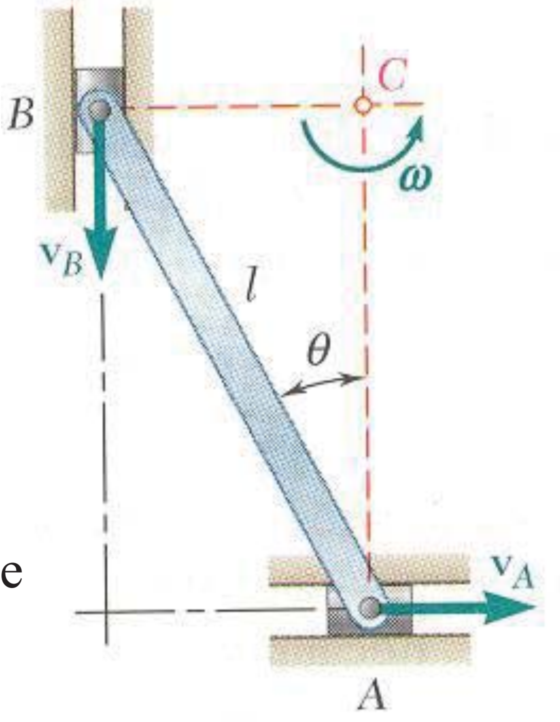


Location of IC
knowing $\mathbf{v}_A$ and $\mathbf{v}_B$

Kinematics of Rigid Bodies

☐ Instantaneous Center of Rotation in Plane Motion

- The instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .



$$\omega = \frac{v_A}{l_{AC}} = \frac{v_A}{l\cos\theta} \quad \Rightarrow \quad \boxed{\omega = \frac{v_A}{l\cos\theta}}$$

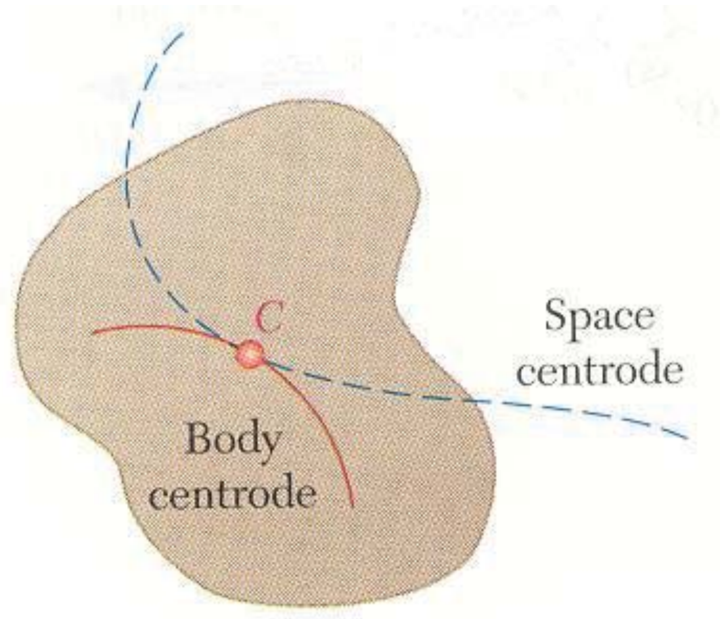
$$v_B = l_{BC}\omega = (l\sin\theta)\frac{v_A}{l\cos\theta} \quad \Rightarrow \quad \boxed{v_B = v_A\tan\theta}$$

- The velocities of all particles on the rod are as if they were rotated about *C*.

Kinematics of Rigid Bodies

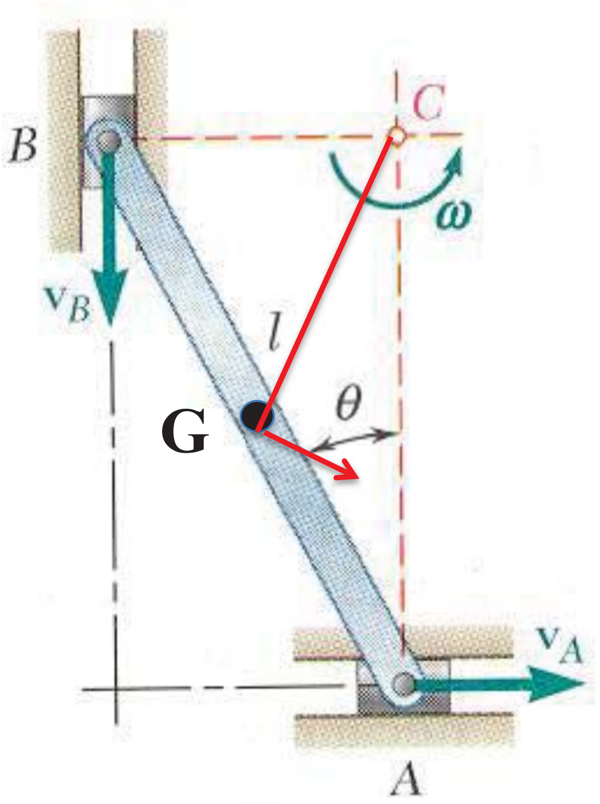
❑ Instantaneous Center of Rotation in Plane Motion

- The particle at the center of rotation has zero velocity.
- The particle coinciding with the center of rotation changes with time and ***the acceleration of the particle at the instantaneous center of rotation is not zero.***
- The acceleration of the particles in the slab cannot be determined as if the slab were simply rotating about *C*.
- The trace of the locus of the center of rotation on the body is the ***body centrode*** and in space is the ***space centrode.***



Kinematics of Rigid Bodies

❑ Group Problem Solving



At the instant shown, what is the approximate direction of the velocity of point G, the center of bar AB?

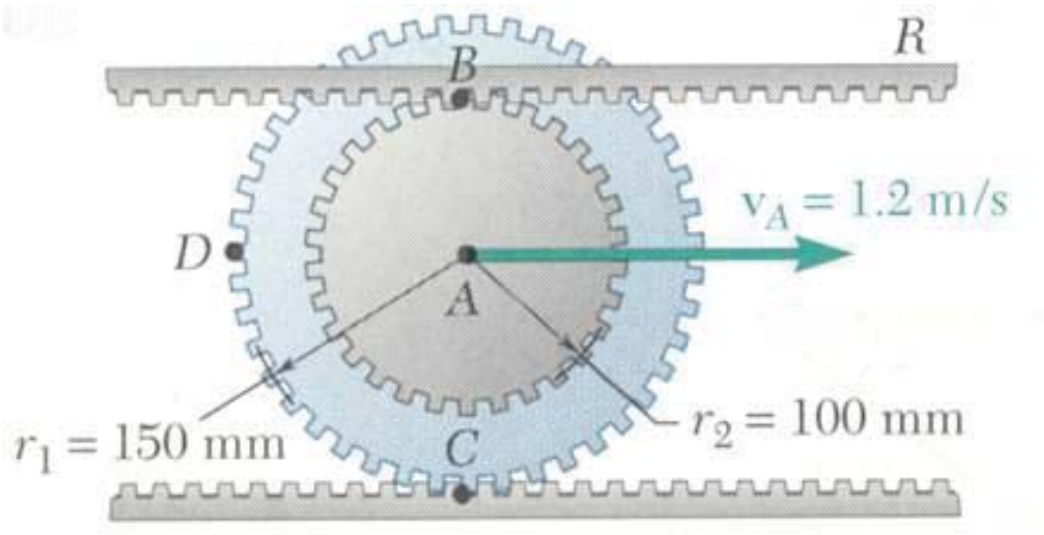
- **a)** →
- **b)** ↗
- [✓ **c)** ↘]
- **d)** ↓

Kinematics of Rigid Bodies

❑ Sample Problem 05

The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s.

Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack *R* and point *D* of the gear.

