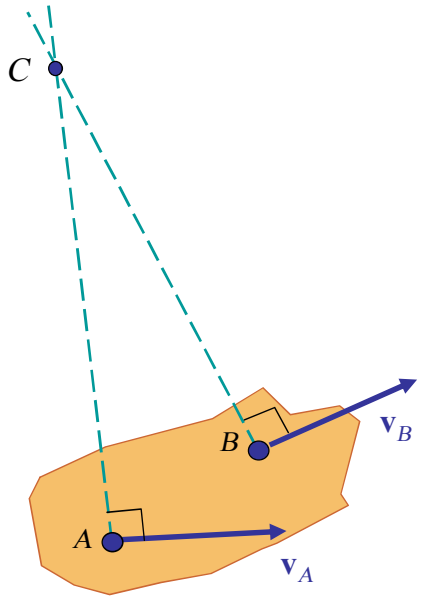
Kinematics and Dynamics of Machines

2. Kinematic Analysis of Mechanisms

(Instantaneous Center Method)

Introduction



- The instantaneous center method of analyzing the motion in a mechanism is based upon the concept that any displacement of a body having motion in one plane, can be considered as a pure rotational motion of a rigid link as whole about some center, known as instantaneous center or virtual center of motion.

Definition

- There are two definitions for instantaneous center:

1. Instantaneous center is a point on a member which another member rotates around, permanently or instantaneously.
2. Instantaneous center is a point in common between two members where the velocities are equal, both in direction and magnitude.

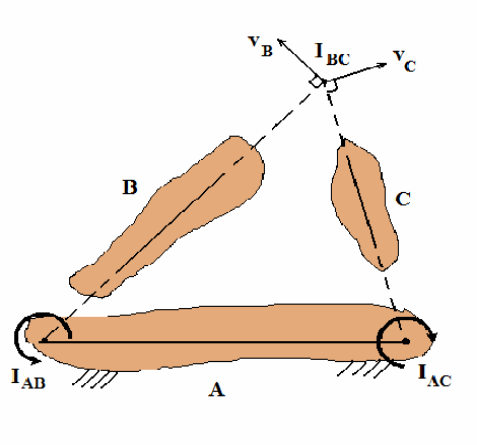
Number of Instantaneous Centers

- The number of instantaneous centers in a considered kinematic chain is equal to number of combinations of two links:

$$N = \frac{n\,(n-1)}{2}, \quad n = Number\ \ of\ links$$

Kennedy's Theorem

- The Kennedy's theorem states that if three bodies move relatively to each other, they have three instantaneous centers that lie on a straight line.



Types of Instantaneous Centers

- The instantaneous centers for a mechanism are of the following three types:

  1- Fixed instantaneous centers

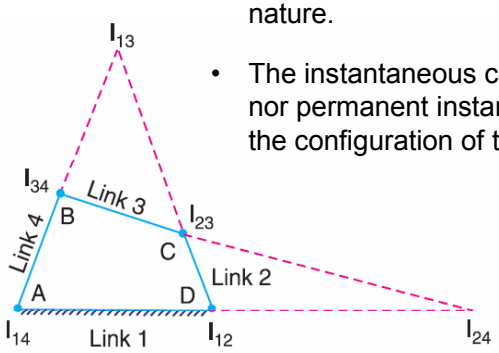
  2- Permanent instantaneous centers

  3- Neither fixed nor permanent instantaneous centers

- The first two types are together known as **primary instantaneous centers** and the third type is known as **secondary instantaneous centers**

