

# Kinematics and Dynamics of Machines

## 9. Balancing of Reciprocating Systems

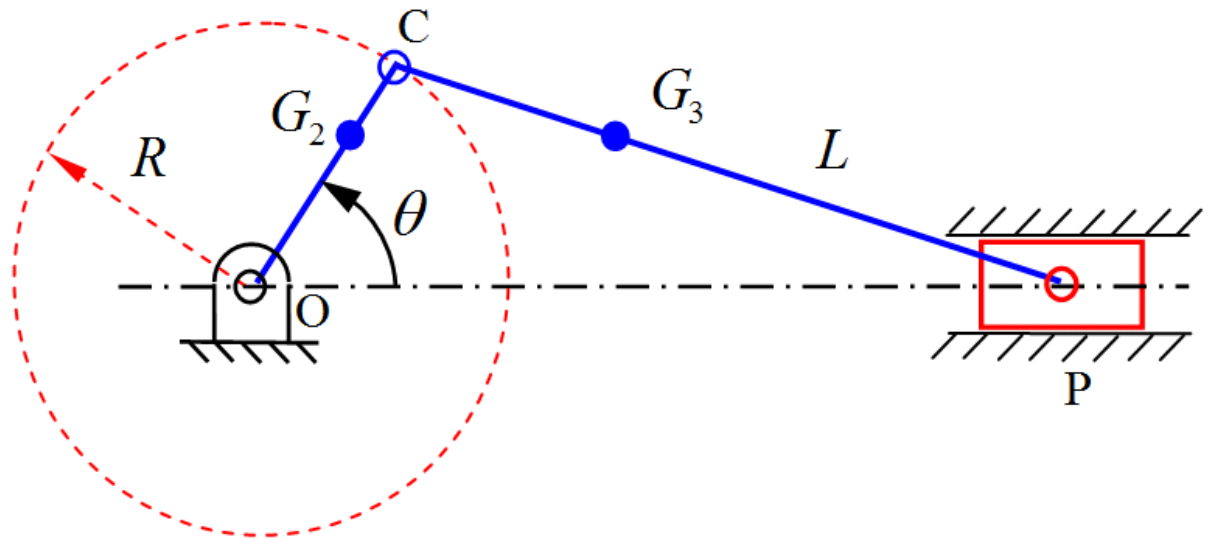
# Introduction

- The resultant of all the forces acting on the body of the engine due to inertia forces is known as ***unbalanced force*** or ***shaking force***.
- If the resultant of all the inertia forces is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple may be present.
- Balancing of a mechanism includes counterbalancing both the inertia forces and the inertia couples. In some cases we can eliminate this effects almost completely. However, in general it is too difficult to prepare a device to balance a mechanism totally.
- In most of cases we can reduce the shaking forces and the shaking couples by adding appropriate balancing masses. In such cases, the mechanism is said to be partially balanced.

# Partial balancing of a reciprocating mechanism

Consider a crank-slider mechanism rotating with a constant velocity. From previous analysis, the acceleration of the slider is obtained as follows:

$$a_P = -R\omega^2 \left( \cos\theta + \frac{R}{L} \cos 2\theta \right)$$



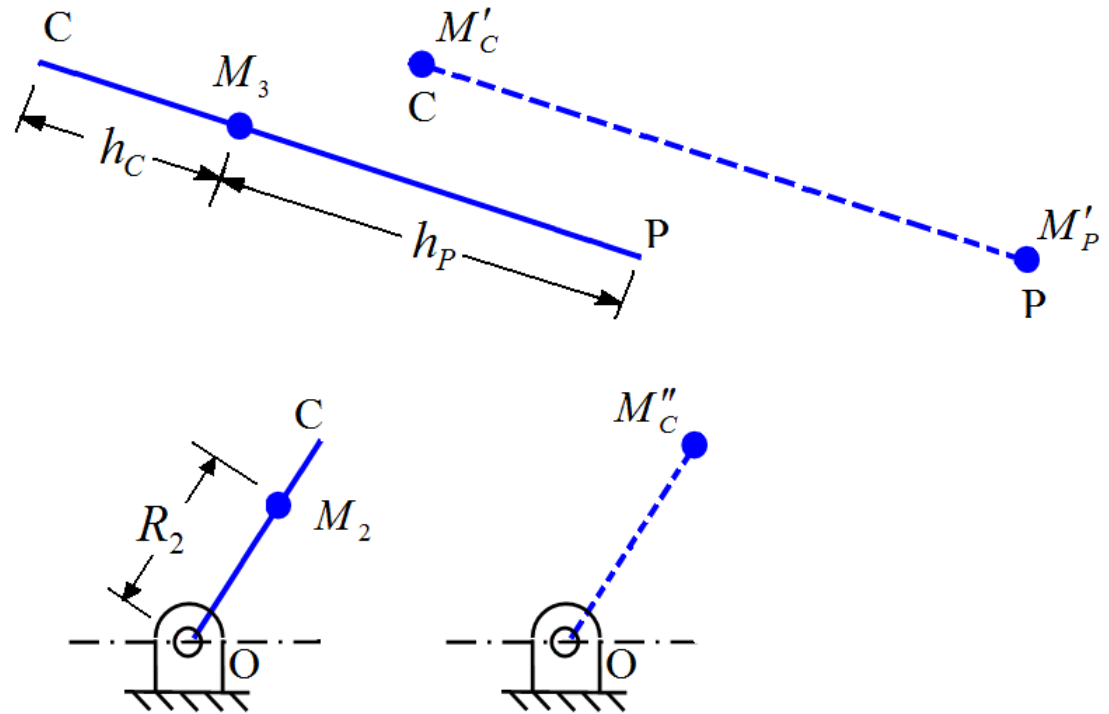
Using the phenomenon of dynamically equivalent bodies, the crank and the coupler can be replaced by lump masses coinciding the joints and mass-less links, as illustrated. From the phenomenon of dynamically equivalent bodies, we get:

$$M'_C + M'_P = M_3$$

$$M'_C h_C = M'_P h_P$$

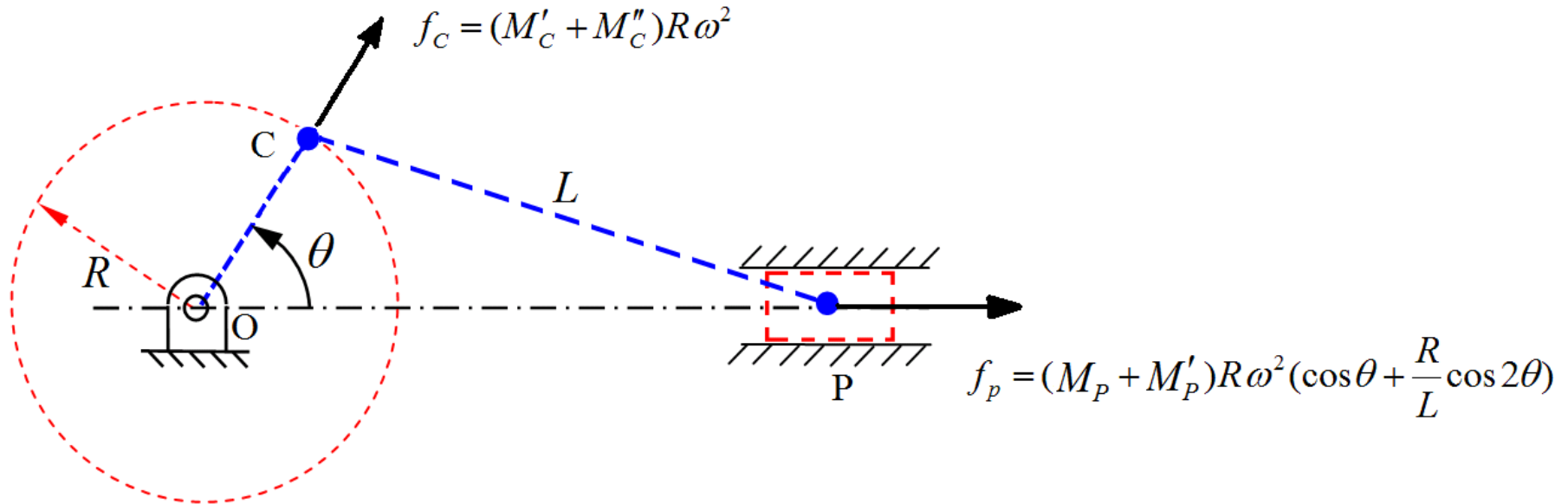
and

$$M''_C = \frac{R_2}{R} M_2$$



Note that there is a small difference between the inertia moments of the new coupler and the original one, since the third condition necessary for equivalent bodies is not satisfied.

For the equivalent mechanism introduced here, the inertia forces are obtained as showed in the figure.