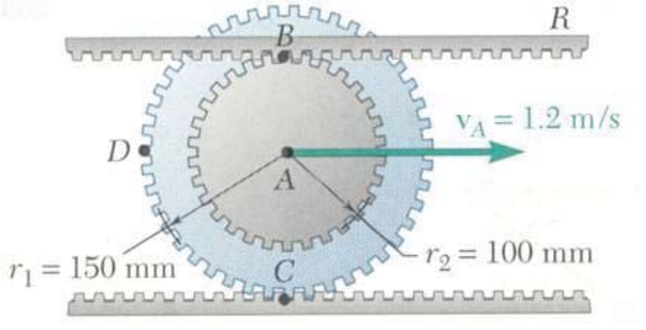


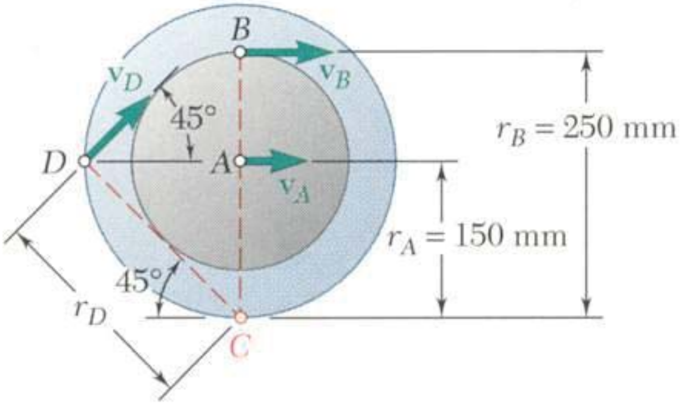
Kinematics of Rigid Bodies

❑ Sample Problem 05

SOLUTION:

- The point *C* is in contact with the stationary lower rack and, instantaneously, has zero velocity. ***It must be the location of the instantaneous center of rotation.***



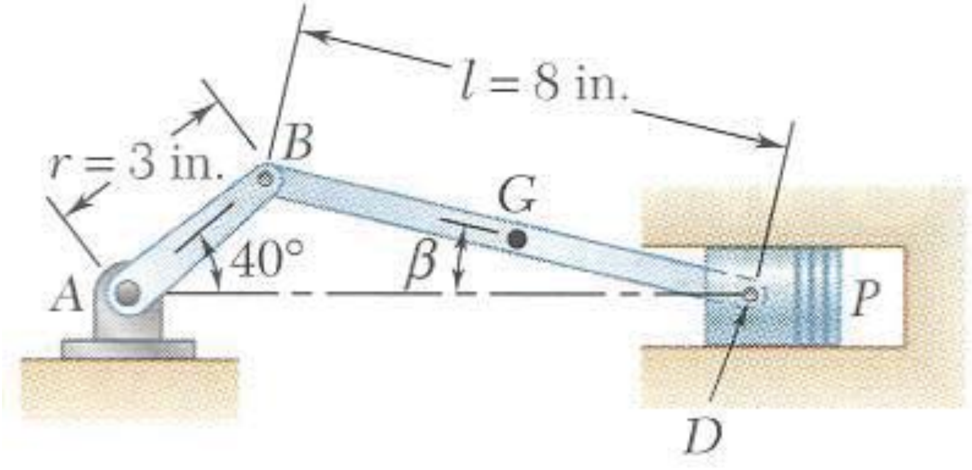


Kinematics of Rigid Bodies

❑ Sample Problem 06

The crank *AB* has a constant clockwise angular velocity of 2000 rpm.

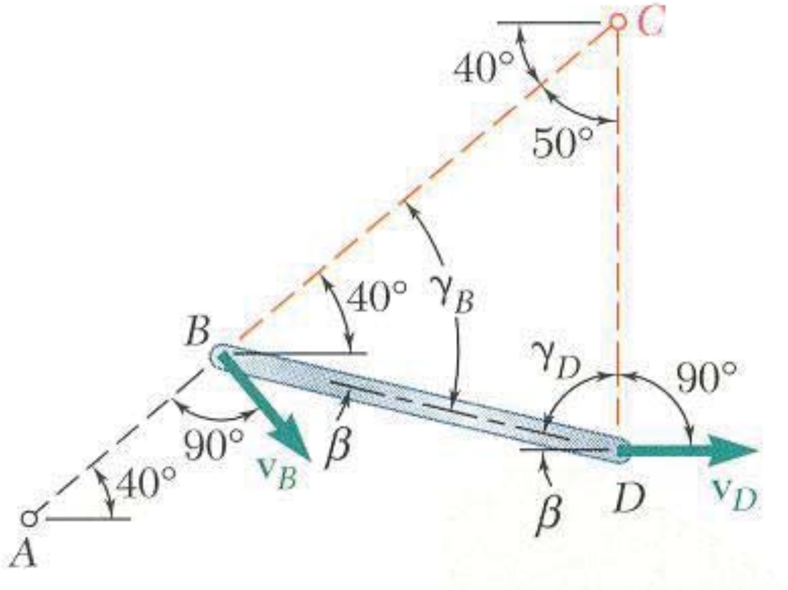
For the crank position indicated, determine *(a)* the angular velocity of the connecting rod *BD*, and *(b)* the velocity of the piston *P*.



Kinematics of Rigid Bodies

❑ Sample Problem 06

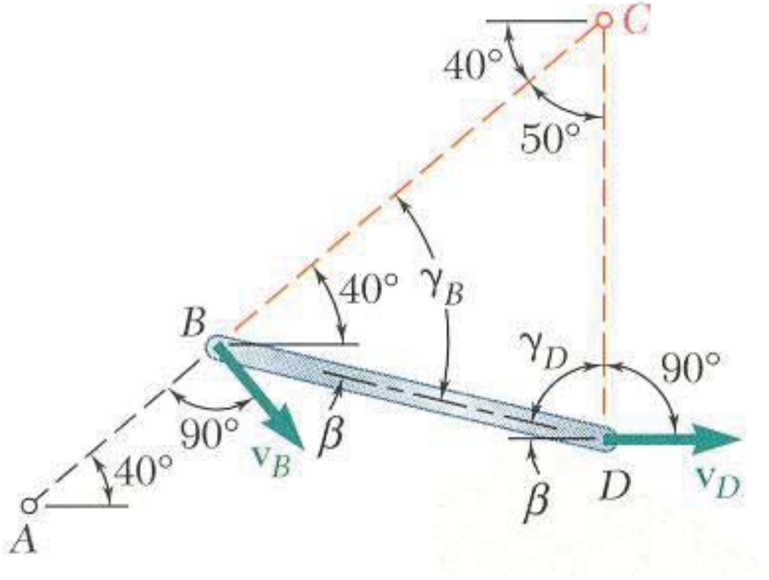
SOLUTION:



Kinematics of Rigid Bodies

❑ Sample Problem 06

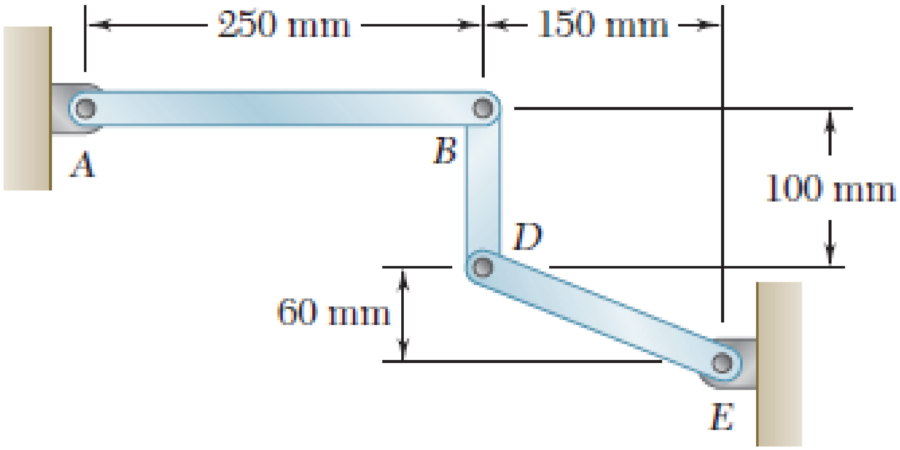
SOLUTION:



Kinematics of Rigid Bodies

☐ Sample Problem 07

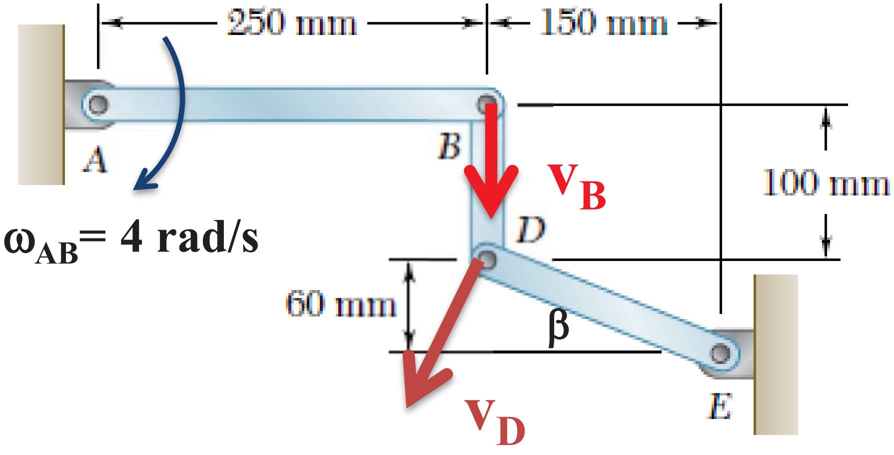
In the position shown, bar AB has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars BD and DE.

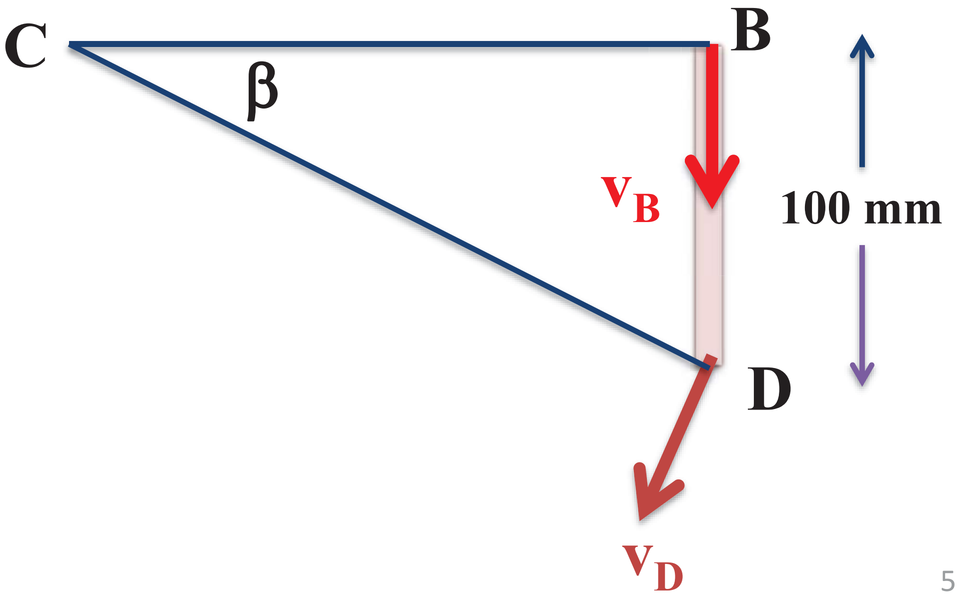


Kinematics of Rigid Bodies

❑ Sample Problem 07

SOLUTION:





Kinematics of Rigid Bodies

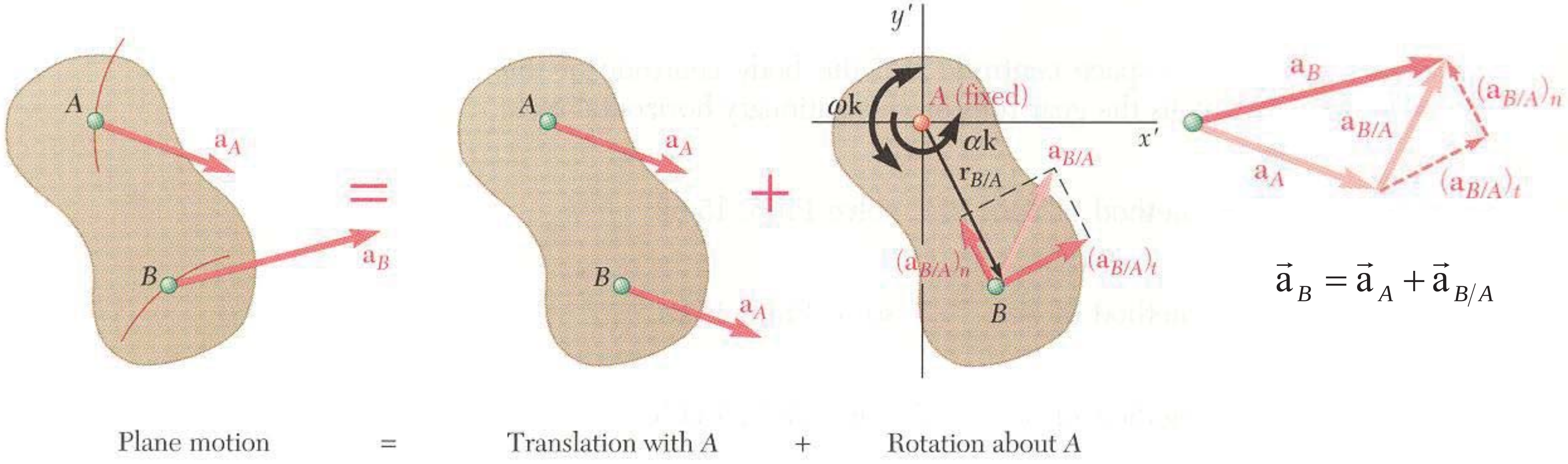
❑ Absolute and Relative Acceleration in Plane Motion

As the bicycle accelerates, a point on the top of the wheel will have acceleration due to the acceleration from the axle (the overall linear acceleration of the bike), the tangential acceleration of the wheel from the angular acceleration, and the normal acceleration due to the angular velocity.

Kinematics of Rigid Bodies

❑ Absolute and Relative Acceleration in Plane Motion

- Absolute acceleration of a particle of the slab,



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

- Relative acceleration $\vec{a}_{B/A}$ associated with rotation about *A* includes tangential and normal components,

$$(\vec{a}_{B/A})_t = \alpha\vec{k} \times \vec{r}_{B/A} \qquad (a_{B/A})_t = r\alpha$$

