Chapter 3: Equilibrium of a Particle

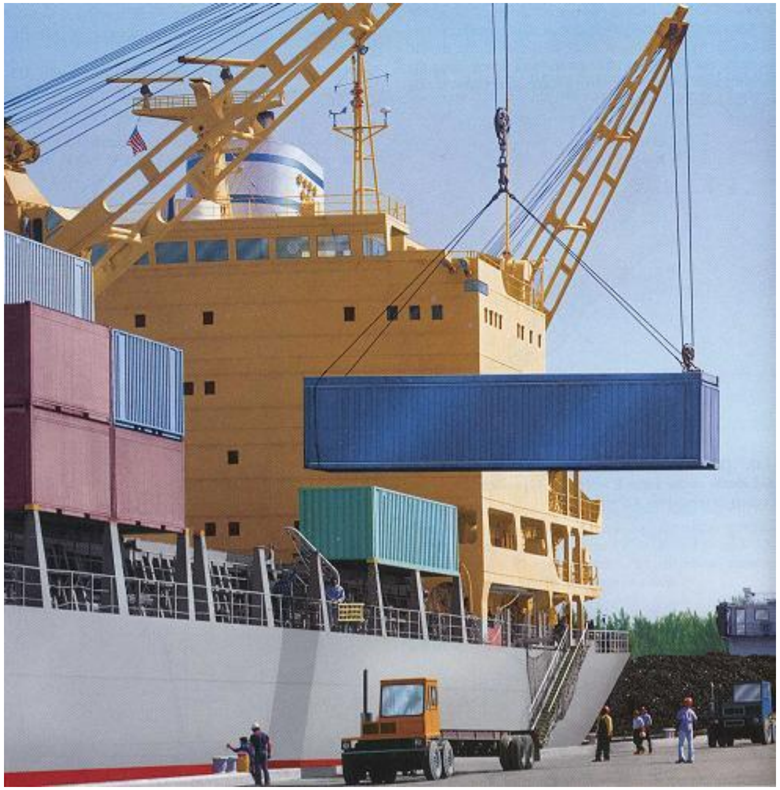
EQUILIBRIUM OF A PARTICLE, THE FREE-BODY DIAGRAM & COPLANAR FORCE SYSTEMS

Section I Objectives:

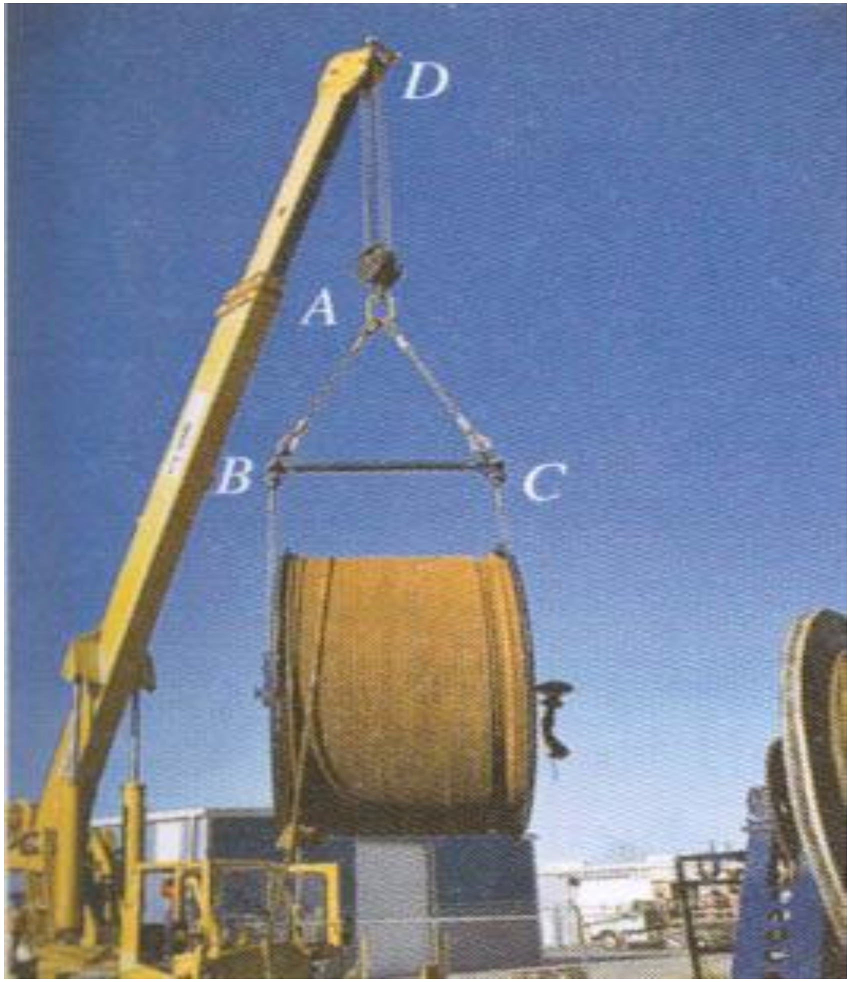
Students will be able to :

a) Draw a free body diagram (FBD), and,

b) Apply equations of equilibrium to solve a 2-D problem.



APPLICATIONS

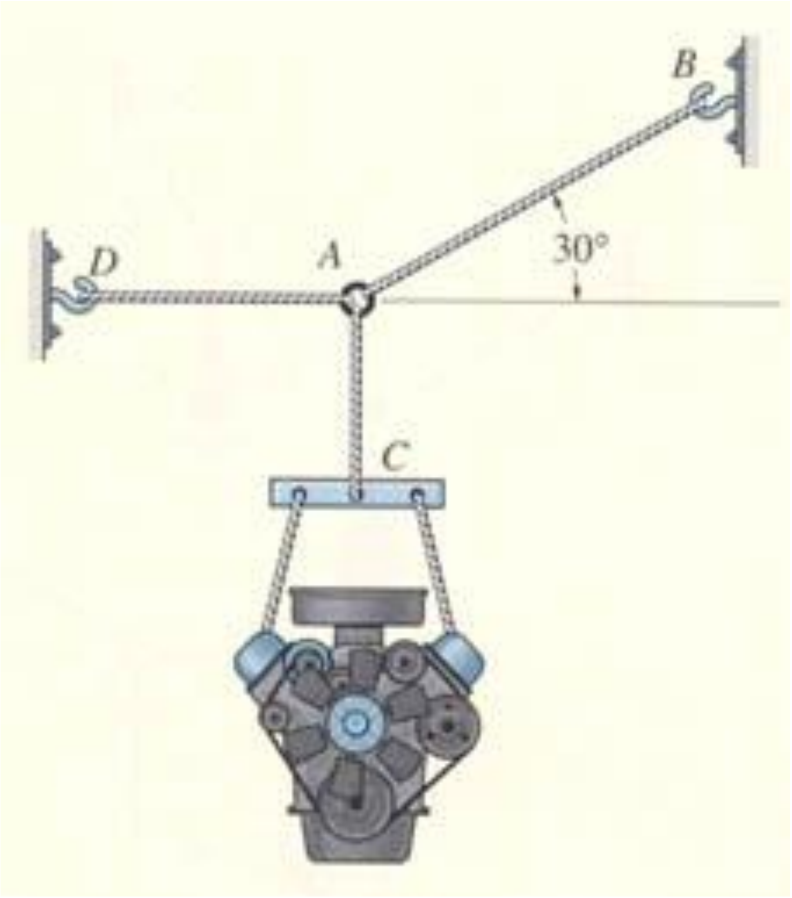


For a spool of given weight, what are the forces in cables AB and AC ?