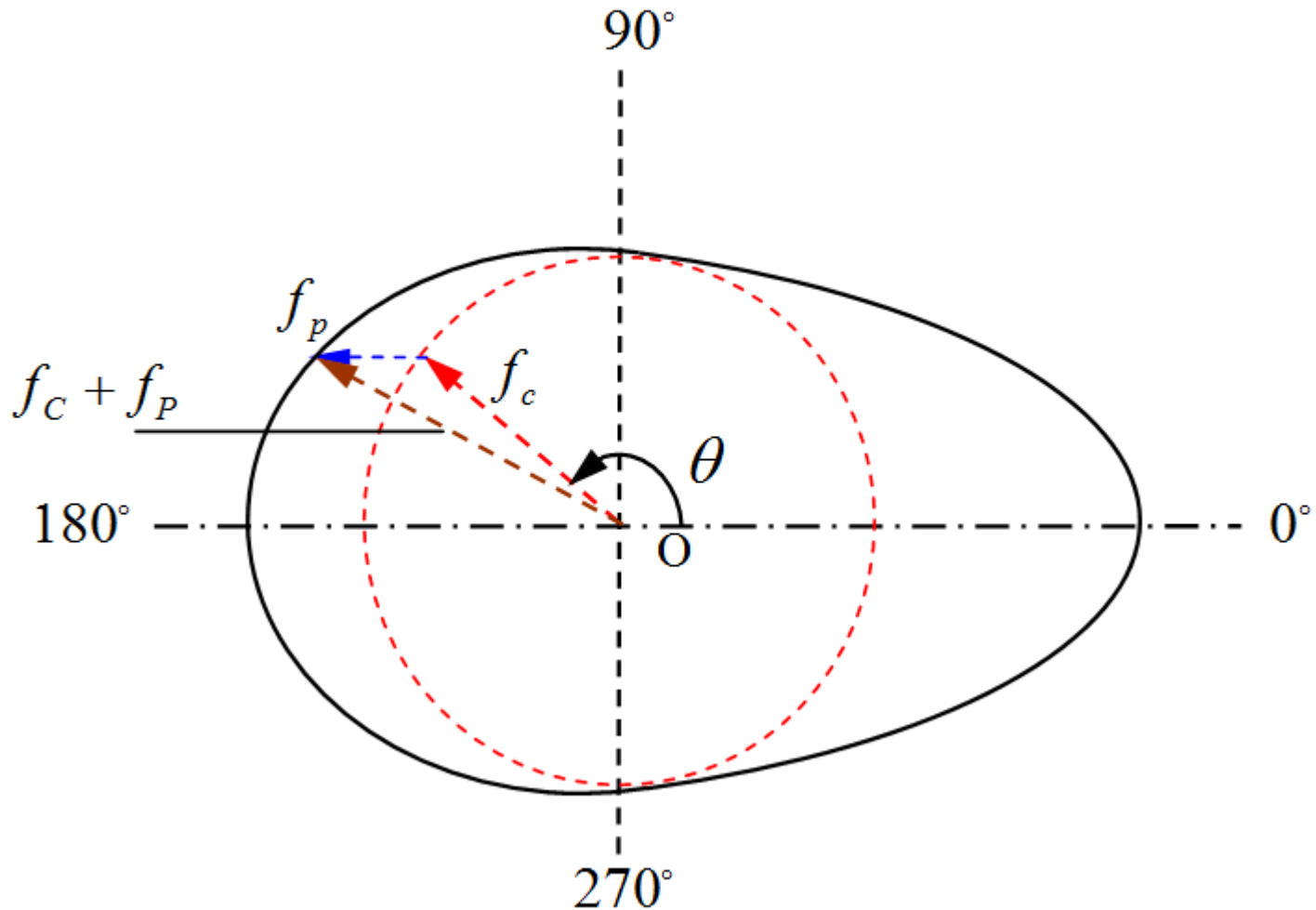


The inertia force of the slider contains to main terms which are:

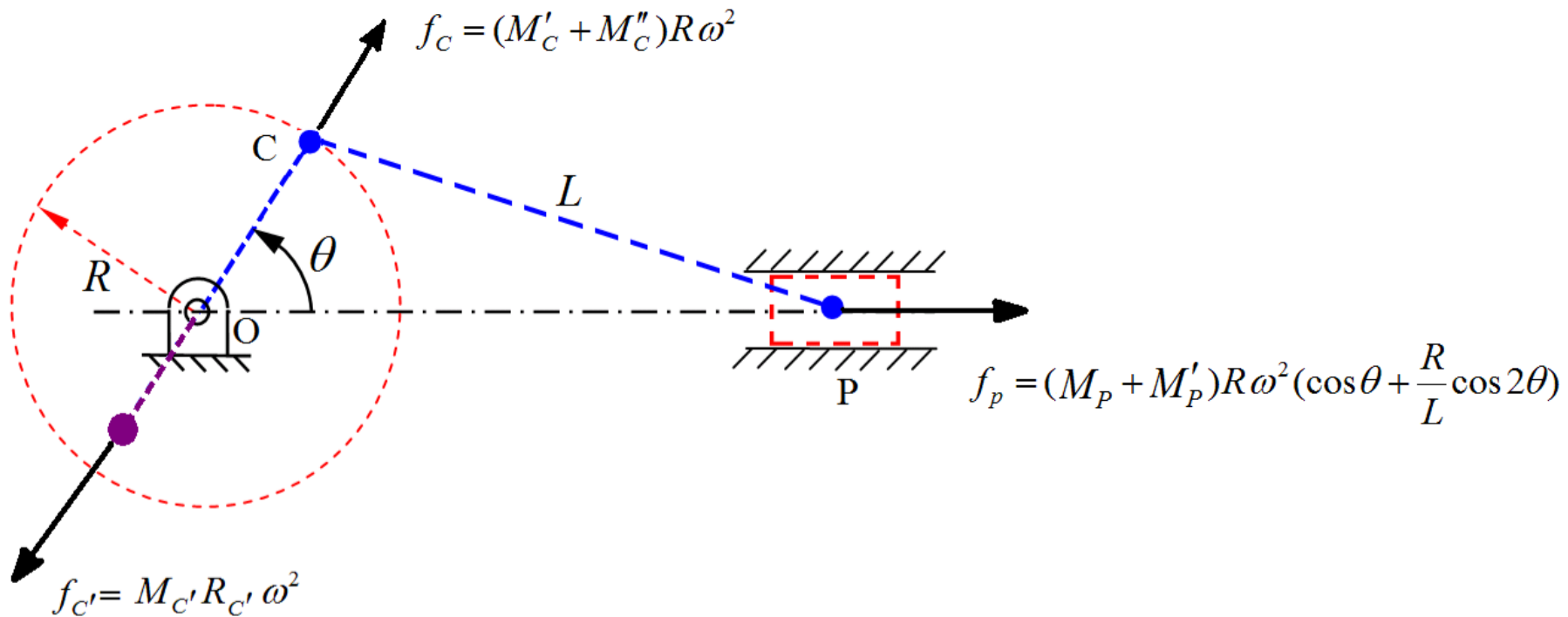
$$f_{p_1} = (M_P + M'_P)R\omega^2 \cos\theta \quad \text{primary unbalanced force}$$

$$f_{p_2} = (M_P + M'_P)\omega^2 \frac{R^2}{L} \cos 2\theta \quad \text{secondary unbalanced force}$$

