Kinematics and Dynamics of Machines

8. Balancing of Rotating Systems

Introduction

- The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore very essential that all the rotating and reciprocating parts should be completely balanced as far as possible.

- If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in various members, but also produce unpleasant and even dangerous vibrations.

- In order to determine the motion of a rigid body, it is usually convenient to replace the rigid body by a dynamically equivalent system.

Dynamically Equivalent systems

- A dynamically equivalent system, is a system of materials which are rigidly connected and exhibit a similar acceleration when subjected to an external force, comparing to the original system.

- In order to determine the motion of a rigid body, it is usually convenient to replace the rigid body by to masses placed at fixed distance apart, in such a way

1. The sum of their masses is equal to the total mass of the body.

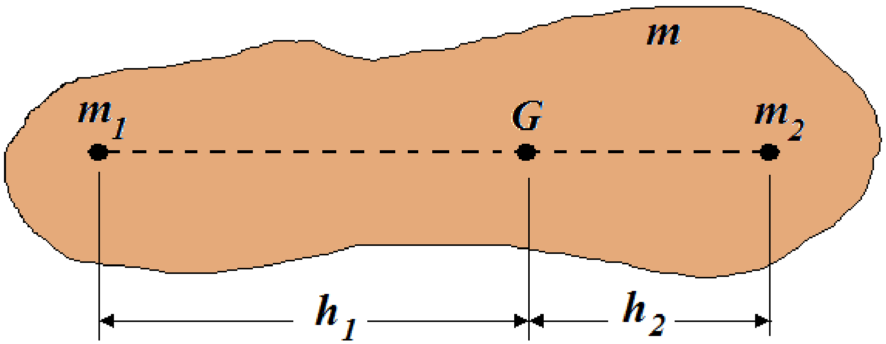
2. The center of gravity of the two masses coincides with that of the original one, and

3. The sum of mass moment of inertia of the masses about their center of gravity is equal to that of original body.

$$m_1 + m_2 = m$$

$$m_1 h_1 = m_2 h_2$$

$$m_1 h_1^2 + m_2 h_2^2 = I$$



Balancing of a single rotating mass

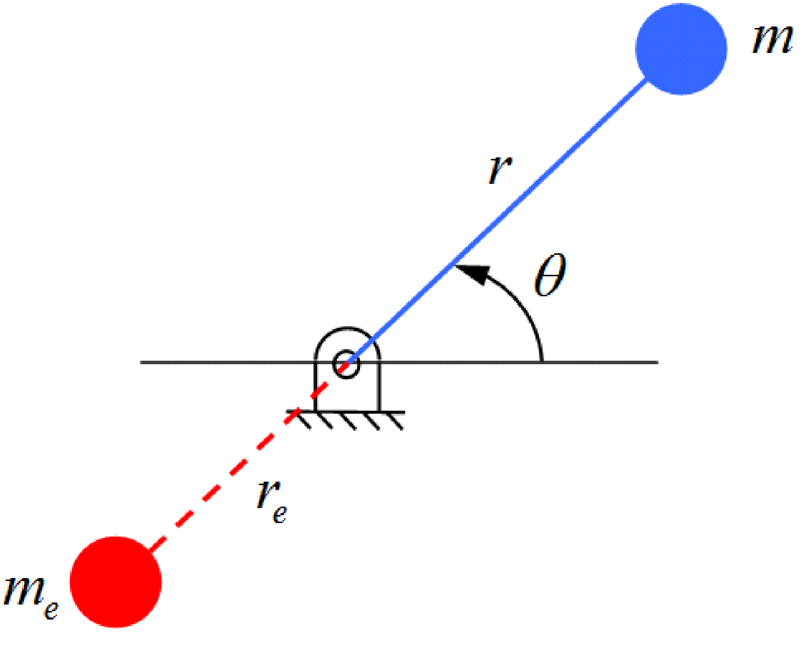
- For the case of a single rotating mass, one can balance the system by adding another mass to the system in the same plane of rotation and at the opposite side of the distributing mass.