APPLICATIONS
(continued)

For a given cable strength, what is the maximum weight that can be lifted ?

COPLANAR FORCE SYSTEMS



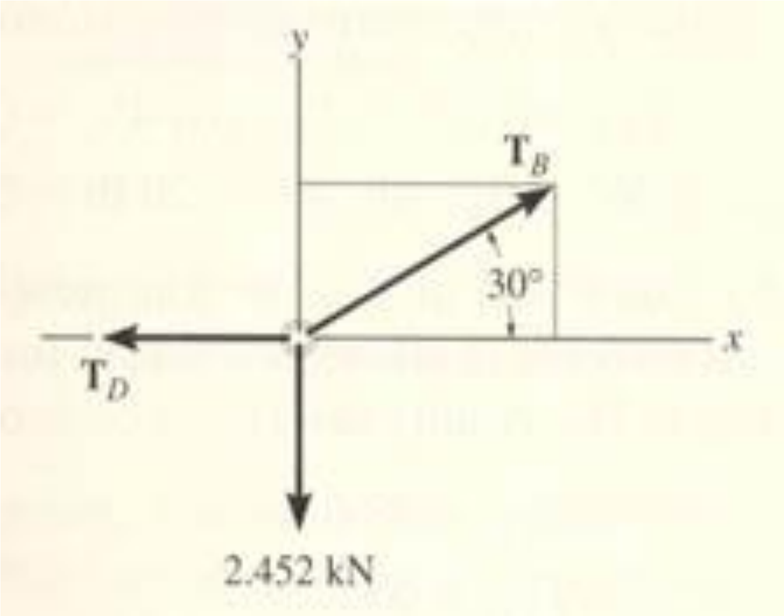
This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle A is also in equilibrium.

To determine the tensions in the cables for a given weight of the engine, we need to learn how to draw a free body diagram and apply equations of equilibrium.

THE WHAT, WHY AND HOW OF A FREE BODY DIAGRAM (FBD)

Free Body Diagrams are one of the most important things for you to know how to draw and use.

What ? - It is a drawing that shows all external forces acting on the particle.



Why ? - It helps you write the equations of equilibrium used to solve for the unknowns (usually forces or angles).

<u>How</u> ?

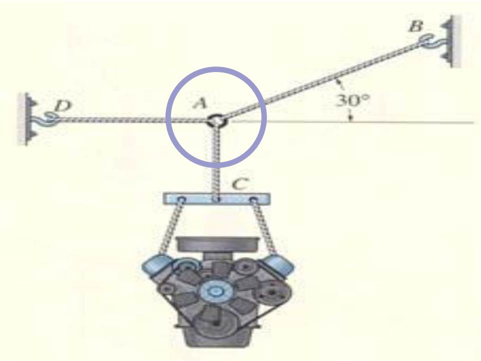
1. Imagine the particle to be isolated or cut free from its surroundings.

2. Show all the forces that act on the particle.

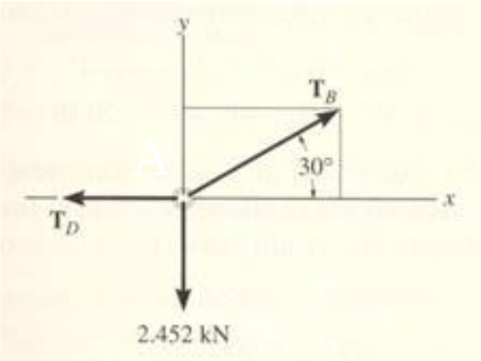
   Active forces: They want to move the particle.

   Reactive forces: They tend to resist the motion.

3. Identify each force and show all known magnitudes and directions. Show all unknown magnitudes and / or directions as variables .

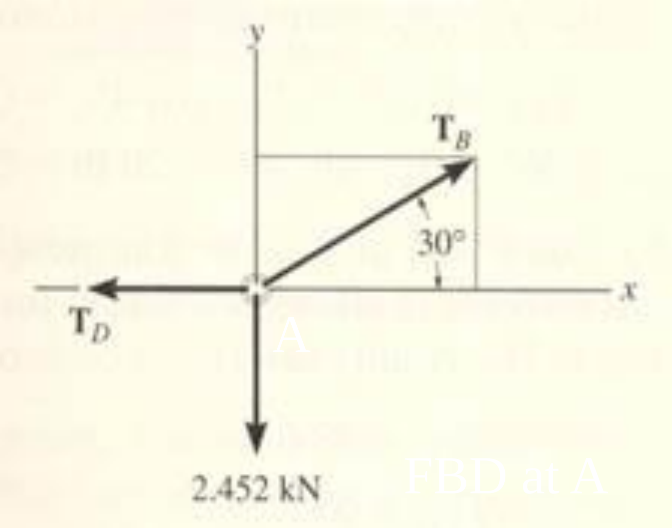


Note : Engine mass = 250 Kg



FBD at A

EQUATIONS OF 2-D EQUILIBRIUM



Since particle A is in equilibrium, the net force at A is zero.

So $\boldsymbol{F}_{AB} + \boldsymbol{F}_{AC} + \boldsymbol{F}_{AD} = 0$

or $\Sigma \boldsymbol{F} = 0$

In general, for a particle in equilibrium, $\Sigma \boldsymbol{F} = 0$ or

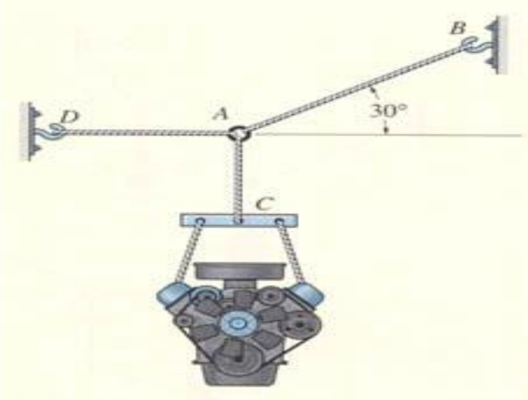
$\Sigma F_x \, \boldsymbol{i} + \Sigma F_y \, \boldsymbol{j} = 0 = 0 \, \boldsymbol{i} + 0 \, \boldsymbol{j}$ (A vector equation)

Or, written in a scalar form,

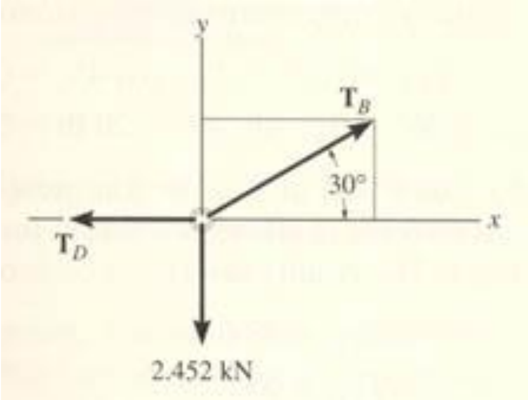
$\Sigma F_x = 0$ and $\Sigma F_y = 0$

These are two scalar equations of equilibrium (EofE). They can be used to solve for up to two unknowns.