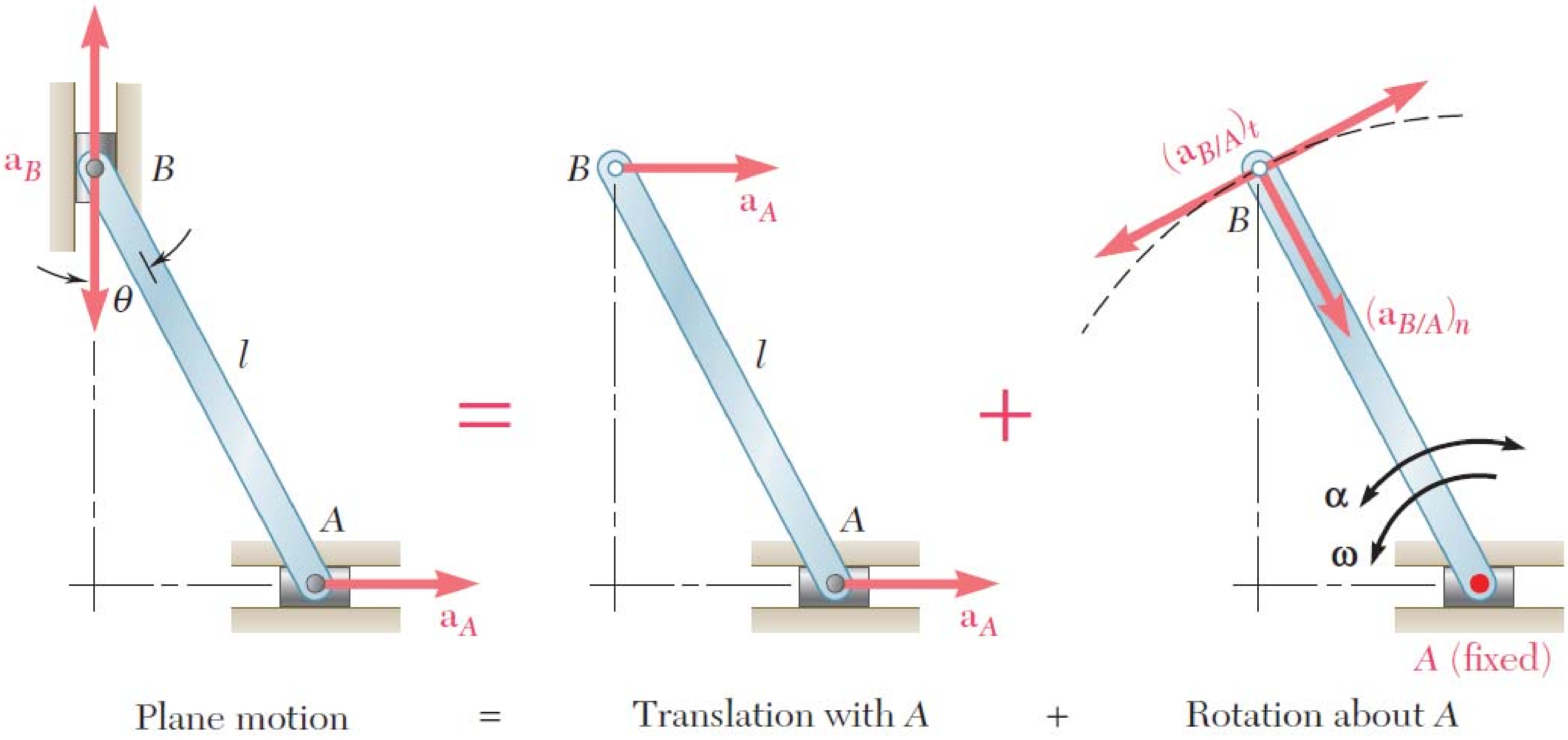
$$(\vec{a}_{B/A})_n = -\omega^2\vec{r}_{B/A} \qquad (a_{B/A})_n = r\omega^2$$

Kinematics of Rigid Bodies

☐ Absolute and Relative Acceleration in Plane Motion

- Given $\vec{a}_A$ and $\vec{v}_A$, determine $\vec{a}_B$ and $\vec{\alpha}$.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \Rightarrow \boxed{\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_n + (\vec{a}_{B/A})_t}$$

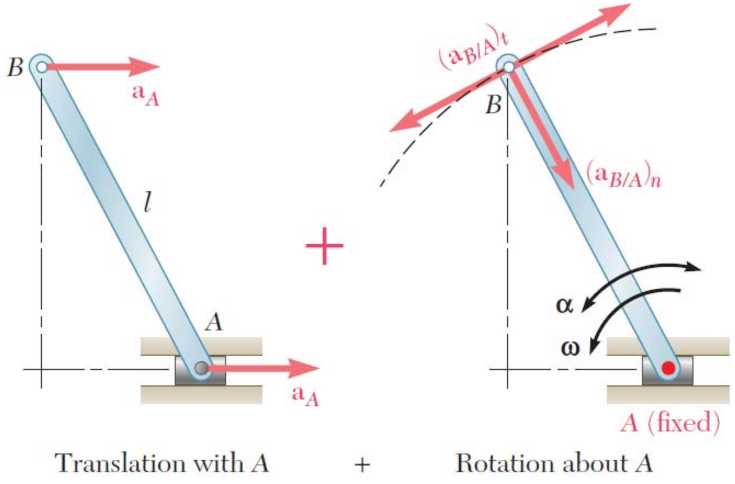


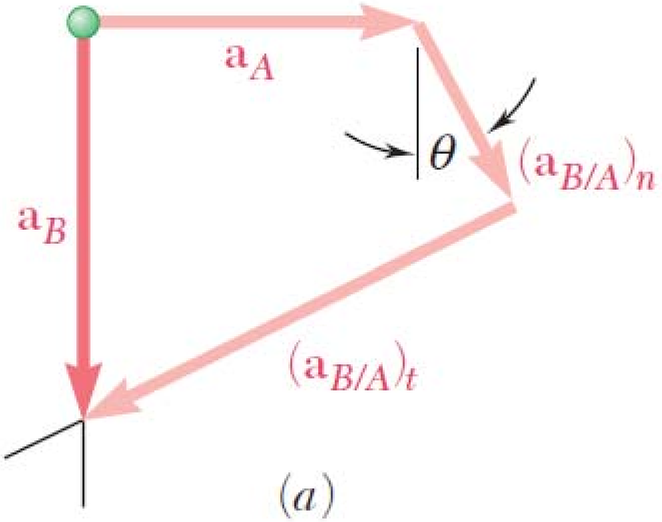
Kinematics of Rigid Bodies

❑ Absolute and Relative Acceleration in Plane Motion

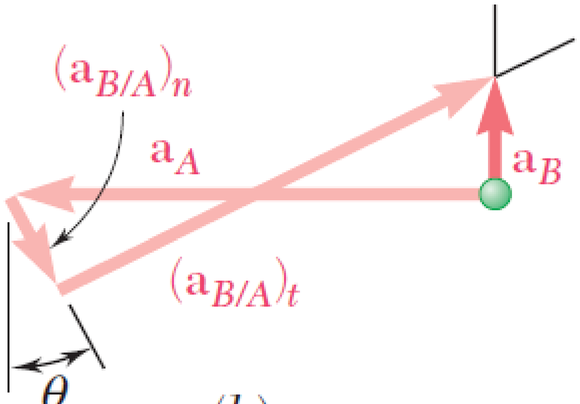
- Vector result depends on sense of $\vec{a}_A$ and the relative magnitudes of a_A and $\left(a_{B/A}\right)_n$

- Must also know angular velocity ω.





(a)

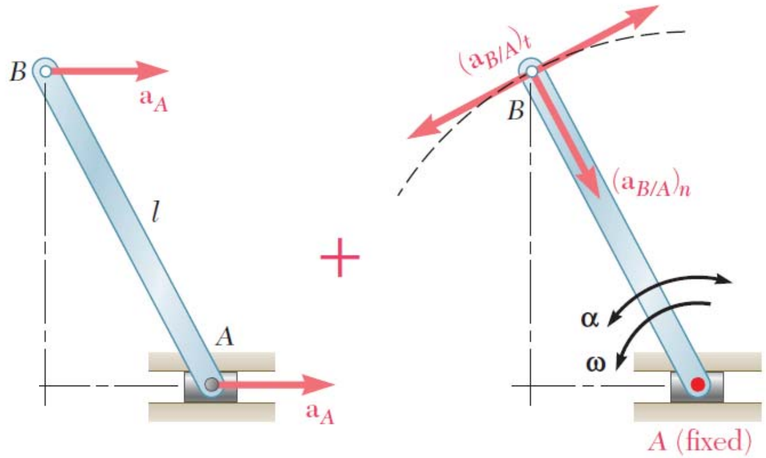


(b)