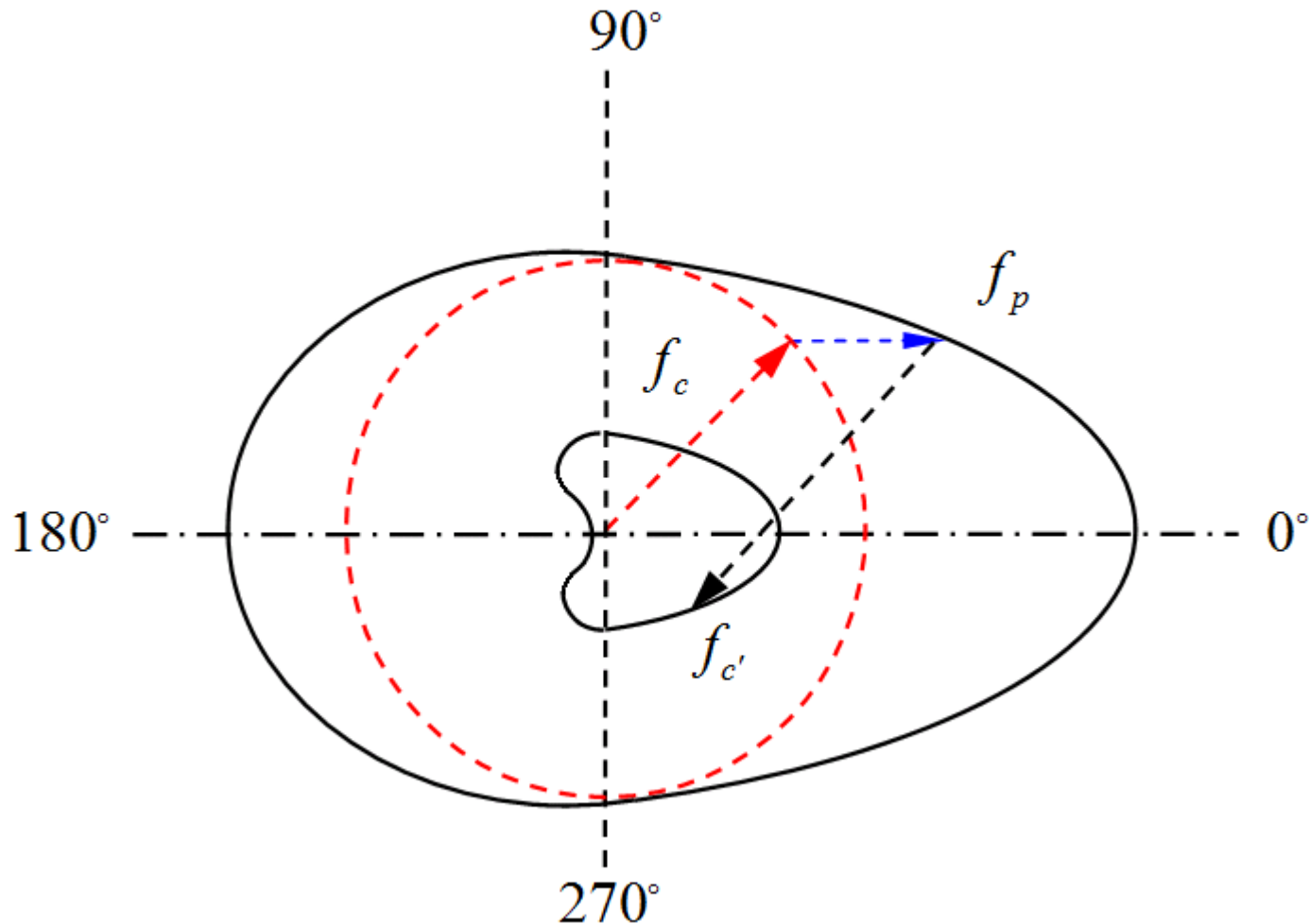
The summation of inertia forces which acts on the body of the engine, is a function of the crank angle. A graphical presentation of this force is showed in the below figure.



In order to partially balance the shaking force, a balancing mass is added to system as presented.



If the balancing mass is selected properly, it can reduce the shaking force considerably. The following figure shows how the shaking force is manipulated by addition of the balancing mass.





# Proper selection of the balancing mass

- According to the previous equations, we have three group of inertia forces, which are:

$$f_{P1} = m_P R \omega^2 \cos \theta,$$

$$f_c = m_c R \omega^2 (\cos \theta i + \sin \theta j),$$

$$f_c' = -m_b r_b \omega^2 (\cos \theta i + \sin \theta j),$$

$$m_P = M_P + M_P', \quad m_c = M_c'' + M_c', \quad m_b r_b = m_{C'} r_{C'}$$

Here we have neglected the secondary part of the piston inertia force. Hence, the shaking force (primary) is equal to

$$\sum_{i=1}^3 F = (m_P R + m_c R - m_B r_B) \omega^2 \cos \theta i \\ + (m_c R - m_B r_B) \omega^2 \sin \theta j$$

The primary force is balanced if

$$m_P R + m_c R = m_B r_B$$

But the centrifugal force produced by the revolving masses has a vertical component also.

It is thus obvious, that effect of above method is to change the direction of unbalanced force from the line of stroke to the perpendicular of line of stroke.

As a compromise let the whole rotating masses and a fraction 'c' of the reciprocating masses is balanced, such that

$$m_B r_B = (c m_P + m_c) R$$

In this case, the unbalanced force along the line of stroke is

$$\sum F_x = (1 - c) m_P R \omega^2 \cos \theta$$

and unbalanced force perpendicular to the line of stroke is

$$\begin{aligned} \sum F_y &= (m_c R - m_B r_B) \omega^2 \sin \theta \\ &= c m_P R \omega^2 \sin \theta \end{aligned}$$