- To have a static balance, the moment of mass weights about the axis must be equal to zero. Thus,

$$-m\,g\,r\cos\theta + m_e\,g\;r_e\cos\theta_e = 0$$

or

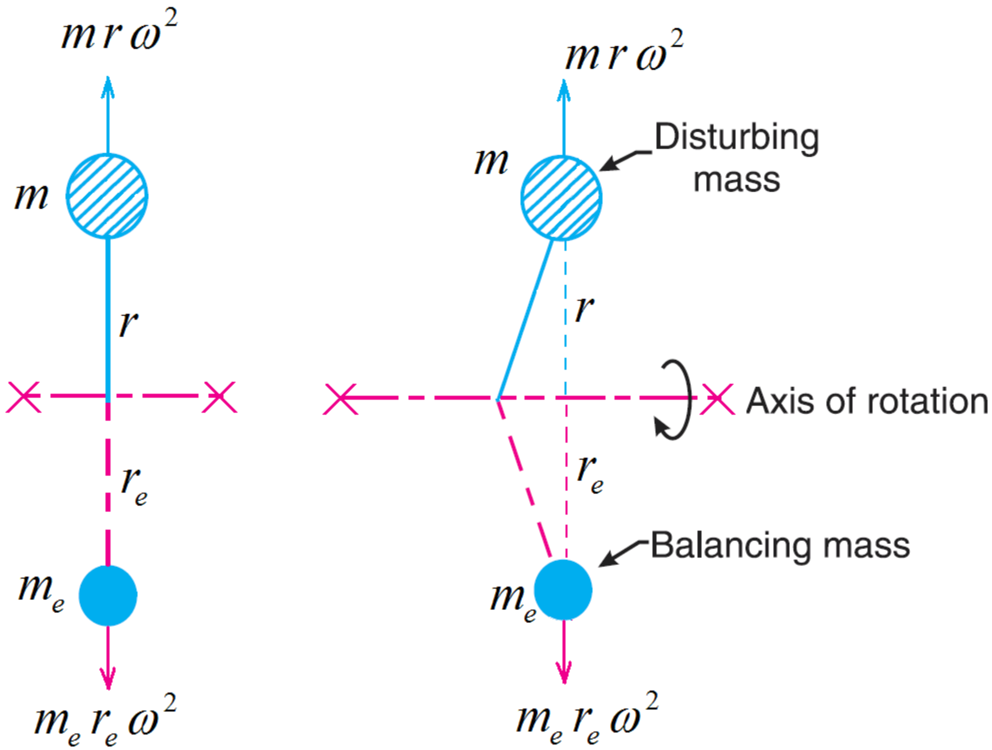
$$m\,r = m_e\,r_e$$



- To have a dynamic balance, the sum of inertia forces (which are centrifugal) and their moments about any specific point must be equal to zero:

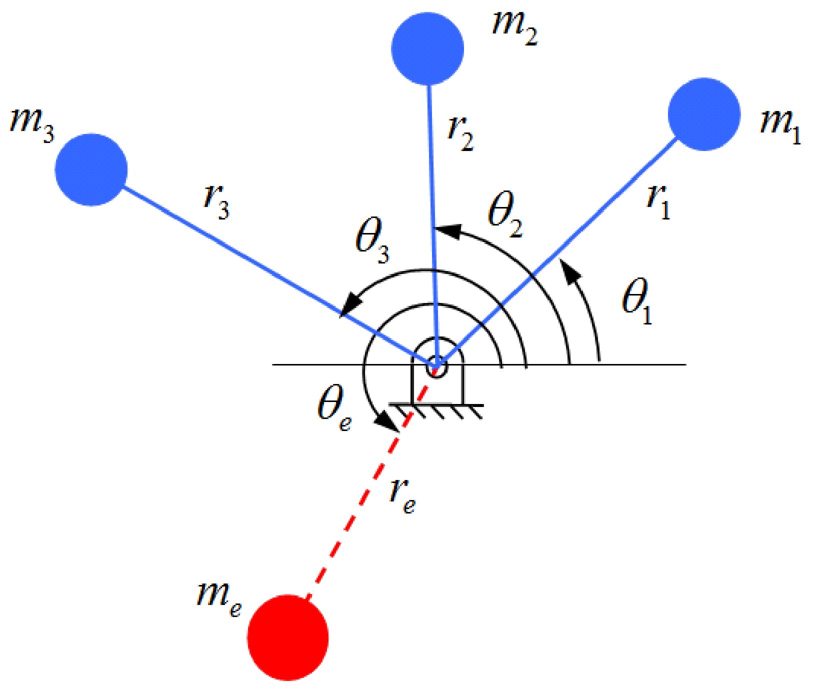
$$m r \omega^2 - m_e r_e \omega^2 = 0 \qquad \text{or} \qquad m r = m_e r_e$$