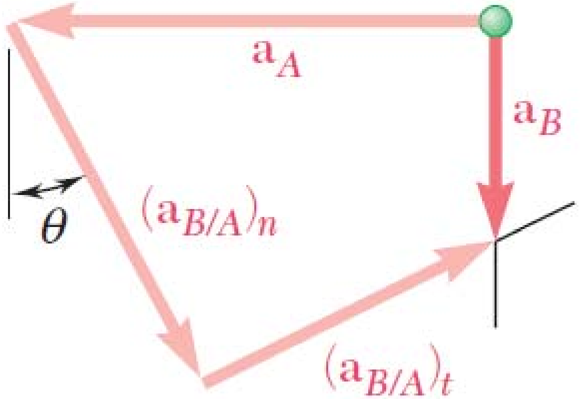
Kinematics of Rigid Bodies

❑ Absolute and Relative Acceleration in Plane Motion

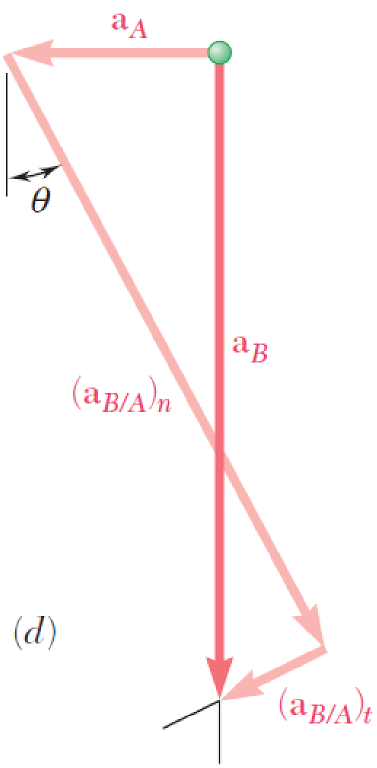
- Vector result depends on sense of $\vec{a}_A$ and the relative magnitudes of a_A and $(a_{B/A})_n$
- Must also know angular velocity ω.



Translation with A + Rotation about A



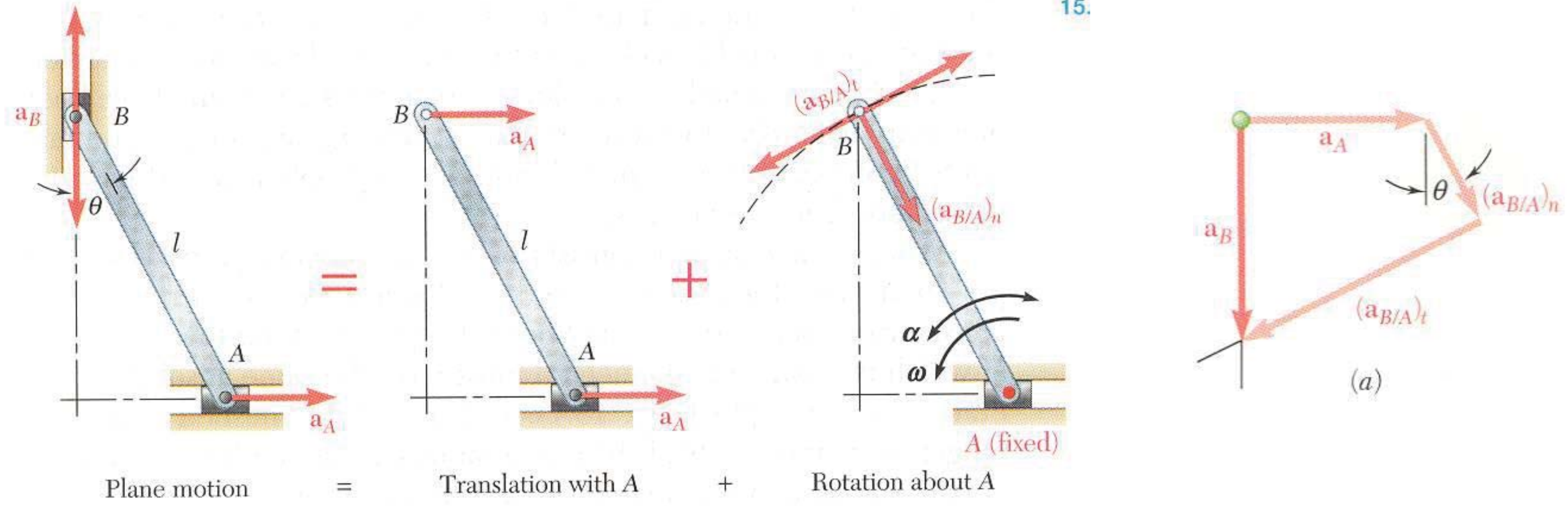
(c)



(d)

Kinematics of Rigid Bodies

❑ Absolute and Relative Acceleration in Plane Motion



- Write $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ in terms of the two component equations,

$\xrightarrow{+}$ x components: $\boxed{0 = a_A + l\omega^2 \sin\theta - l\alpha\cos\theta}$

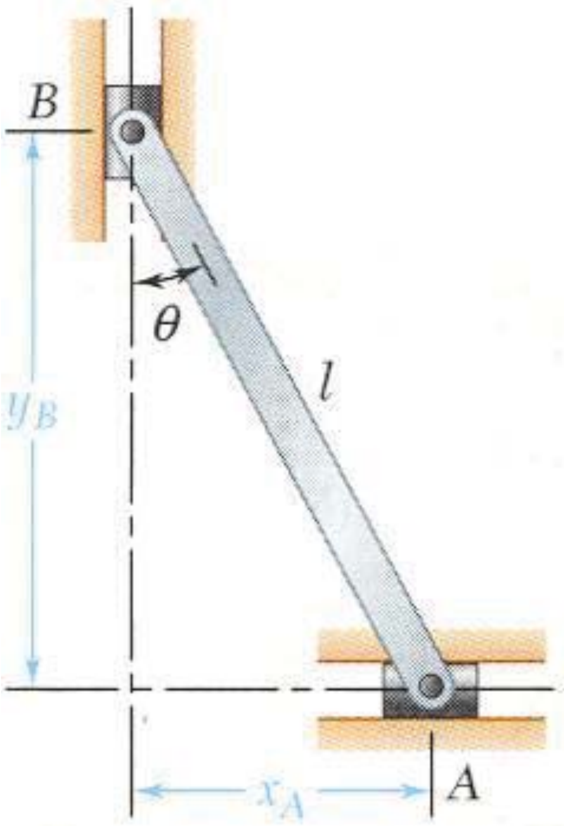
$+\uparrow$ y components: $\boxed{-a_B = -l\omega^2 \cos\theta - l\alpha\sin\theta}$

- Solve for a_B and α.

Kinematics of Rigid Bodies

❑ Analysis of Plane Motion in Terms of a Parameter

- In some cases, it is advantageous to determine the absolute velocity and acceleration of a mechanism directly.



$$\boxed{x_A = l\sin\theta}$$

$$v_A = \dot{x}_A = l\dot{\theta}\cos\theta \quad \Rightarrow \quad \boxed{v_A = l\omega\cos\theta}$$

$$a_A = \ddot{x}_A = -l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta \quad \Rightarrow \quad \boxed{a_A = -l\omega^2\sin\theta + l\alpha\cos\theta}$$

$$\boxed{y_B = l\cos\theta}$$

$$v_B = \dot{y}_B = -l\dot{\theta}\sin\theta \quad \Rightarrow \quad \boxed{v_B = -l\omega\sin\theta}$$

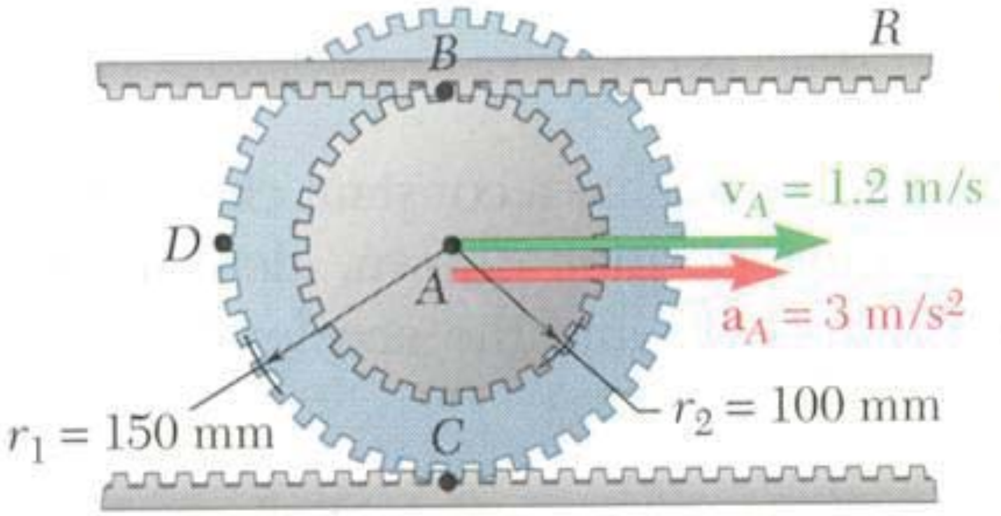
$$a_B = \ddot{y}_B = -l\dot{\theta}^2\cos\theta - l\ddot{\theta}\sin\theta \quad \Rightarrow \quad \boxed{a_B = -l\omega^2\cos\theta - l\alpha\sin\theta}$$

Kinematics of Rigid Bodies

❑ Sample Problem 08

The center of the double gear has a velocity and acceleration to the right of 1.2 m/s and 3 m/s^2, respectively. The lower rack is stationary.