EXAMPLE



Note : Engine mass = 250 Kg



FBD at A

Write the scalar EofE:

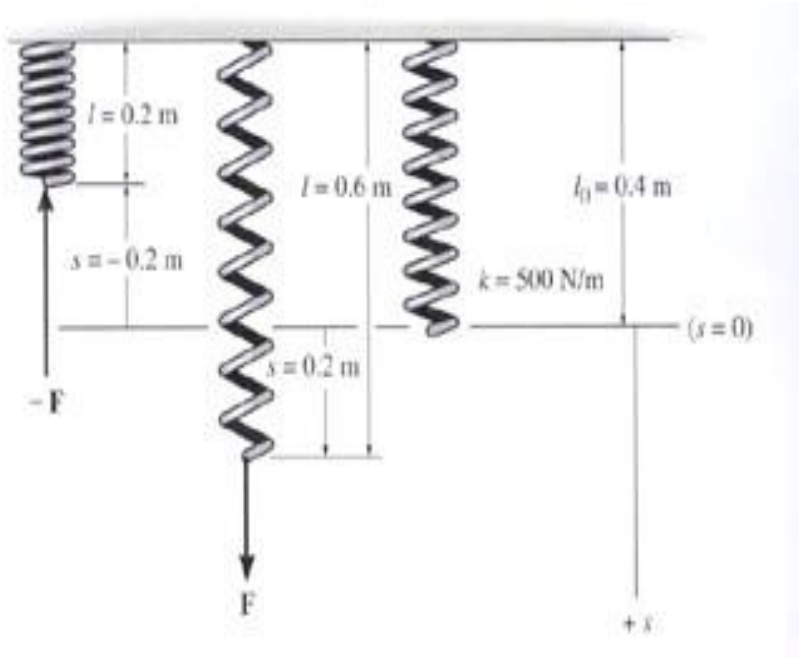
$$+\rightarrow \Sigma F_x = T_B \cos 30^o - T_D = 0$$

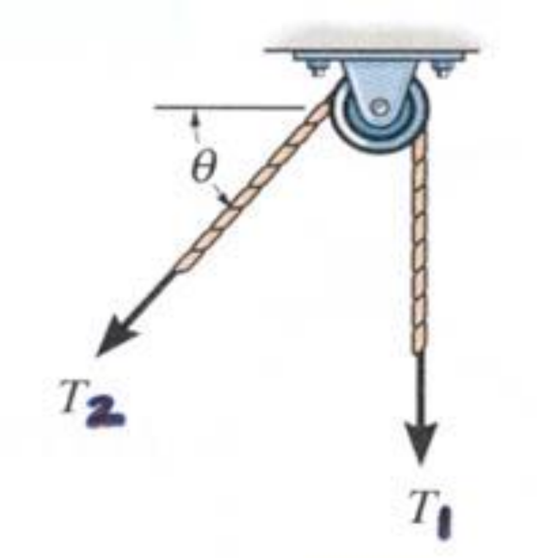
$$+\uparrow \Sigma F_y = T_B \sin 30^o - 2.452 \text{ kN} = 0$$

Solving the second equation gives: $T_B = 4.90$ kN

From the first equation, we get: $T_D = 4.25$ kN

SPRINGS, CABLES, AND PULLEYS



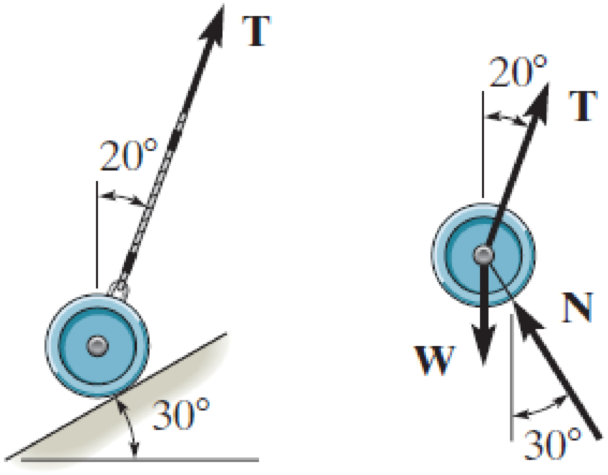


Spring Force = spring constant * deformation, or

F = k * S

With a frictionless pulley, $T_1 = T_2$.

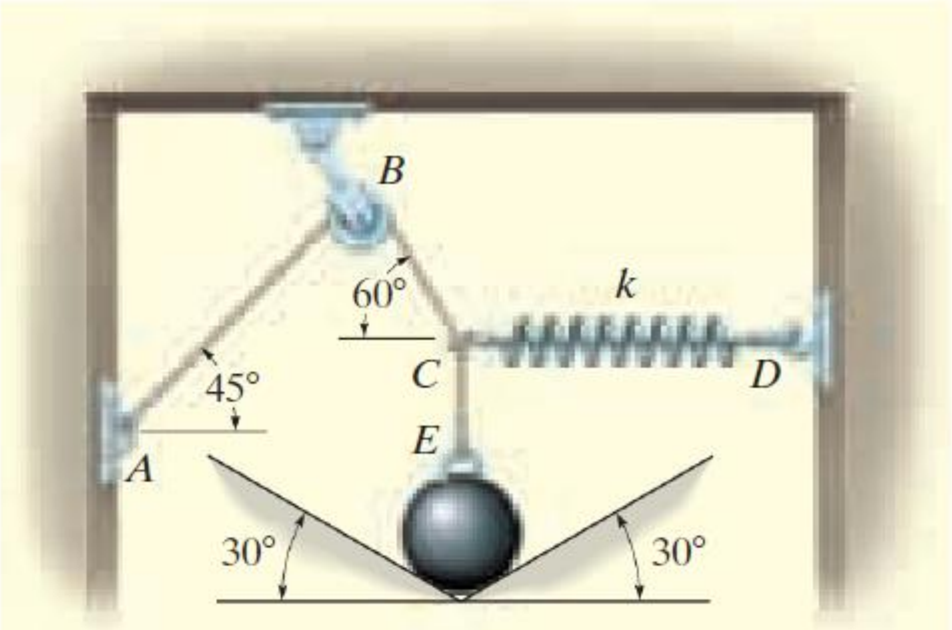
Smooth Contact



(a) (b)

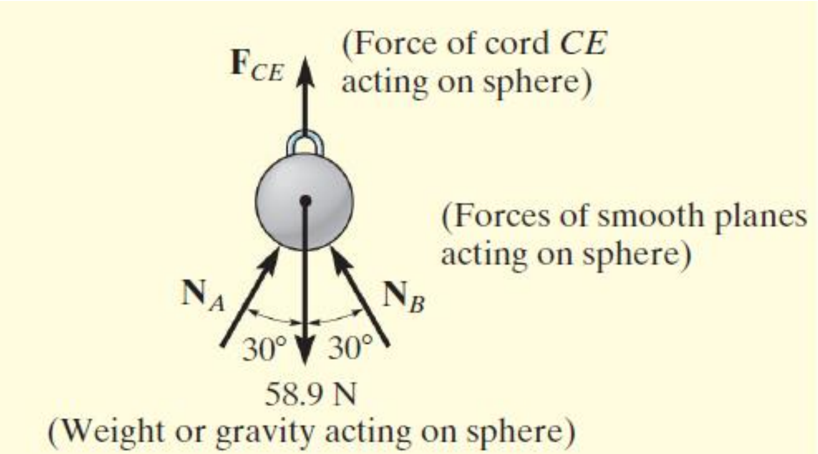
EXAMPLE

The sphere in Fig. 3–3*a* has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord *CE*, and the knot at *C*.