- Balancing of a single rotating mass by attaching two balancing masses in two different planes parallel to the of rotation of the distributing mass. However, this is not suggested in general, since bending moments and stresses would be produced in the rotating shaft



Balancing of single-plane systems

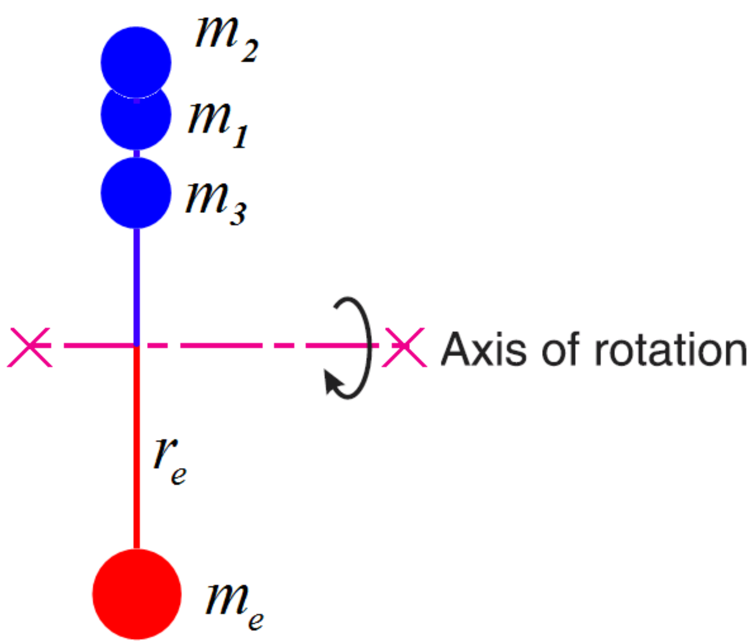
- Consider any number of masses rotating in the same plane as shown in the figure. In order to balance the system dynamically, one can attach a balancing mass to the same plane where the distributed masses are located. The centrifugal force of the balancing mass must counterbalance the centrifugal forces of the distributed masses both in vertical plane and horizontal plane. This gives

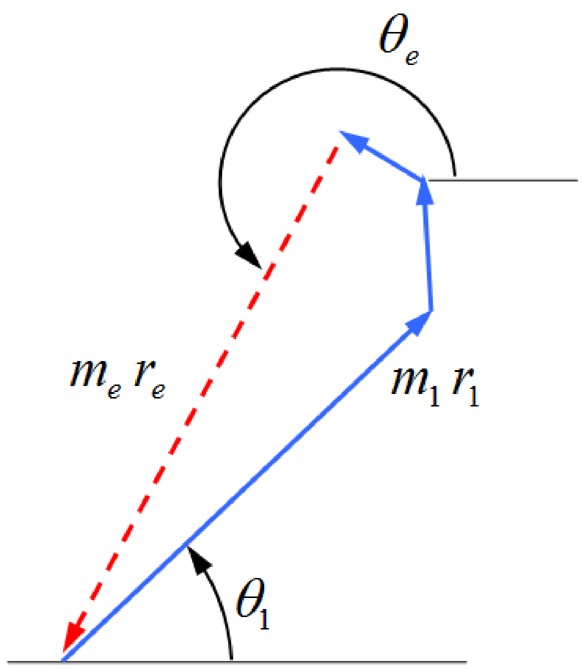


$$\sum_{i=1}^{n} m_i\, r_i\, \omega^2 \cos\theta_i + m_e\, r_e\, \omega^2 \cos\theta_e = 0$$

$$\sum_{i=1}^{n} m_i\, r_i\, \omega^2 \sin\theta_i + m_e\, r_e\, \omega^2 \sin\theta_e = 0$$

where the first equation of the above equations is the necessary condition for static balancing.