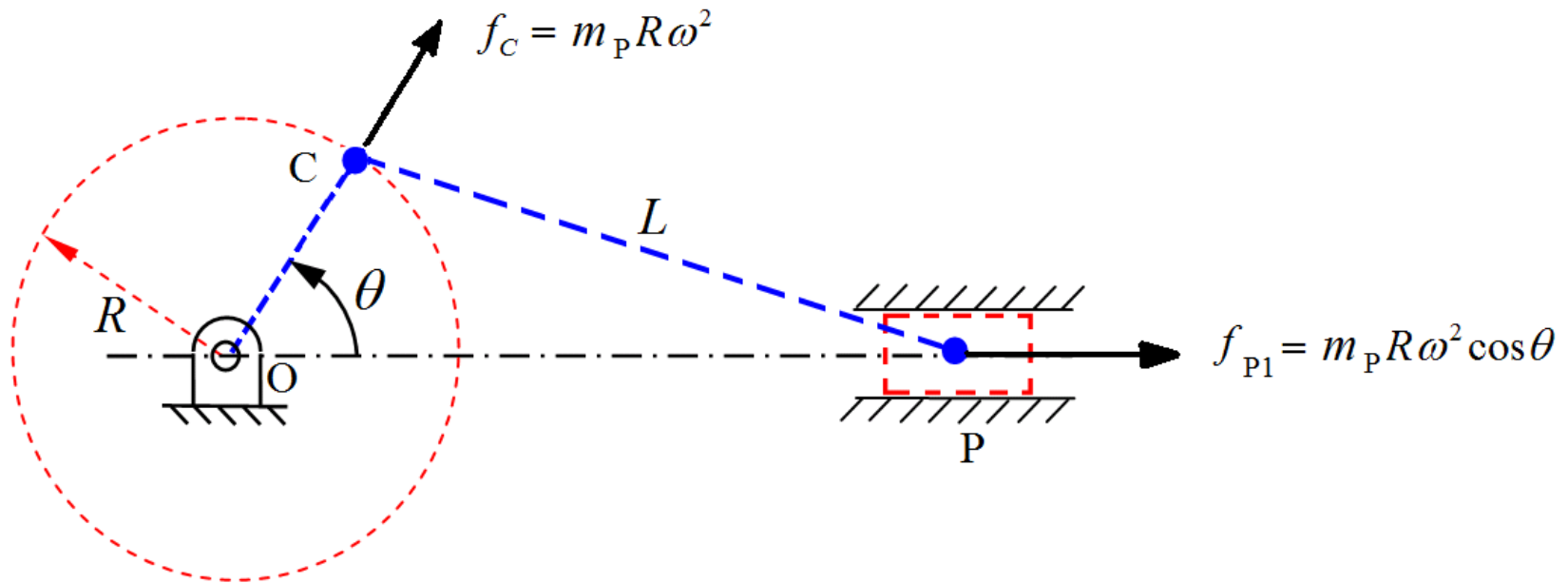
Thus, the resultant unbalanced force is

$$= m_P R \omega^2 \sqrt{(1 - c)^2 \cos^2 \theta + c^2 \sin^2 \theta}$$

# Primary Balancing of Multi-cylinder In-line engines

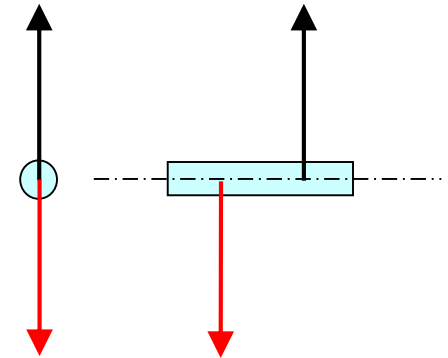
- The multi-cylinder engines with the cylinder center lines in the same plane and on the same side of the center line of the crank-shaft, are known as in-line engines.
- Two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine:
  1. The algebraic sum of the primary forces must be equal to zero.
  2. The algebraic sum of the couples about any point in the plane of primary forces must be equal to zero.

- The primary unbalanced force may be considered as the component of centrifugal force produced by rotating mass  $m_P$  placed at crank radius  $R$  along the line of stroke, as shown in the figure

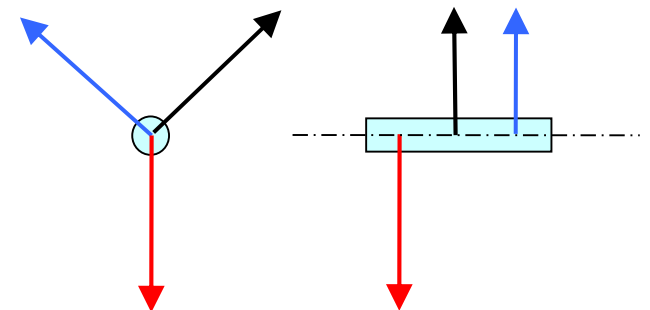


- In order to give the primary balance of a multi-cylinder engine, it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of the revolving masses.

- For a two cylinder engine with cranks at  $180^\circ$ , the first condition may be satisfied, but this will result in an unbalanced couple. Thus, the above method of balancing can not be applied in this case.



- For a three cylinder engine with cranks at  $120^\circ$ , the first condition may be satisfied. However, by taking a reference plane through one of cylinder center lines, two couples with non-parallel axes will remain. Hence, the above method of balancing fails in this case also.



- For a four cylinder engine, similar reasoning will show that complete primary balance is possible and it follows that:
- For a multi-cylinder engine, the primary forces may be completely balanced by suitably arranging the crank angles, provided that the number of cranks are not less than four.

Lets consider the algebraic sum of the primary forces be equal to zero:

$$F_I = \sum_{i=1}^n F_{1i} = 0 \quad \text{at any arbitrary time}$$

For this, we should have:

$$\sum_{i=1}^n m_i R_i \omega^2 \cos(\theta_{ti}) = 0 \quad \text{at any arbitrary time}$$

On the other hand

$$\theta_{ti} = \omega t + \theta_i$$

therefore, we get



$$\sum_{i=1}^n m_i R_i \omega^2 \cos(\omega t + \theta_i) = 0 \quad \text{at any arbitrary time}$$

Substituting to the primary forces, it gives

$$\sum_{i=1}^n m_i R_i \omega^2 \cos(\omega t + \theta_i) = 0 \quad \text{at any arbitrary time}$$

Expanding the trigonometric terms, it yields

$$\left( \sum_{i=1}^n m_i R_i \omega^2 \sin(\theta_i) \right) \cos \omega t + \left( \sum_{i=1}^n m_i R_i \omega^2 \cos(\theta_i) \right) \sin \omega t = 0$$