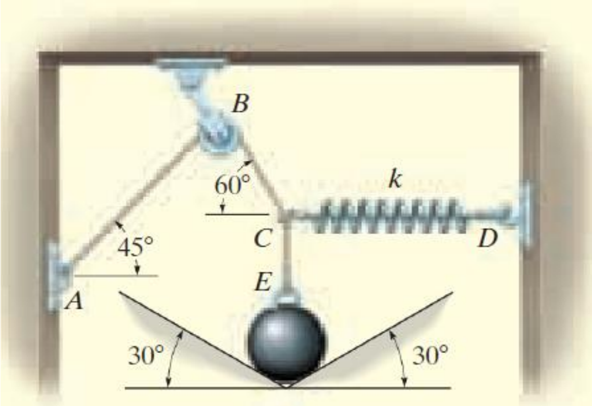
(a)

EXAMPLE



F_{CE}
(Force of cord *CE* acting on sphere)
(Forces of smooth planes acting on sphere)
N_A
N_B
30°
30°
58.9 N
(Weight or gravity acting on sphere)

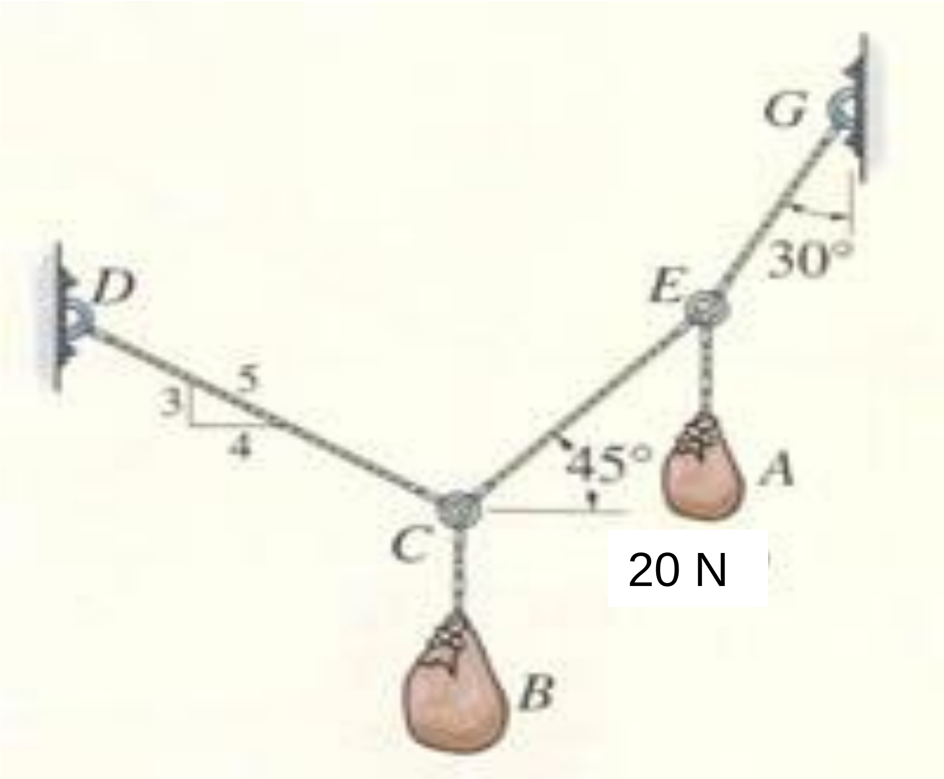
(b)



B
60°
k
45°
C
D
E
A
30°
30°

(a)

EXAMPLE



Given: Sack A weighs 20 N. and geometry is as shown.

Find: Forces in the cables and weight of sack B.

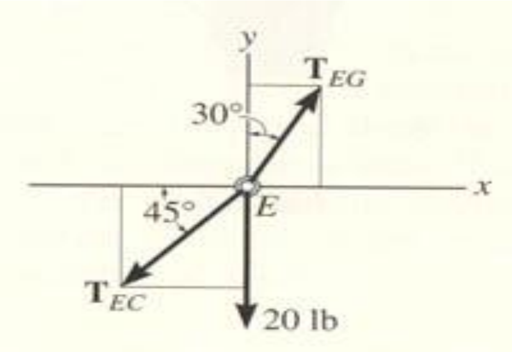
Plan:

1. Draw a FBD for Point E.

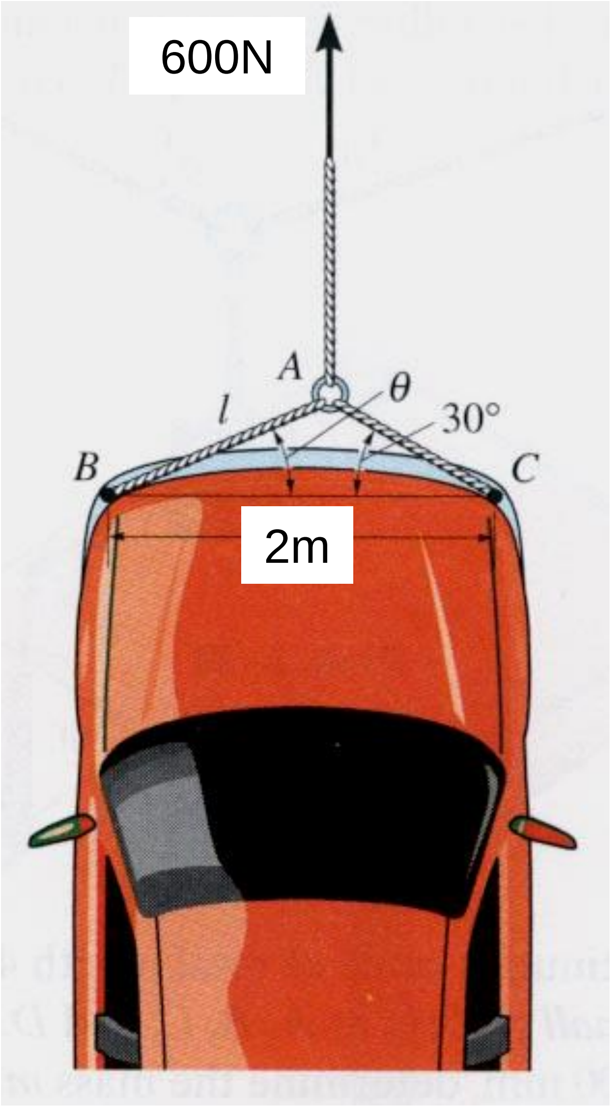
2. Apply EofE at Point E to solve for the unknowns (T_{EG} & T_{EC}).

3. Repeat this process at C.

EXAMPLE
(continued)



GROUP PROBLEM SOLVING



Given: The car is towed at constant speed by the 600 N force and the angle θ is 25°.

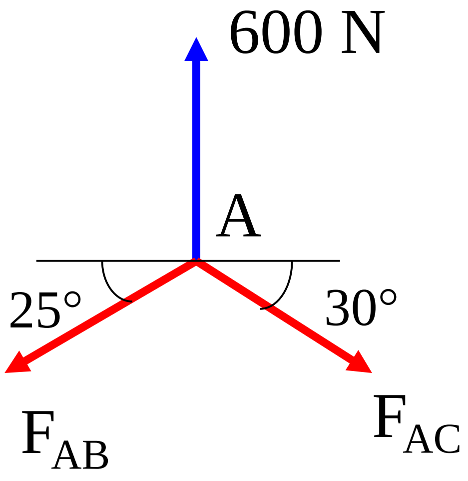
Find: The forces in the ropes AB and AC.

