A simple model for gas bubble drag reduction

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A simple stress model has been developed to explain the observations of reduced drag when small gas bubbles are introduced into a turbulent boundary layer. The drag reduction is caused by a combination of density reduction and turbulence modification. The maximum reduction is obtained when the gas volume fraction approaches the bubble packing limit; the medium viscosity also increases markedly in this limit and becomes the important factor in restricting further reduction in drag. The derived analytical expression represents experimental data well.

Drag reduction caused by the introduction of small gas bubbles (microbubbles) in a water turbulent boundary layer has been observed in several experiments conducted in the last decade.[1–6] Even though these measurements have been well documented, no attempts at modeling the physical behavior have appeared in the literature. The present paper is an initial attempt in this direction. A simple stress model, which suggests that the drag reduction occurs because of a combination of density reduction and turbulence modification, is derived and compared favorably with experimental data.

The two U.S. experiments[1,2] sandwich several studies[3–6] undertaken in the Soviet Union. All of the bubble experiments witnessed a decrease in drag ranging from about 50% to 90% compared with nonbubble flows. McCormick and Bhattacharyya[1] introduced hydrogen bubbles into their towed-body boundary layer by the novel means of water electrolysis. At their slowest tow speed they observed approximately 50% drag reduction; however, they experienced difficulties with flow separation because of the body geometry and flow disturbance effects caused by their wire windings. All the other experiments[2–6] introduced bubbles into the turbulent boundary layer by forcing air through a porous plate at the bottom of the boundary layer. These investigators measured 80% to 90% reduction in skin friction under the maximum air injection conditions.

The study by Bogdevich and Evseev[4] is perhaps the most interesting of the Soviet studies. They describe drag reduction results and turbulence modification caused by the presence of *small* bubbles in *large* concentration in water turbulent boundary layers. The size of the bubbles generated by the porous wall are particularly important for their boundary-layer control experiments. Porous plates with pores 1–3 μm in diameter provided optimum results; the use of 50–100 μm pores did not produce drag reduction. The packing or concentration of bubbles is the other essential feature.[4] Their best drag reduction results ($C_f/C_{f0} \approx 0.2$) were achieved when the peak concentration of bubbles approached the packing limit. Under such conditions, they claim that their concentrated bubble layer forms a stable structure promoting turbulence modification.

The common threads from all of the experimental studies—small bubbles ($< 100\ \mu$m), high concentration of bubbles, turbulent boundary layer flow—are now incorporated into a simple model for "microbubble" drag reduction. We will treat the two-phase flow as a medium with molecular and turbulent viscosities. Both viscosities are important in the turbulent flow considered herein. The medium density is also an essential feature; in fact, the decrease of the medium density as the gas concentration increases provides the primary drag reduction mechanism.

We begin the analysis by examining a schematic of the boundary layer (gas) concentration profile in Fig. 1. Since the gas bubbles are injected into the bottom of the boundary layer ($y = 0$), one would expect the gas concentration α_1 to be highest near the wall and decay toward the boundary-layer edge. The maximum concentration , $\alpha_1 = \alpha_{\max}$, is denoted by α. The Soviet measurements,[4] taken with a concentration probe, reveal such a peaked boundary layer distribution near the wall. The concentration profile near the wall is approximated to be the constant maximum concentration α as shown in Fig. 1. How does such a bubble layer reduce the shear in the turbulent boundary layer?