*at any arbitrary time*

Finally, we get

$$\sum_{i=1}^n m_i R_i \omega^2 \sin(\theta_i) = 0$$

$$\sum_{i=1}^n m_i R_i \omega^2 \cos(\theta_i) = 0$$

Similarly, to balance the inertia couples, it is necessary to have

$$\sum_{i=1}^n m_i R_i a_i \omega^2 \sin(\theta_i) = 0$$

$$\sum_{i=1}^n m_i R_i a_i \omega^2 \cos(\theta_i) = 0$$

# Secondary Balancing of Multi-cylinder In-line engines

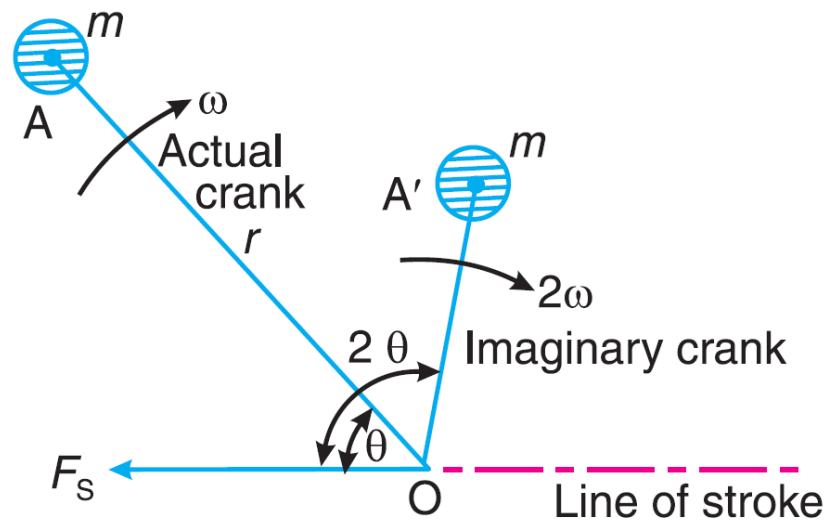
- When the connecting rod is not too long (comparing to crank), then the secondary inertia force due to reciprocating mass arises.
- From previous analysis, the secondary force is

$$F_S = F_{2i} = m \omega^2 \frac{R^2}{L} \text{Cos}(2\theta),$$

- This expression may be written as

$$F_S = F_{2i} = m (2\omega)^2 \frac{R^2}{4L} \text{Cos}(2\theta),$$

- As in the case of primary forces, the secondary forces may be considered as equivalent to the component, parallel to the line of stroke, of the centrifugal force produced by an equal mass placed at the imaginary crank of length  $R^2/4L$  and revolving at twice the speed of actual crank, as shown in the following figure.



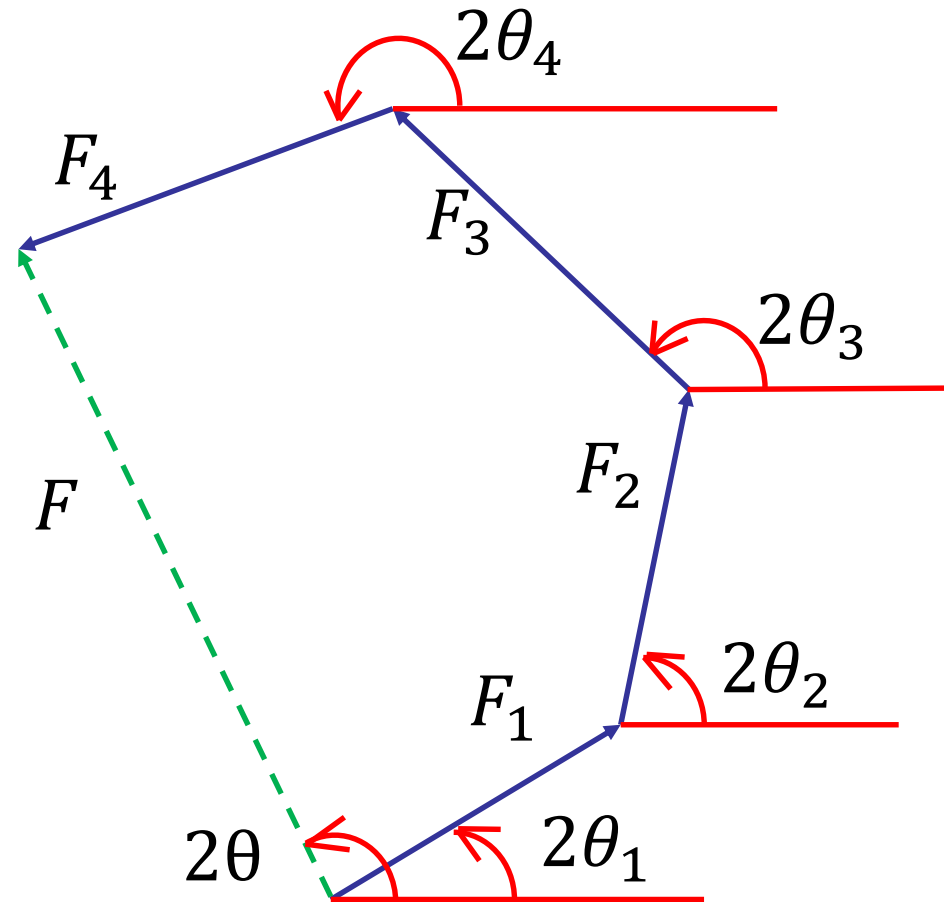
Secondary force.