Plan:

1. Draw a FBD for point A.

2. Apply the E-of-E to solve for the forces in ropes AB and AC.

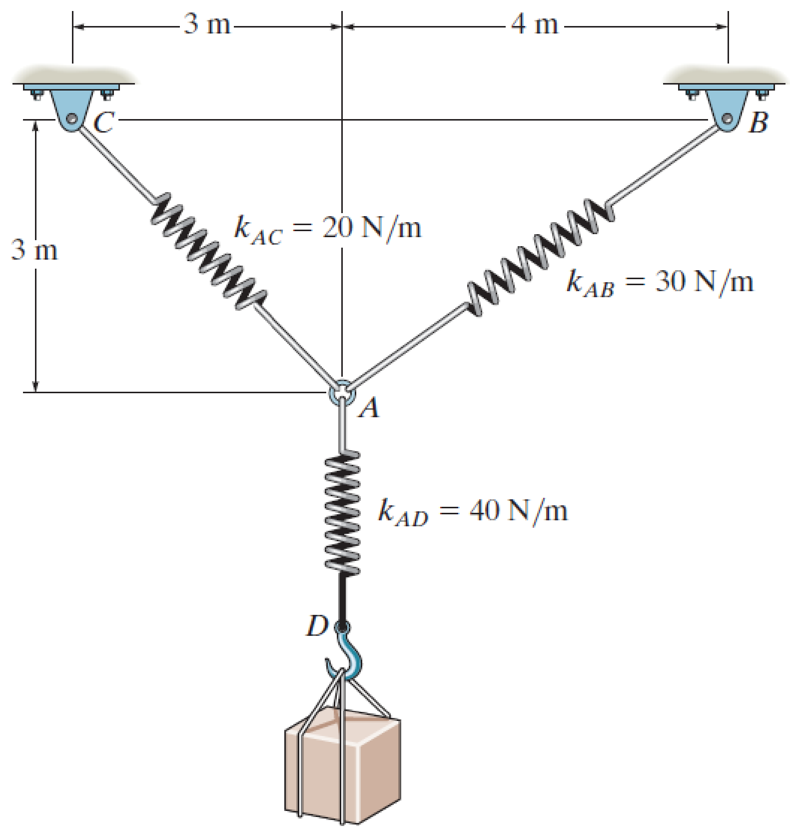
GROUP PROBLEM SOLVING
(continued)



FBD at point A

Quiz

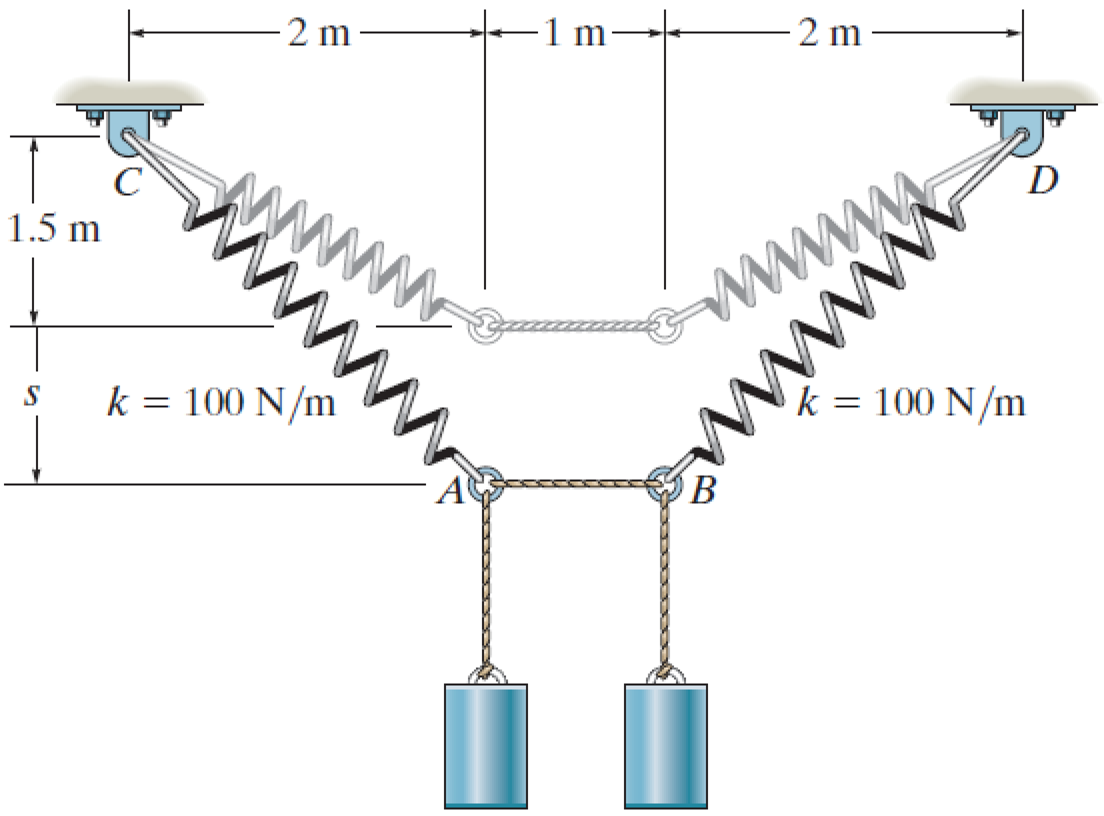
3–15. The unstretched length of spring AB is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at D.



Quiz

Quiz

*3–16. Determine the mass of each of the two cylinders if they cause a sag of $s = 0.5$ m when suspended from the rings at A and B. Note that $s = 0$ when the cylinders are removed.



Quiz

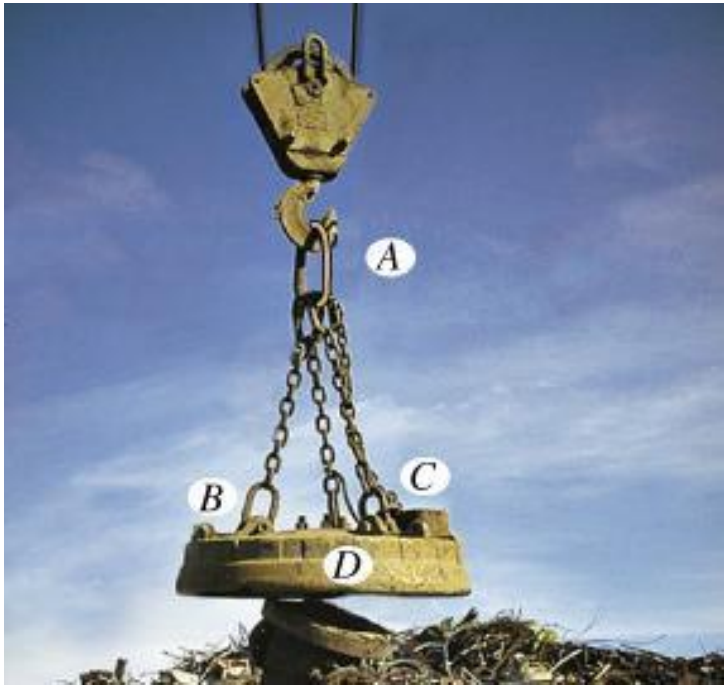
THREE-DIMENSIONAL FORCE SYSTEMS

Section II Objectives:

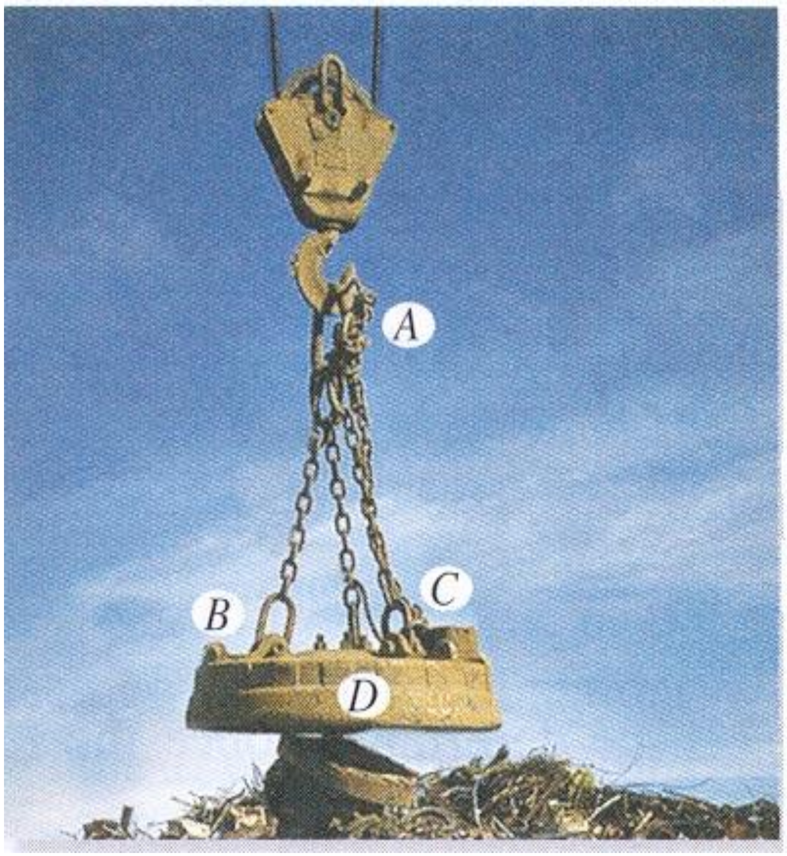
Students will be able to solve 3-D particle equilibrium problems by

a) Drawing a 3-D free body diagram, and,

b) Applying the three scalar equations (based on one vector equation) of equilibrium.



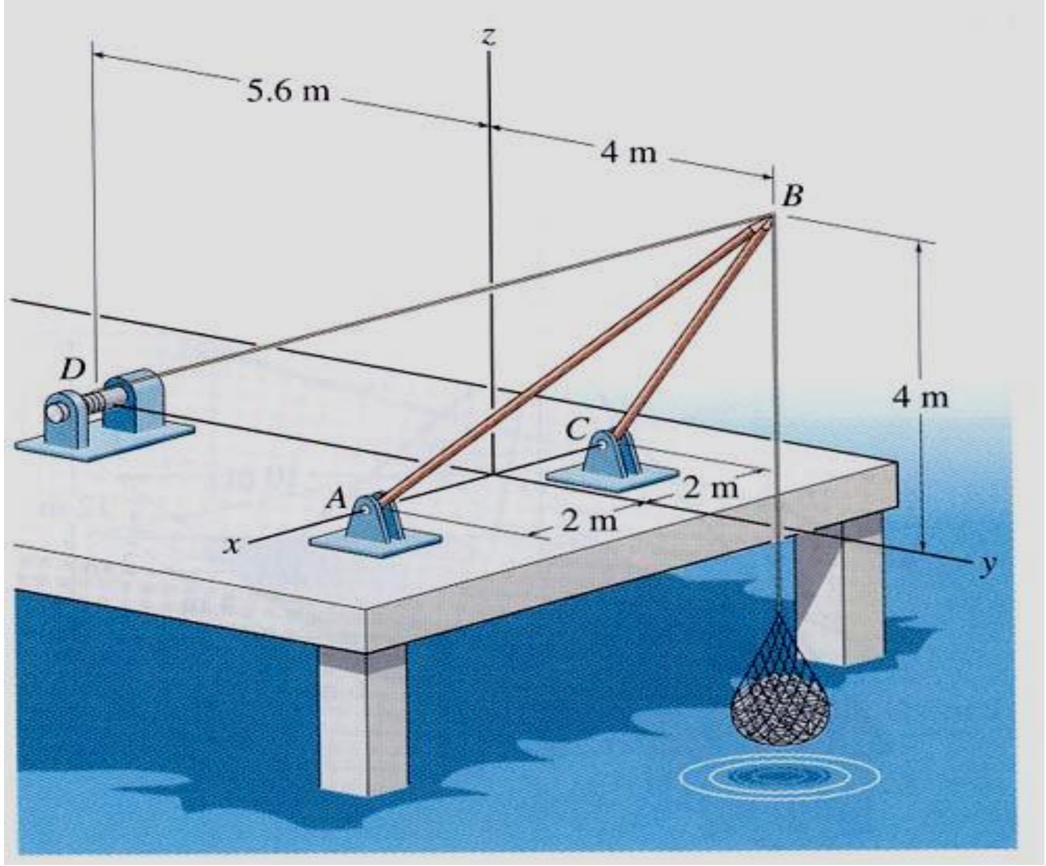
APPLICATIONS



The weights of the electromagnet and the loads are given.

Can you determine the forces in the chains?

APPLICATIONS
(continued)

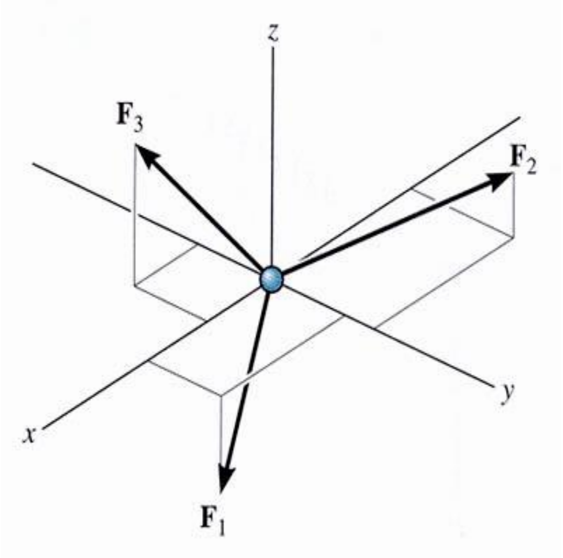


The shear leg derrick is to be designed to lift a maximum of 500 kg of fish.

What is the effect of different offset distances on the forces in the cable and derrick legs?

THE EQUATIONS OF 3-D EQUILIBRIUM

When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero $(\Sigma \boldsymbol{F} = 0)$.



This equation can be written in terms of its x, y and z components. This form is written as follows.

$$(\Sigma F_x)\,\boldsymbol{i} + (\Sigma F_y)\,\boldsymbol{j} + (\Sigma F_z)\,\boldsymbol{k} = 0$$

This vector equation will be satisfied only when

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

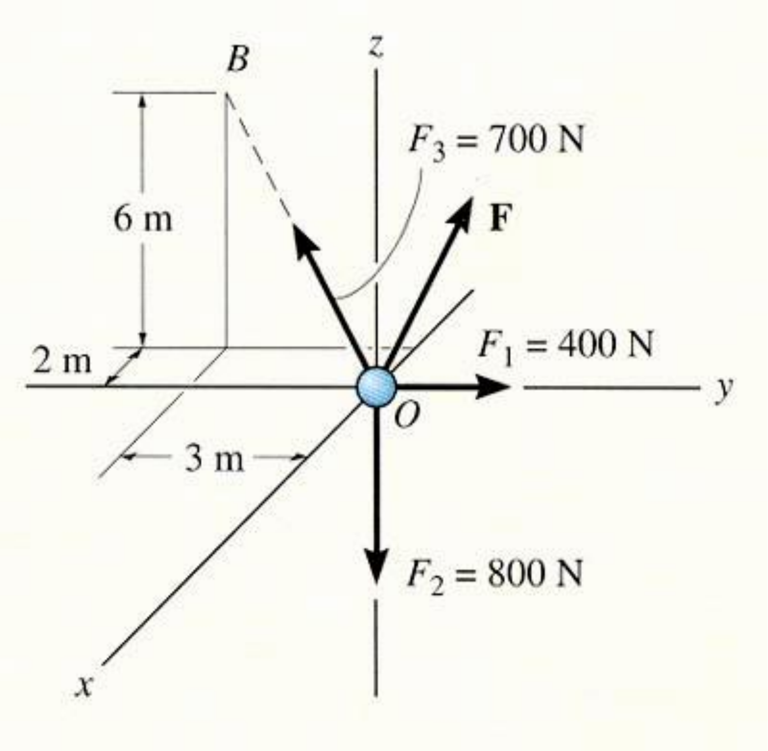
$$\Sigma F_z = 0$$

These equations are the three scalar equations of equilibrium. They are valid at any point in equilibrium and allow you to solve for up to three unknowns.