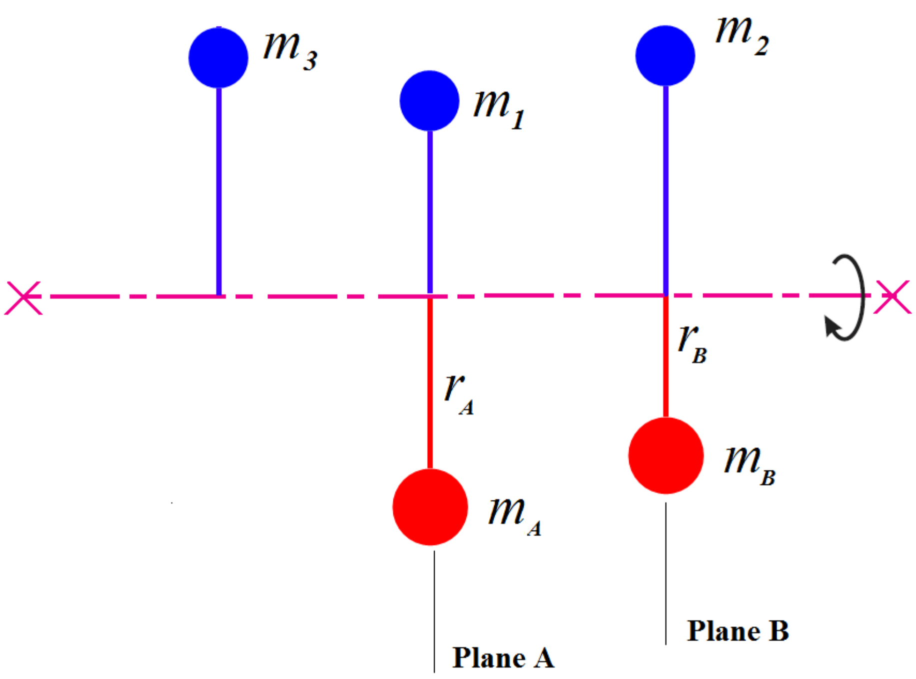




Balancing of multi-plane systems

- When the distributed rotating masses locate in different planes perpendicular to the axis of rotation, dynamical balance of inertia forces is not enough to balance the system totally, since the inertia forces still can generate inertia moments about any specific point of the axis.

- The total rotor system can be balanced by attaching two balancing masses to two arbitrary planes such as planes A and B.

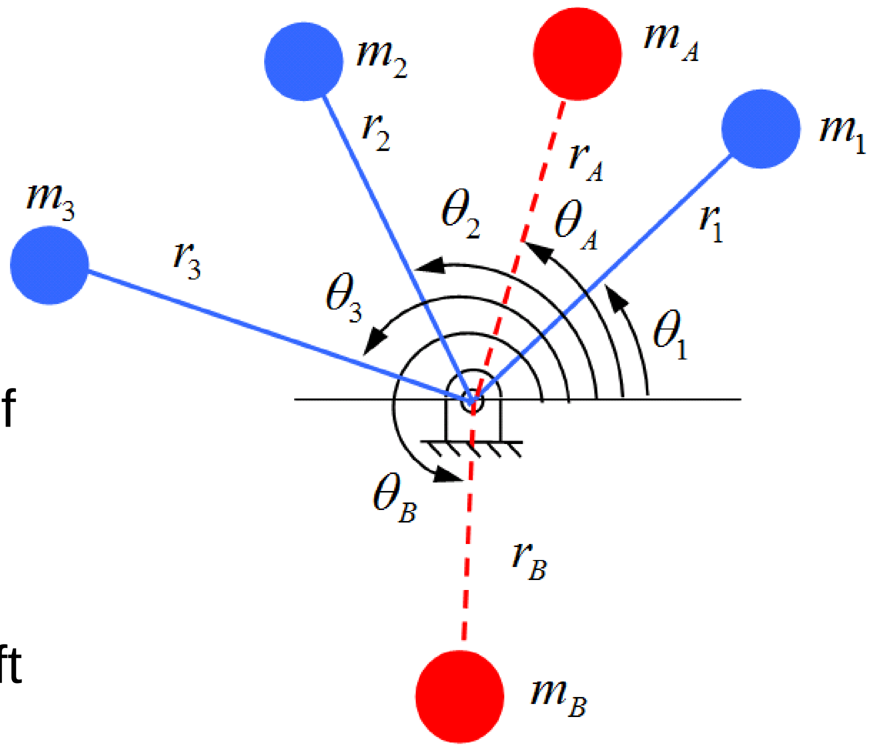


The balancing procedure is done in two steps:

1. A balancing mass is attached to Plane B to counterbalance the inertia moments about vertical and horizontals axes of Plane A. This requirement is satisfied by the following equations:

$$\sum_{i=1}^{n} m_i\, r_i\, a_i \cos\theta_i + m_B\, r_B\, a_B \cos\theta_B = 0$$

$$\sum_{i=1}^{n} m_i\, r_i\, a_i \sin\theta_i + m_B\, r_B\, a_B \sin\theta_B = 0$$



where a_i gives the relative position of distributed masses with respect to Plane A and sign of a_i defines that if the corresponding plane is located in right hand side of Plane A or in the left hand side of it.

2. In order to balance the inertia forces, a balancing mass is attached to Plane A which counterbalances the vertical and horizontal inertia forces. This mass must necessarily be added to Plane A to keep the balance of inertia moments about vertical and horizontal axes of Plane A. The corresponding balancing equations are:

$$\sum_{i=1}^{n} m_i\, r_i \cos\theta_i + m_B\, r_B \cos\theta_B + m_A\, r_A \cos\theta_A = 0$$

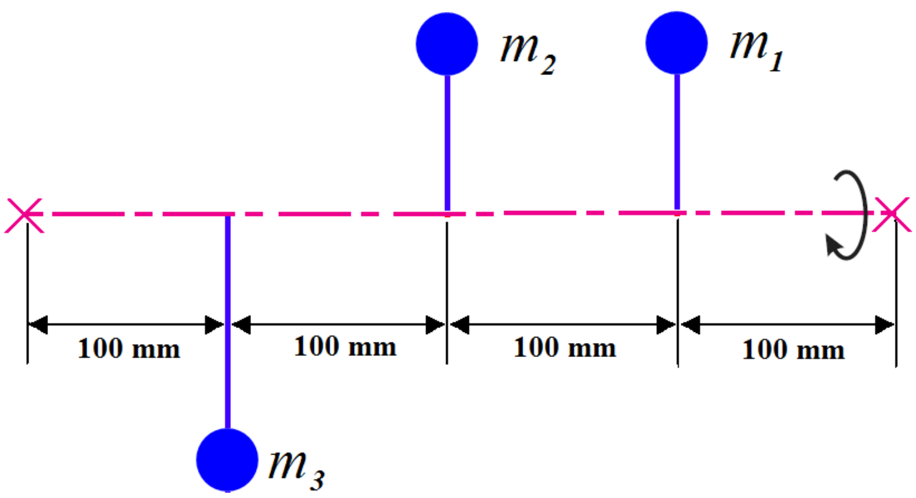
$$\sum_{i=1}^{n} m_i\, r_i \sin\theta_i + m_B\, r_B \sin\theta_B + m_A\, r_A \sin\theta_A = 0$$

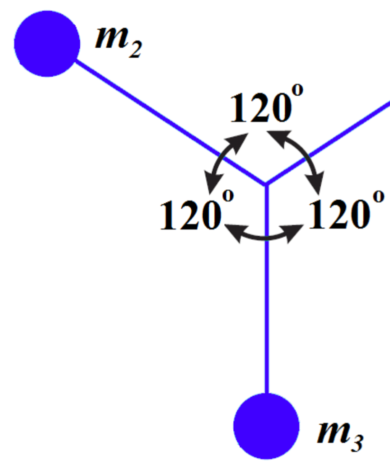
It is usually suggested to place the balancing masses as far as possible from rotor shaft. This, reduces the weight of balancing masses and subsequently reduces the total weight of the system.

Moreover, selecting two distant planes for balancing masses, reduces the weight of required balancing masses.

Although, each rotating system can be balanced by attaching two balancing masses to two different arbitrary points, it is preferred to balance each unbalanced mass in its own plane of rotation t avoid bending moments.

Example: balance the illustrated rotating system by adding to balancing masses





$r_1 = r_2 = r_3 = 0.1\ m$

$m_1 = m_2 = m_3 = 0.2\ kg$