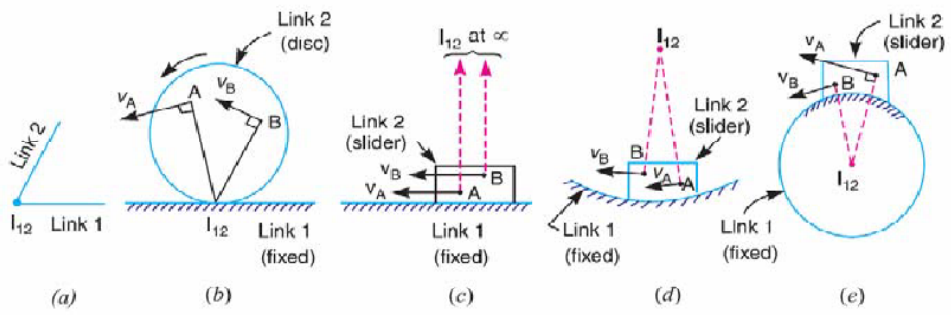
Example: Consider a four-bar ABCD mechanism as shown in the figure:

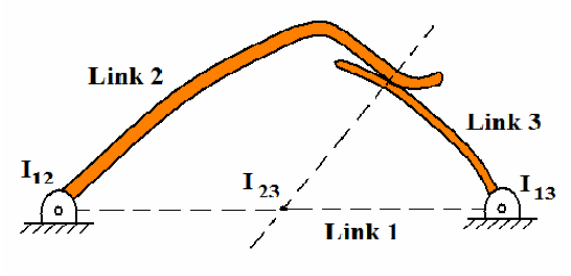
- The instantaneous centers I_{12} and I_{14} are called the fixed instantaneous centers.
- The instantaneous centers I_{23} and I_{34} are the permanent instantaneous centers as they move when the mechanism moves, but the joints are of the permanent nature.
- The instantaneous centers I_{13} and I_{24} are neither fixed nor permanent instantaneous centers as the vary with the configuration of the mechanism.



Location of Instantaneous Centers

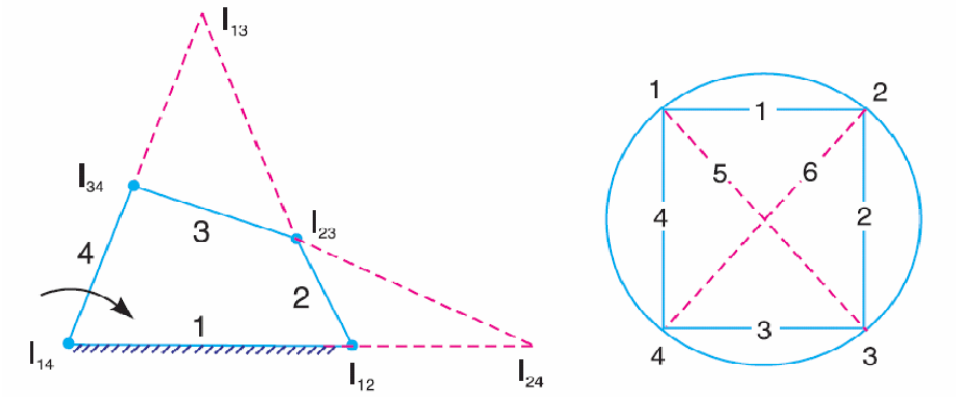
1. When the two links are connected by a pin joint, the instantaneous center lies on the center of the pin
2. When the two links have a pure rolling contact, the instantaneous point lies on their point of contact
3. When the two points have a sliding contact, the instantaneous center lies on the common normal at the point of contact (the members must have the same component of velocity in the direction normal to the sliding surface).