# Conditions of Secondary Balance

- The following two conditions must be satisfied in order to give a complete secondary balance of an engine:
  1. The algebraic sum of the secondary forces must be equal to zero. In the other word the **secondary force polygon** must be **closed**.
  2. The algebraic sum of the couples about any point in the plane of primary forces must be equal to zero. In the other word the secondary couple polygon must be closed.

# Secondary force polygon

- The closing side of the secondary force polygon ( $F$ ) gives the **maximum unbalanced secondary force**
- The closing side of the secondary couple polygon gives the maximum unbalanced secondary couple.



The algebraic sum of the secondary forces is equal to

$$F_{II} = \sum_{i=1}^n F_i \cos(2\theta_i), \quad F_i = m_i \omega^2 \frac{R_i^2}{L_i}$$

According to the previous figure, the closing side of the secondary forces polygon is defined as follows:

$$F = \left( \sum_{i=1}^n F_i \cos(2\theta_i) \right) i + \left( \sum_{i=1}^n F_i \sin(2\theta_i) \right) j$$

Therefore, it is obvious that  $F_{II}$  is equal to the horizontal component of  $F$ . when the crankshaft rotates, in some time, the angle  $2\theta$  equals to  $180^\circ$  and  $F_{II}$  takes its maximum value.

$$\text{Max}(F_{II}) = |F| =$$

$$\sqrt{\left( \sum_{i=1}^n F_i \cos(2\theta_i) \right)^2 + \left( \sum_{i=1}^n F_i \sin(2\theta_i) \right)^2}$$

# Mathematical derivation

The algebraic sum of the secondary forces is calculated as follows:

$$F_{II} = \sum_{i=1}^n F_{2i} = \sum_{i=1}^n m_i \omega^2 \frac{R_i^2}{L_i} \text{Cos}(2\theta_{ti}),$$

On the other hand

$$\theta_{ti} = \omega t + \theta_i$$

therefore, we get

$$F_{II} = \sum_{i=1}^n F_{2i} = \sum_{i=1}^n m_i \omega^2 \frac{R_i^2}{L_i} \text{Cos}(2\omega t + 2\theta_i),$$



Expanding the trigonometric terms, it yields

$$F_{II} = \sum_{i=1}^n F_{2i} = \left( \sum_{i=1}^n m_i \omega^2 \frac{R_i^2}{L_i} \text{Cos}(2\theta_i) \right) \text{Cos}(2\omega t) \\ + \left( \sum_{i=1}^n m_i \omega^2 \frac{R_i^2}{L_i} \text{sin}(2\theta_i) \right) \text{sin}(2\omega t)$$

Differentiating with respect to time t, after some algebraic operations, we obtain:

$$\text{Max} (F_{II}) = \sqrt{\left( \sum_{i=1}^n m_i \omega^2 \frac{R_i^2}{L_i} \text{Cos}(2\theta_i) \right)^2 + \left( \sum_{i=1}^n m_i \omega^2 \frac{R_i^2}{L_i} \text{sin}(2\theta_i) \right)^2}$$

Similarly we can show

$$M_{II} = \sum_{i=1}^n M_{2i} = \left( \sum_{i=1}^n m_i a_i \omega^2 \frac{R_i^2}{L_i} \cos(2\theta_i) \right) \cos(2\omega t) \\ + \left( \sum_{i=1}^n m_i a_i \omega^2 \frac{R_i^2}{L_i} \sin(2\theta_i) \right) \sin(2\omega t)$$

and

$$\text{Max} (M_{II}) = \sqrt{\left( \sum_{i=1}^n m_i a_i \omega^2 \frac{R_i^2}{L_i} \cos(2\theta_i) \right)^2 + \left( \sum_{i=1}^n m_i a_i \omega^2 \frac{R_i^2}{L_i} \sin(2\theta_i) \right)^2}$$

**Example 22.8.** A four crank engine has the two outer cranks set at  $120^\circ$  to each other, and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. If the engine is to be in complete primary balance, find the reciprocating mass and the relative angular position for each of the inner cranks.

If the length of each crank is 300 mm, the length of each connecting rod is 1.2 m and the speed of rotation is 240 r.p.m., what is the maximum secondary unbalanced force ?

<i>Plane</i> (1)	<i>Mass</i> (m) kg (2)	<i>Radius</i> (r) m (3)	<i>Cent. force</i> $\div \omega^2$ (m.r) kg-m (4)	<i>Distance from</i> plane (2) (l) m (5)	<i>Couple</i> $\div \omega^2$ (m.r.l.) kg-m <sup>2</sup> (6)
1	400	0.3	120	- 0.45	- 54
2(R.P.)	$m_2$	0.3	$0.3 m_2$	0	0
3	$m_3$	0.3	$0.3 m_3$	0.75	$0.225 m_3$
4	400	0.3	120	1.35	162

- Since the engine is to be in complete primary balance, the primary couple polygon must be closed. It gives

$$0.225 m_3 = 196 \text{ kg-m}^2 \quad \text{or} \quad m_3 = 871 \text{ kg}$$

and

$$\theta_3 = 326^\circ$$

- Now, drawing the primary force polygon, we get

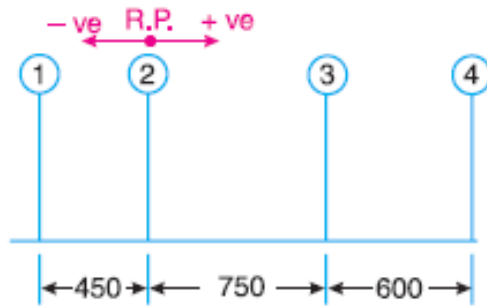
$$0.3 m_2 = 284 \text{ kg-m} \quad \text{or} \quad m_2 = 947 \text{ kg}$$