The shear stress in a two-dimensional turbulent flow is given by

$$\tau = \mu \frac{\partial \overline{U}}{\partial y} - \rho\, \overline{U'V'}, \tag{1}$$

where μ is the molecular viscosity, $\overline{U}$ is the mean velocity, ρ is the density, and $-\overline{U'V'}$ is the Reynolds stress. If the Reynolds stress is postulated to be governed by a gradient diffusion approximation,

$$\overline{U'V'} = -q\Lambda \frac{\partial \overline{U}}{\partial y}, \tag{2}$$

where $q\Lambda$ is an eddy viscosity with q^2 the turbulence energy and Λ the energy-containing turbulent scale, then by substitution,

$$\tau = (\mu + \rho q \Lambda) \frac{\partial \overline{U}}{\partial y}. \tag{3}$$

Note that μ is retained in the present context; in most single phase turbulent flows the turbulent diffusivity dominates that laminar value (i.e., $\rho\, q\, \Lambda \gg \mu$) and the viscosity term is neglected. We will show that the shear stress is substantially reduced in concentrated bubble-layer boundary layers because the reduction in both density and the (kinematic) eddy viscosity $q\Lambda$ overwhelms the increase in medium viscosity μ under concentrated bubble conditions.

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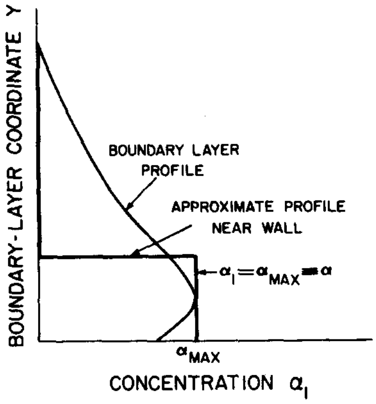

FIG. 1. Bubble layer gas concentration profile.

Equation (3) is typically used in common single-phase turbulent flows. We shall also apply (3) to the bubble-modified turbulent boundary layer under study and indicate so by adding a subscript (1) to all the elements in Eq. (3). Furthermore, the bubble-modified stress τ_1 can be ratioed to the standard hydrodynamic boundary layer (unsubscripted) stress τ as follows:

$$\frac{\tau_1}{\tau} = \frac{(\mu_1/\mu + \rho_1 q_1 \Lambda_1/\mu)(\partial \bar{U}/\partial y)_1}{(1 + \rho q \Lambda/\mu)(\partial \bar{U}/\partial y)}. \tag{4}$$

The critical postulate of the model is now made. It is postulated that the molecular and turbulent medium is altered when large bubble concentrations are introduced into the turbulent boundary layer, but that the velocity gradient near the wall is unchanged. This appears to be true near the outer portions of the boundary layer,[6] but evidence near the wall is not available to substantiate or deny this approximation.

The invariant velocity gradient postulate necessitates some additional discussion since it is such a bold approximation. How can it be improved and validated? An initial rational approach to this issue should employ boundary-layer theory in the form of the Karman integral equation and a separate mass-balance integral to relate the injected gas flow to the concentration profile. The use of these relations would transform the phenomenological model into a predictive one permitting the assessment of parameter variations such as free-stream velocity and injected gas flow rate. Boundary-layer modeling would also serve to identify another matter not considered in the present model: the significance of the location of the maximum gas concentration within the layer. It is useful to remember these points as we pursue the phenomenological model using this key approximation.

In order to evaluate Eq. (4) with the velocity gradients cancelled, the standard and bubble-modified medium must be characterized. The mixture density is defined as

$$\rho_1 = (1-\alpha)\rho + \alpha\rho_g, \tag{5}$$

where ρ is the liquid density and ρ_g is the gas density. Since the gas density in the bubbles is a thousand times smaller than the liquid density, $\rho_g \ll \rho$, Eq. (5) becomes simply $\rho_1 = (1-\alpha)\rho$. The next approximation involves the denominator of Eq. (4). For fully developed turbulent boundary layers, the eddy viscosity $\rho q \Lambda$ exceeds the molecular viscosity by several orders of magnitude, i.e., unity can be neglected in comparison to the viscosity ratio. Consequently,

$$\frac{\tau_1}{\tau} = \frac{(\mu_1/\mu)}{(\rho q \Lambda/\mu)} + (1-\alpha)\frac{q_1\Lambda_1}{q\Lambda}. \tag{6}$$