Determine *(a)* the angular acceleration of the gear, and *(b)* the acceleration of points *B*, *C*, and *D*.

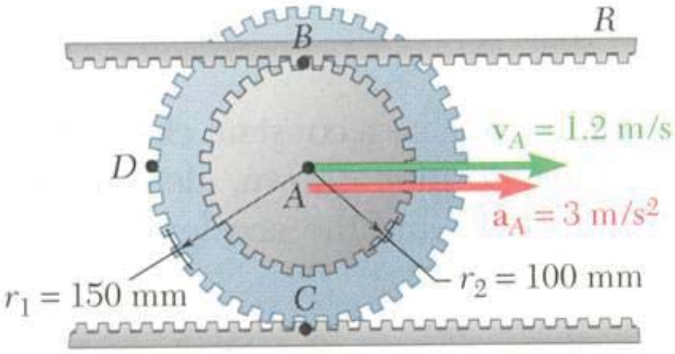


Kinematics of Rigid Bodies

❑ Sample Problem 08

SOLUTION:

- The expression of the gear position as a function of θ is differentiated twice to define the relationship between the translational and angular accelerations.

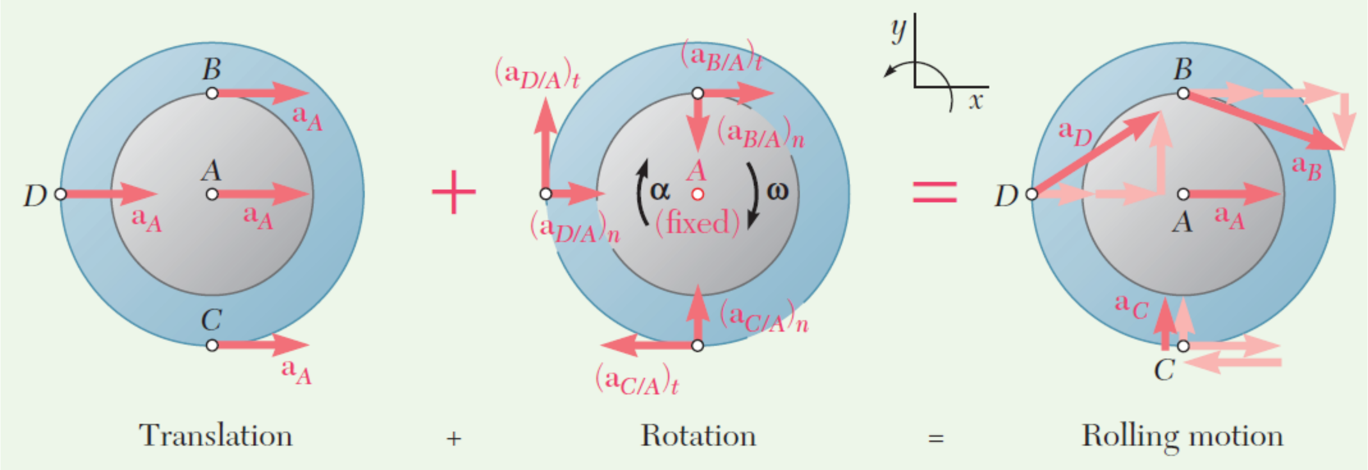


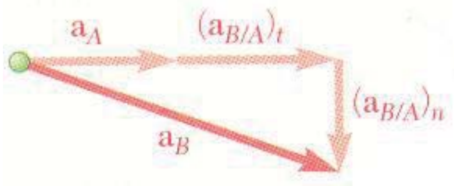
Kinematics of Rigid Bodies

❑ Sample Problem 08

SOLUTION:

- The acceleration of each point is obtained by adding the acceleration of the gear center and the relative accelerations with respect to the center. The latter includes normal and tangential acceleration components.

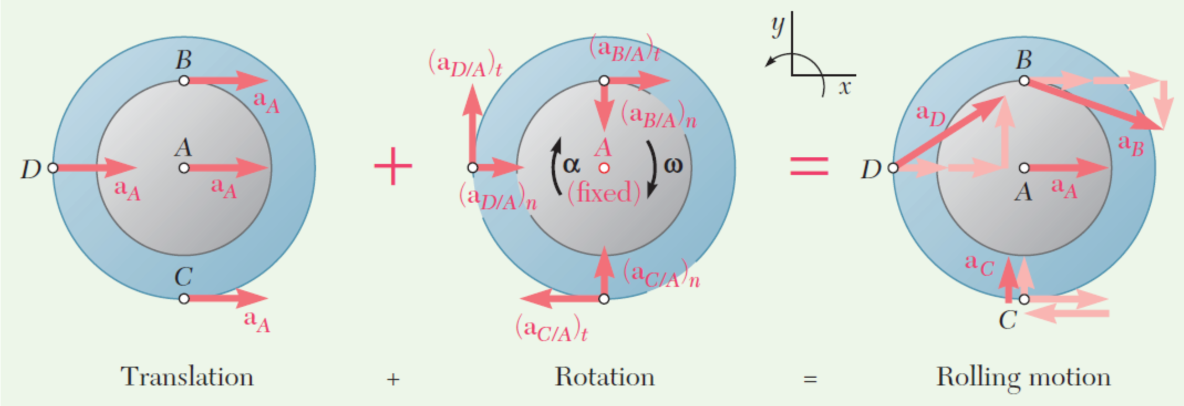


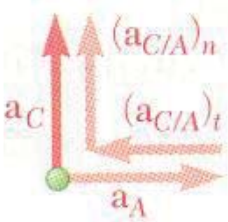


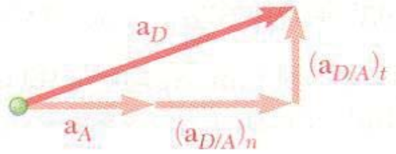
Kinematics of Rigid Bodies

❑ Sample Problem 08

SOLUTION:





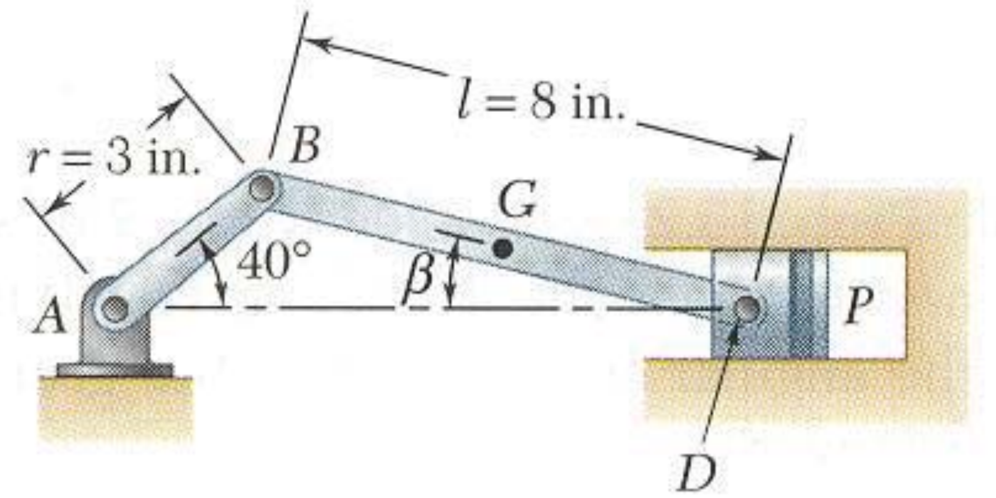


Kinematics of Rigid Bodies

❑ Sample Problem 09

Crank *AB* of the engine system has a constant clockwise angular velocity of 2000 rpm.