and

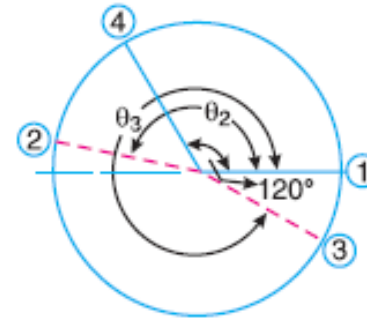
$$\theta_2 = 168^\circ$$

- Thus, maximum unbalanced secondary force is

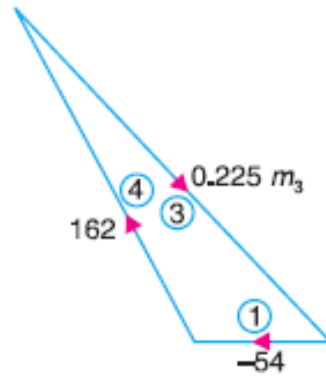
$$= 582 \times \frac{\omega^2}{n} = \frac{582(25.14)^2}{1.2/0.3} = 91\,960 \text{ N} = 91.96 \text{ kN}$$



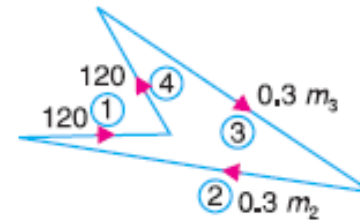
(a) Positions of planes.



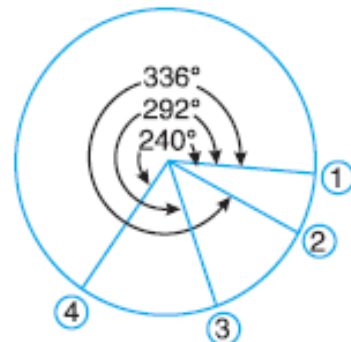
(b) Primary crank positions.



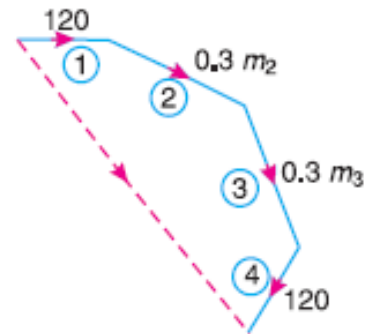
(c) Primary couple polygon.



(d) Primary force polygon.

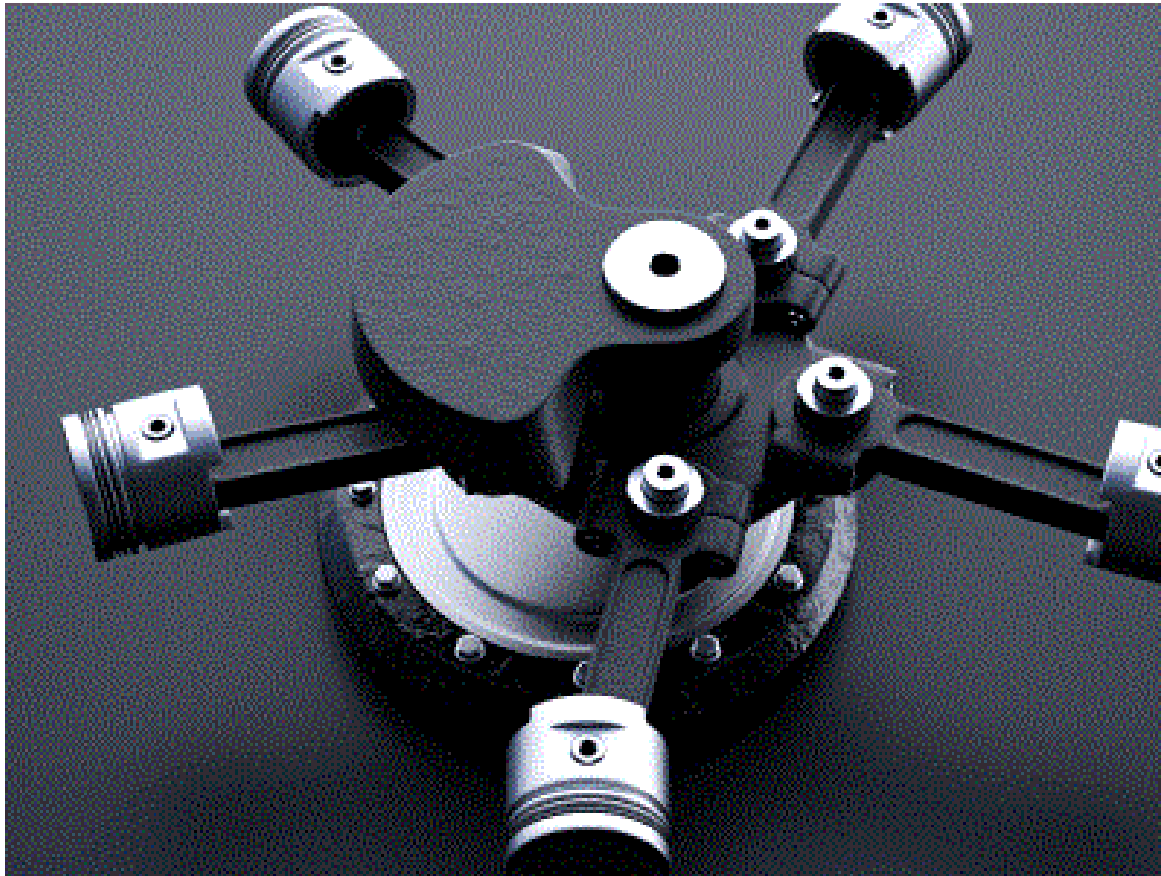


(e) Secondary crank positions.



(f) Secondary force polygon.

# Balancing of V-engines



# Balancing of Radial engines