A typical value for the eddy viscosity/molecular viscosity $(\rho q \Lambda/\mu)$ is 10^3. The viscosity of the modified medium is more difficult to evaluate. Models of bubble behavior under concentrated conditions with the gas volume fraction approaching the packing limit have not been developed. Even the simpler ideal solid sphere suspension case lacks a theoretical solution. Experimental relative viscosity data[7] reveals that μ_1/μ for solid particle suspensions can approach 100 at a particle volume fraction of 0.7. Similar results are found for gas bubble dispersions. An empirical relationship obtained by Sibree[8] is used in the present analysis:

$$\mu_1/\mu = 1/(1 - 1.09\alpha^{1/3}). \tag{7}$$

Equation (7) is valid for froths and for dispersions of gas bubbles of wide size distribution in liquids. At a value of $\alpha = 0.75$ (near the packing limit), $\mu_1/\mu = 119$ from Eq. (7). Consequently, it seems evident that molecular viscosity effects could potentially dominate the turbulent behavior under limiting conditions. The resulting expression for the shear stress then becomes

$$\frac{\tau_1}{\tau} = 10^{-3}\left(\frac{1}{1-1.09\alpha^{1/3}}\right) + (1-\alpha)\frac{q_1\Lambda_1}{q\Lambda}. \tag{8}$$

This relation shows that the shear stress reduction depends on the maximum concentration α and the turbulence modification q_1/q and Λ_1/Λ. The first term in Eq. (8) is referred to as the viscosity term, whereas the second term is the density-turbulence term. The two terms of Eq. (8) are shown as a dashed line (density-turbulence term) for two different cases and a dotted line (viscosity term) in Fig. 2. The solid lines (case 1 and case 2) represent the sum of the two terms. Case 1 considers the shear stress variation with no turbulence modification, whereas case 2 involves a specific approximation for

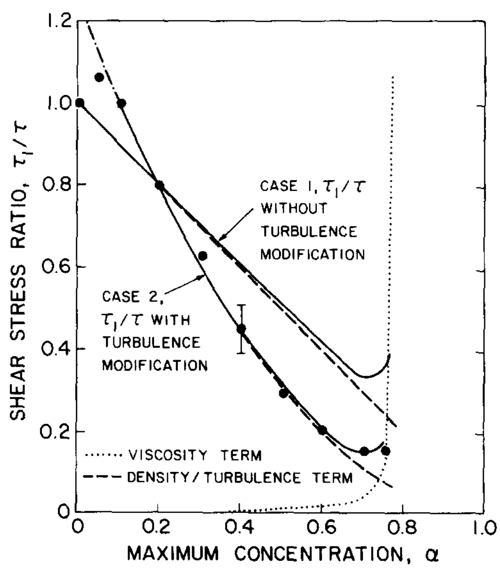

FIG. 2. Drag reduction model compared with experimental data of Bogdevich and Evseev[4] under their highest Reynolds number conditions ($Re = 8 \times 10^6$).

the turbulence modification.

The situation without turbulence modification (case 1) illustrates that a reduction in medium density as the bubble concentration increases provides a shear reduction in direct relation with the density except near the limiting concentration. As α approaches the packing limit, i.e., $\alpha \approx 0.6$, the Sibree viscosity term begins to influence the shear stress variation. In fact, the viscosity serves to limit the drag reduction. The minimum value of τ/τ_1 for this case is about 0.32, or a 68% drag reduction. The comparison between case 1 and the highest Reynolds number experimental data of Bogdevich and Evseev[4] is encouraging but lacks an essential ingredient: the influence of turbulence modification in the boundary layer because of the presence of a high concentration of bubbles.