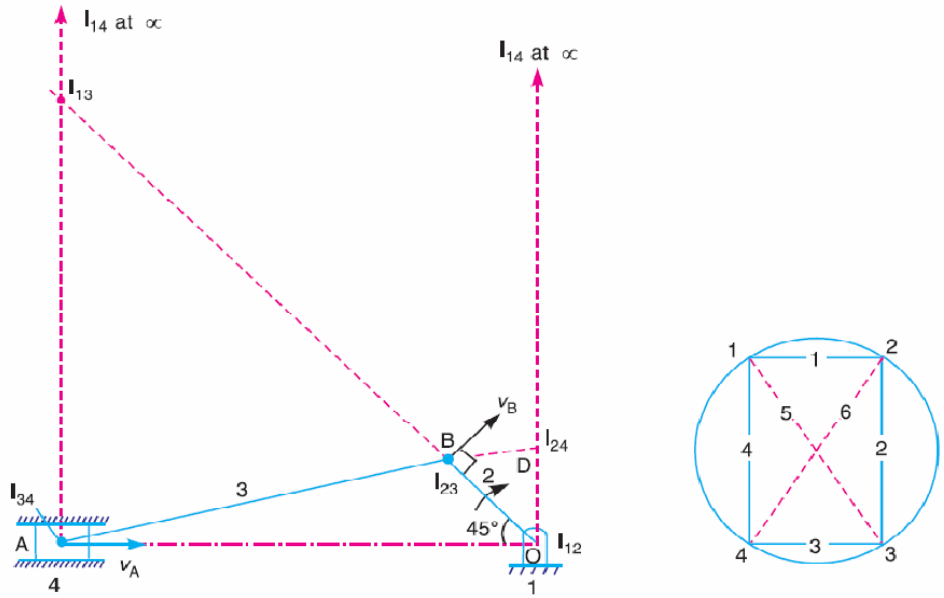


Locating Instantaneous Centers in a Mechanism

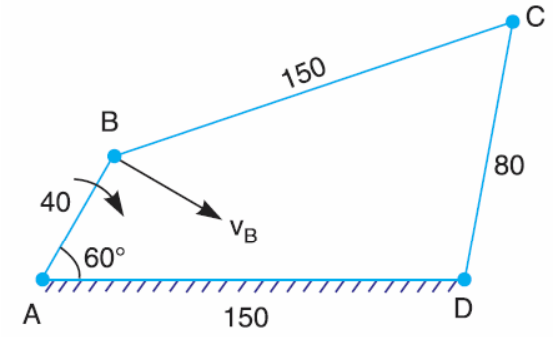
1. First of all, determine the number of instantaneous centers.
2. Locate the fixed and permanent centers by inspection.
3. Locate the remaining neither fixed nor permanent centers by Kennedy's theorem (this is done by circle diagram)
4. On the circle diagram, join the points by solid lines to show that these centers are already found
5. To find the other instantaneous centers, join the two corresponding points. The line joining them forms two adjacent triangles in the circle diagram.



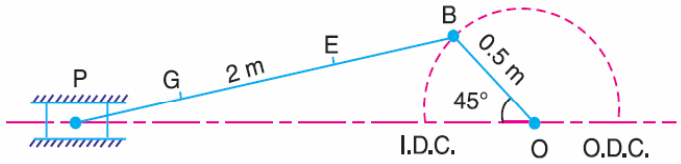
(*a*) Four bar mechanism. (*b*) Circle diagram.



Example : *In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.*

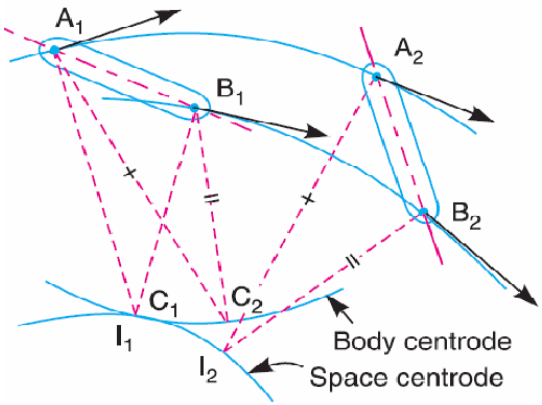


Example : *The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine : 1. velocity of piston, 2. angular velocity of connecting rod, 3. velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4. velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.*



Space and body centrode

- The locus of the instantaneous center in space during a definite motion of the body is called **space centode**
- The locus of instantaneous center relative to the body itself is named the **body centrode**



Equivalent Mechanisms

- Two Mechanisms are kinematical and instantly equivalent if the velocity and the acceleration of the drivers and the followers are equal in both mechanisms