For the crank position shown, determine the angular acceleration of the connecting rod *BD* and the acceleration of point *D*.

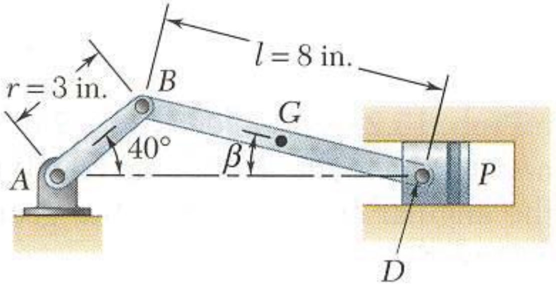


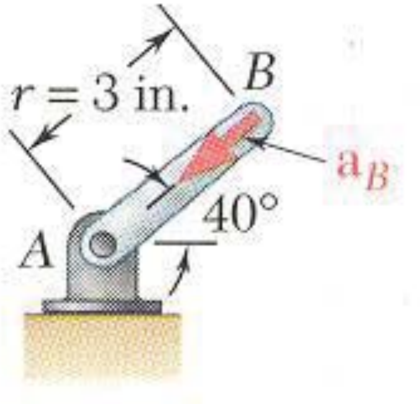
Kinematics of Rigid Bodies

❑ Sample Problem 09

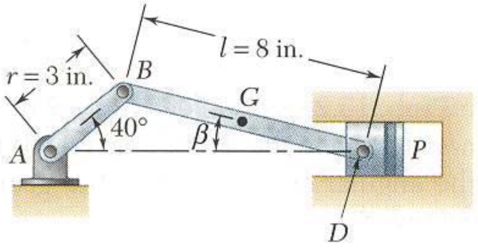
SOLUTION:

- The angular acceleration of the connecting rod *BD* and the acceleration of point *D* will be determined from





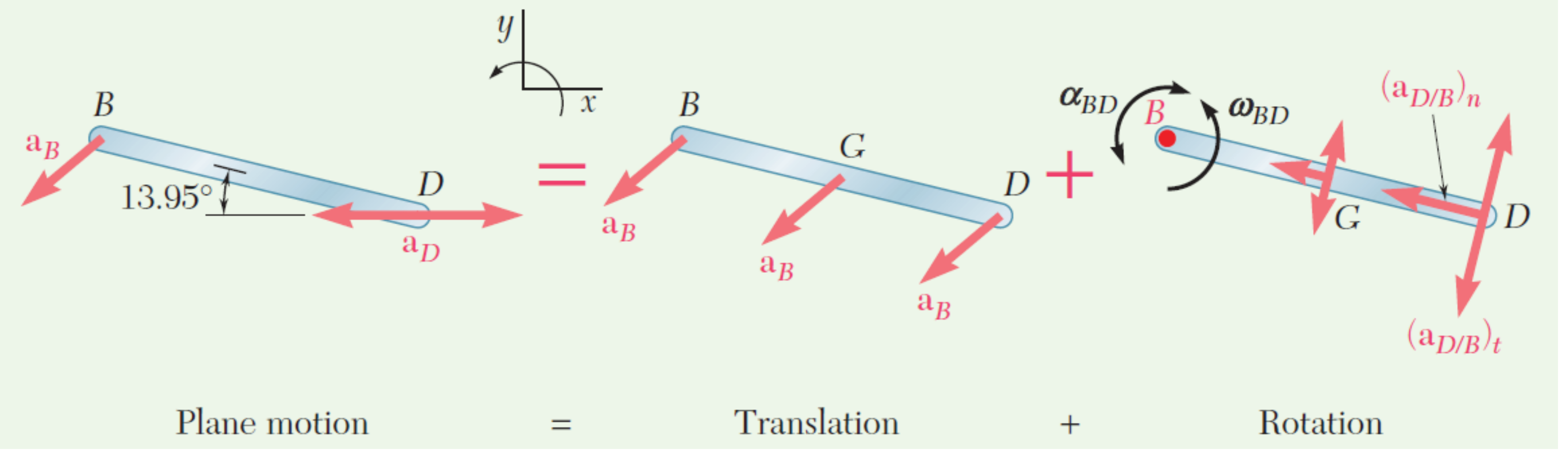
Kinematics of Rigid Bodies



❑ Sample Problem 09

SOLUTION:

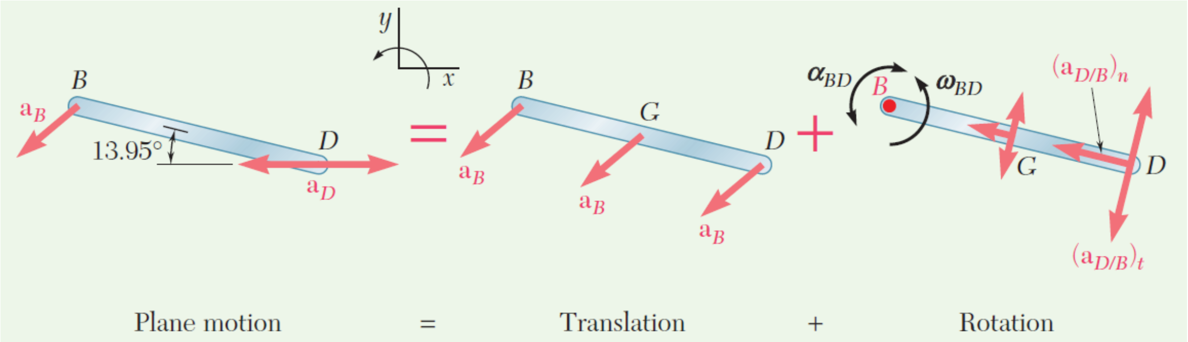
- The directions of the accelerations $\vec{a}_D, (\vec{a}_{D/B})_t$, and $(\vec{a}_{D/B})_n$ are determined from the geometry.

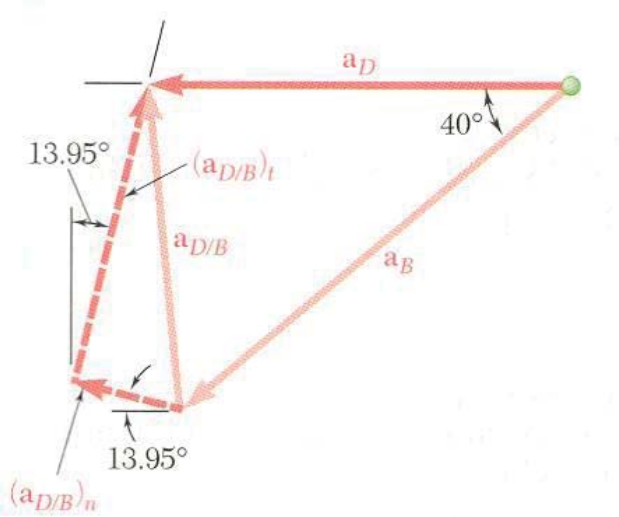


Kinematics of Rigid Bodies

❑ Sample Problem 09

SOLUTION:



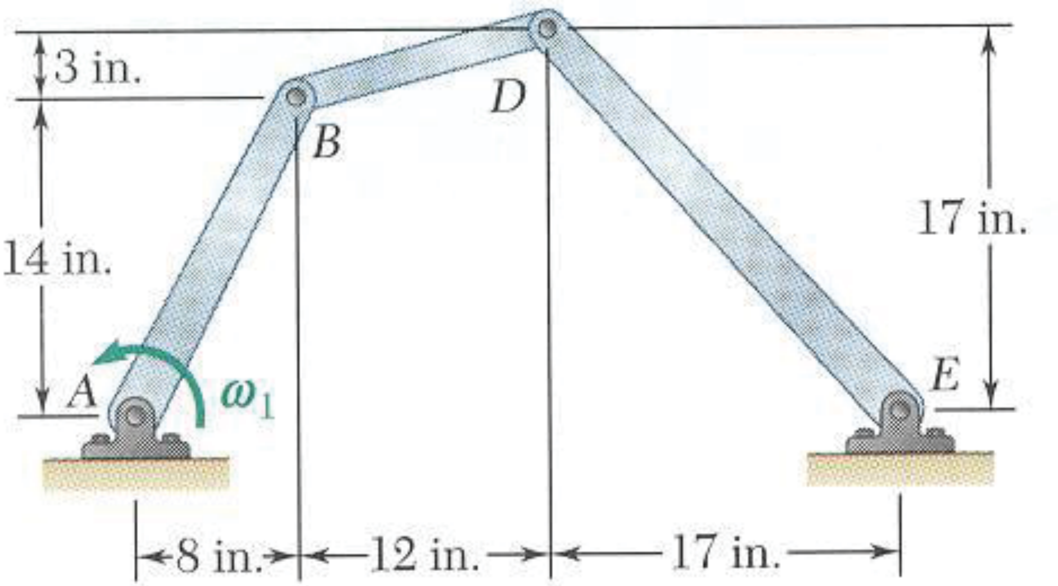


Kinematics of Rigid Bodies

❑ Sample Problem 10

In the position shown, crank *AB* has a constant angular velocity $\omega_1 = 20$ rad/s counterclockwise.

Determine the angular velocities and angular accelerations of the connecting rod *BD* and crank *DE*.

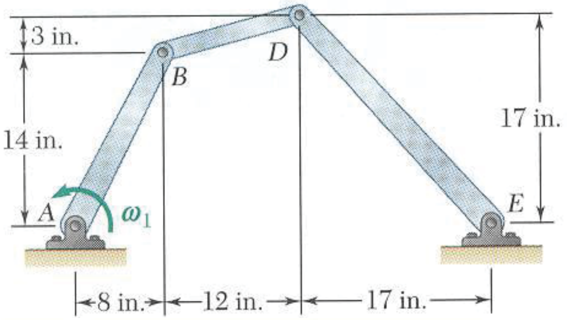


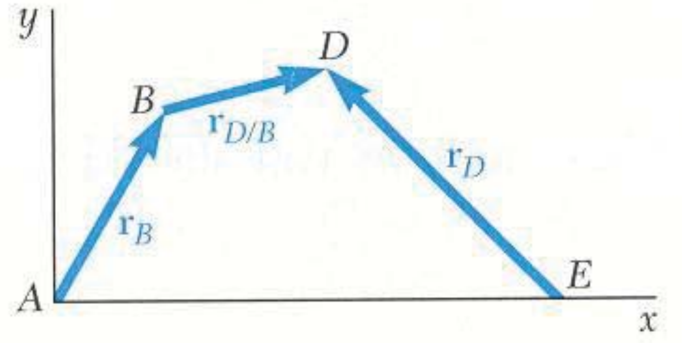
Kinematics of Rigid Bodies

❑ Sample Problem 10

SOLUTION:

- The angular velocities are determined by simultaneously solving the component equations for $\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$



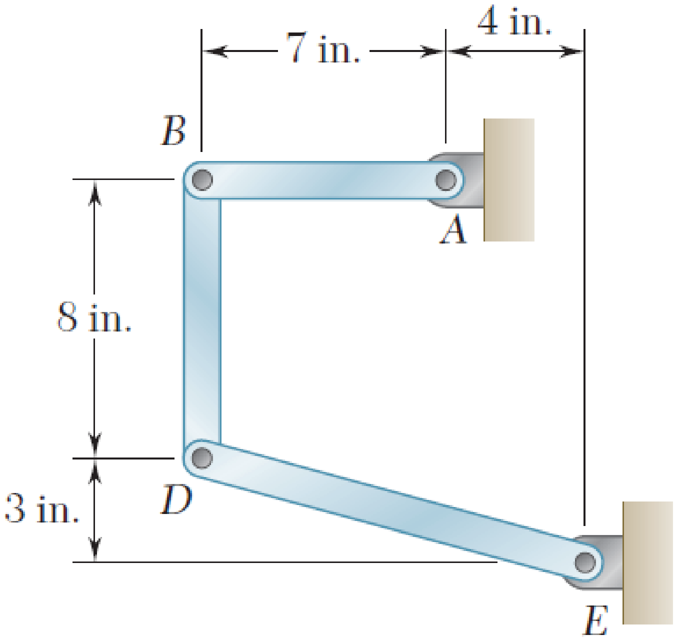


$$\mathbf{r}_B = 8\mathbf{i} + 14\mathbf{j}$$
$$\mathbf{r}_D = -17\mathbf{i} + 17\mathbf{j}$$
$$\mathbf{r}_{D/B} = 12\mathbf{i} + 3\mathbf{j}$$

Kinematics of Rigid Bodies

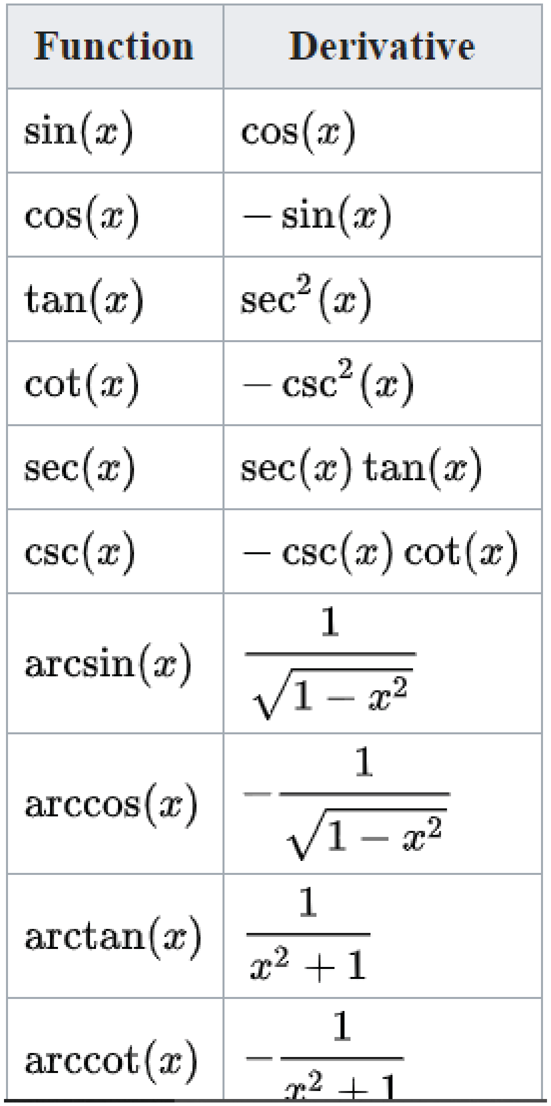
❑ Sample Problem 11

Knowing that at the instant shown bar *AB* has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration of bars *BD* and *DE*.



Kinematics of Rigid Bodies

❑ Sample Problem 12



Function	Derivative
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\cot(x)$	$-\csc^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos(x)$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan(x)$	$\frac{1}{x^2+1}$
$\operatorname{arccot}(x)$	$-\frac{1}{x^2+1}$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = 1+\tan^2(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = \left(\frac{\cos(x)}{\sin(x)}\right)' = \frac{-\sin^2(x)-\cos^2(x)}{\sin^2(x)} = -(1+\cot^2(x)) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \left(\frac{1}{\cos(x)}\right)' = \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)} = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = \left(\frac{1}{\sin(x)}\right)' = -\frac{\cos(x)}{\sin^2(x)} = -\frac{\cos(x)}{\sin(x)}\cdot\frac{1}{\sin(x)} = -\cot(x)\csc(x)$$