Without turbulence modification, $q_1\Lambda_1/q\Lambda$ is unity and the shear stress depends on the sum of the viscosity and density terms as just described. How would the turbulence be expected to change under high bubble concentration conditions? The picture is one of bubbles in close contact with several neighbors, i.e., a close packed bubble medium with liquid squeezed between the bubbles. Such a medium would possess a great resistance to bulk motion and, as Taylor[9] has stated, confer volume elasticity and volume viscosity on the fluid. Consequently, it is realistic to suggest that the eddy viscosity $(q_1\Lambda_1)$ becomes modified by a bulk viscosity mechanism with a value that decreases in direct proportion to the density of the (bulk) medium, i.e., with $(1-\alpha)$. Hence

$$\frac{q_1}{q}\frac{\Lambda_1}{\Lambda} = \frac{1-\alpha}{\alpha_{\lim}}, \tag{9}$$

where the constant of proportionality is the inverse of the limiting packing fraction ($\alpha_{\lim} = 4/5$). The aphysical limit of $q_1\Lambda_1/q\Lambda = 5/4$ for $\alpha = 0$ results because the model curves were matched at $\alpha = 0.2$ in concert with the experimental data. Nevertheless, this choice even leads to a reasonable representation of the data for α less than 0.2.

Because of the presence of the bubbles, the bulk viscosity of the medium becomes significant. Furthermore, the presence of a substantial bulk viscosity term in the governing fluid dynamic equations leads to a turbulence energy equation with a new term that is wholly dissipative. Hence, the turbulence energy q_1^2 is suppressed and the modified eddy viscosity is reduced with increasing bubble concentration. Equation (9) exhibits this behavior. It is rather interesting to point out that the specific form for (9) appears to have theoretical merit based on use of the Prandtl mixing-length concept and appeal to the continuity equation for the fluctuating two phase medium.

The Prandtl mixing-length concept implies that the eddy viscosity ratio in (9) can be rewritten as a turbulence energy ratio if use of the constant velocity gradient approximation is made. Furthermore, if the dominant term in the turbulence energy equation is taken to be the bulk viscosity term [i.e., $\nu_{\text{bulk}}\ \overline{(\Delta')^2}$], where Δ' is the fluctuating dilatation and ν_{bulk} is the bulk viscosity, then employing the Batchelor[10] relation for ν_{bulk} of a dilute suspension of identical spherical bubbles provides for $q_1^2 \sim K\nu\, d\alpha/dt$, where K is a constant and ν is the kinematic viscosity. Appealing to the continuity equation for the two-phase medium and using an approximate expression for the standard q^2 then leads to $q_1^2/q^2 \sim (1-\alpha)$ and Eq. (9).

The comparison between case 2, which includes the bulk turbulence modification, and the experimental data[4] is excellent. The incorporation of Eq. (9) into the shear stress relation (8) results in a "rotation" of the model (case 1) in a manner that appears entirely consistent with data including the viscosity cutoff because the packing limit is approached. The final relation for the bubble-modified shear stress is

$$\frac{\tau_1}{\tau} = 10^{-3}\left(\frac{1}{1-1.09\alpha^{1/3}}\right) + \frac{(1-\alpha)^2}{\alpha_{\lim}}. \tag{10}$$

Figure 2 shows that microbubble drag reduction is the result of three factors: (i) reduction of density, (ii) increase of bubble suspension viscosity as the concentration approaches the packing limit, and (iii) turbulence modification of the boundary layer. The density factor $(1-\alpha)$ controls the drag reduction; only at the largest values of α does the viscosity become important enough to limit the drag reduction. Turbulence modification through the additional $(1-\alpha)$ factor in Eq. (10) alters the picture sufficiently to corroborate the available data.[4] Microbubbles play a special role in the drag reduction mechanisms just described. They are needed as the "carriers" that reduce the medium density in a stable manner.[4,11]

In summary, the simple model presented should be valuable for two important purposes. First, Eq. (10) represents an excellent empirical correlation of drag reduction data. And second, since it is based on sound physical thinking, it should also be useful in suggesting approaches to more sophisticated models of gas bubble drag reduction. A rather noteworthy aspect of the modeling is the apparent success of the *bulk* viscosity argument in explaining the turbulence modification part of the drag reduction. This concept deserves further detailed study since it is a purely dissipative mechanism leading to a reduction of turbulence energy without increasing shear in the boundary layer.

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