EXAMPLE #1

Given: $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$.

Find: The force ***F*** required to keep particle O in equilibrium.



<u>Plan</u>:

1) Draw a FBD of particle O.

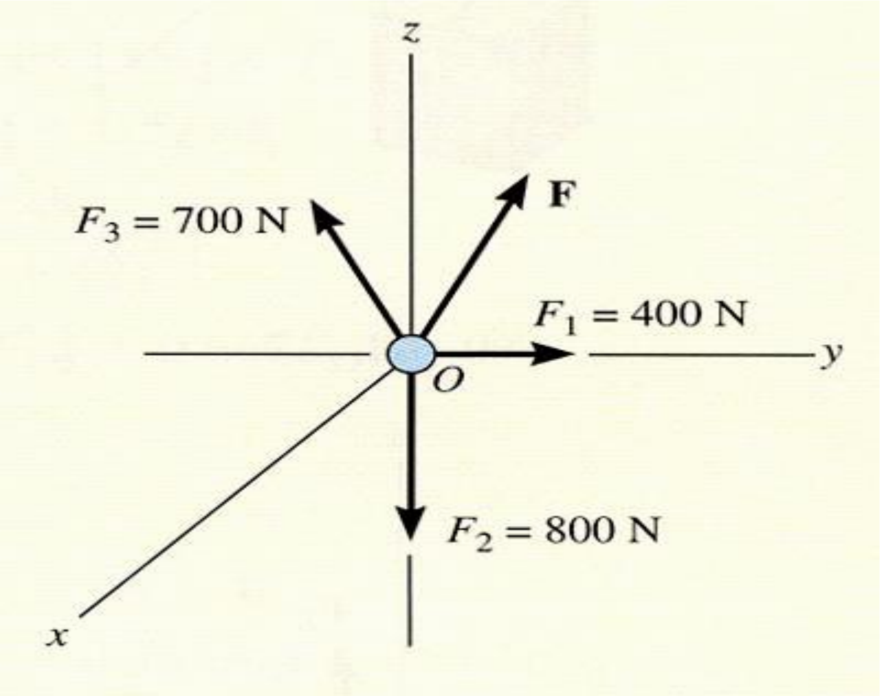
2) Write the unknown force as

$$\boldsymbol{F} = \{F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j} + F_z\,\boldsymbol{k}\}\text{ N}$$

3) Write $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ in Cartesian vector form.

4) Apply the three equilibrium equations to solve for the three unknowns F_x, F_y, and F_z.

EXAMPLE #1
(continued)



EXAMPLE #1
(continued)

Equating the respective ***i***, ***j***, ***k*** components to zero, we have

$\Sigma F_x = -200 + F_X = 0$; solving gives $F_x = 200$ N

$\Sigma F_y = 400 - 300 + F_y = 0$; solving gives $F_y = -100$ N

$\Sigma F_z = -800 + 600 + F_z = 0$; solving gives $F_z = 200$ N

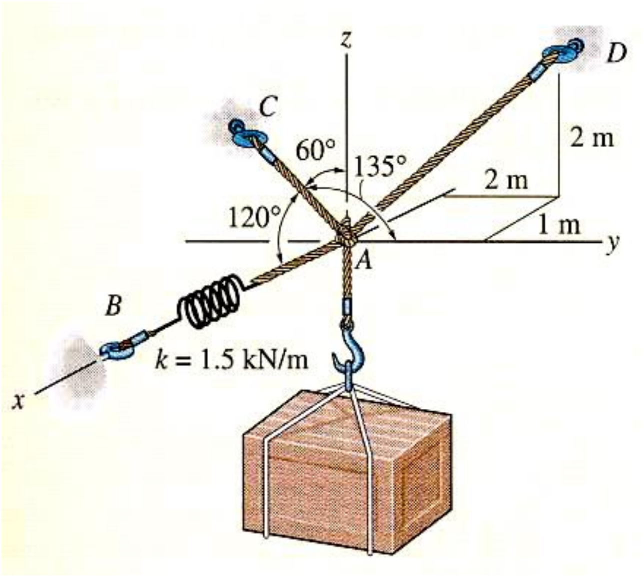
Thus, $\boldsymbol{F} = \{200\,\boldsymbol{i} - 100\,\boldsymbol{j} + 200\,\boldsymbol{k}\}$ N

Using this force vector, you can determine the force's magnitude and coordinate direction angles as needed.

EXAMPLE

Given: A 100 Kg crate, as shown, is supported by three cords. One cord has a spring in it.

Find: Tension in cords AC and AD and the stretch of the spring.



<u>Plan</u>:

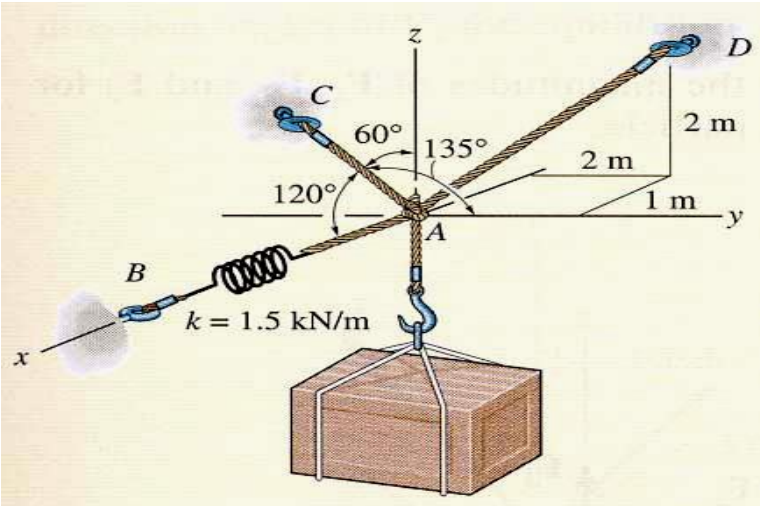
1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

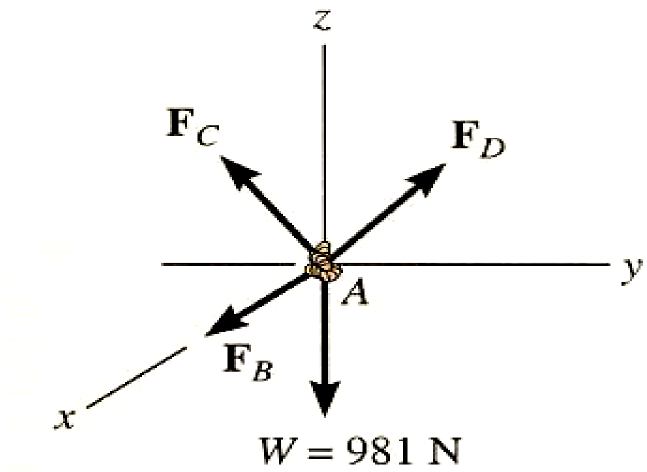
2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

4) Find the spring stretch using $F_B = K * S$.

EXAMPLE #2 (continued)





FBD at A

$\boldsymbol{F_B} = F_B \text{ N } \boldsymbol{i}$

$\boldsymbol{F_C} = F_C \text{ N } (\cos 120° \, \boldsymbol{i} + \cos 135° \, \boldsymbol{j} + \cos 60° \, \boldsymbol{k})$

$= \{- 0.5 \, F_C \, \boldsymbol{i} - 0.707 \, F_C \, \boldsymbol{j} + 0.5 \, F_C \, \boldsymbol{k}\} \text{ N}$

$\boldsymbol{F_D} = F_D(\boldsymbol{r_{AD}}/r_{AD})$

$= F_D \text{ N}[(-1 \, \boldsymbol{i} + 2 \, \boldsymbol{j} + 2 \, \boldsymbol{k})/(1^2 + 2^2 + 2^2)^{½}]$

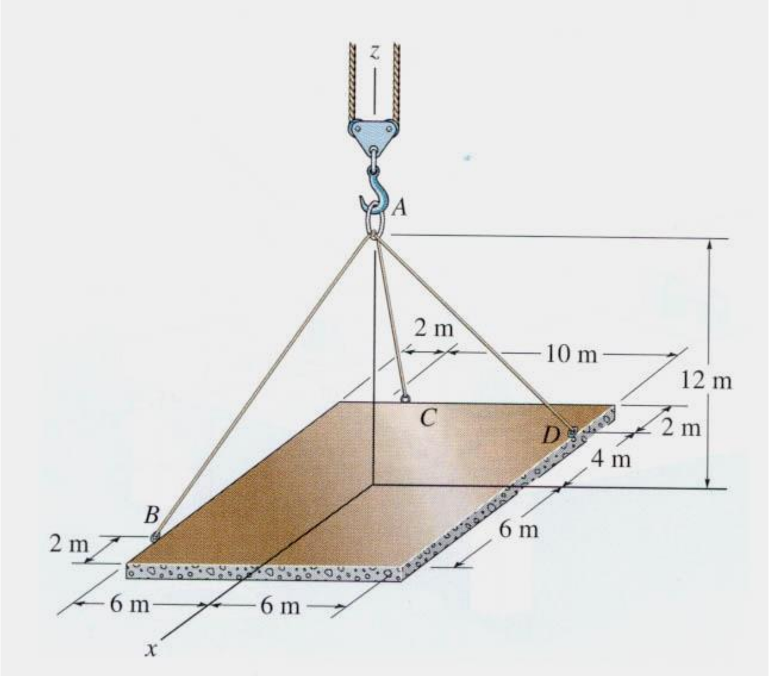
$= \{- 0.3333 \, F_D \, \boldsymbol{i} + 0.667 \, F_D \, \boldsymbol{j} + 0.667 \, F_D \, \boldsymbol{k}\}\text{N}$

EXAMPLE #2 (continued)

GROUP PROBLEM SOLVING

Given: A 150 Kg plate, as shown, is supported by three cables and is in equilibrium.

Find: Tension in each of the cables.



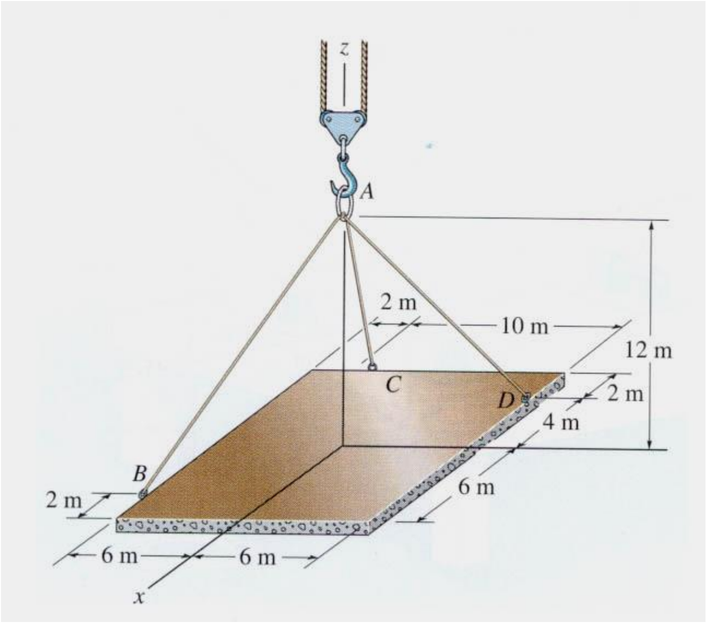
Plan:

1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

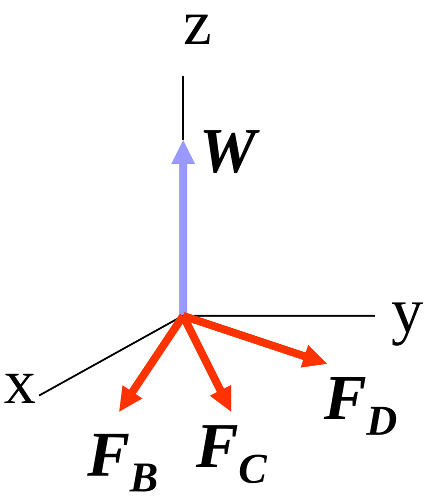
2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

GROUP PROBLEM SOLVING (continued)

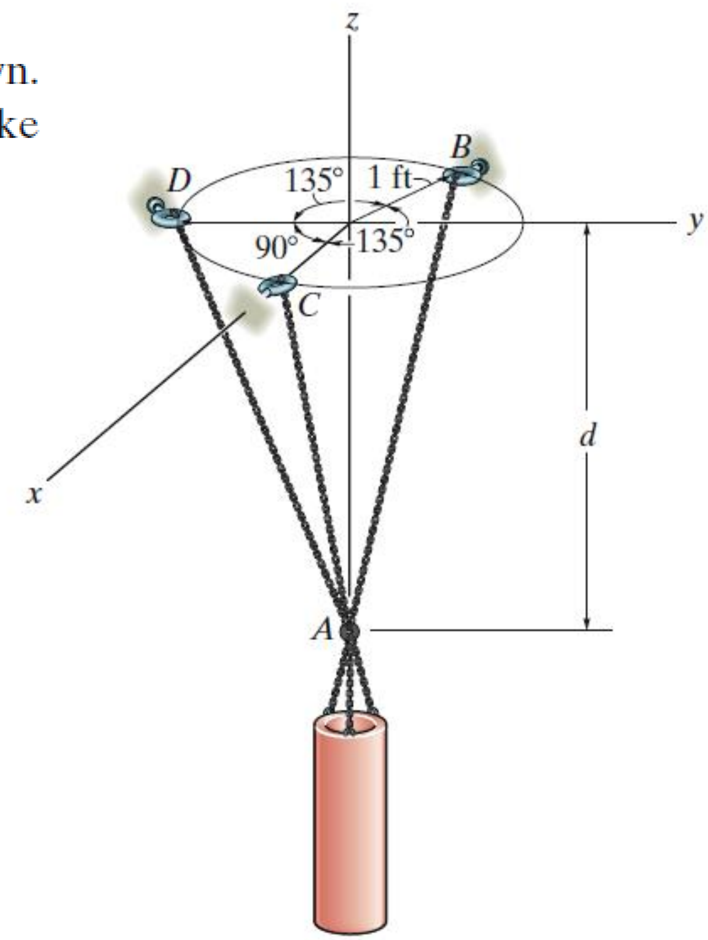


FBD of Point A:



Quiz

The 800-lb cylinder is supported by three chains as shown. Determine the force in each chain for equilibrium. Take $d = 1$ ft.



Quiz