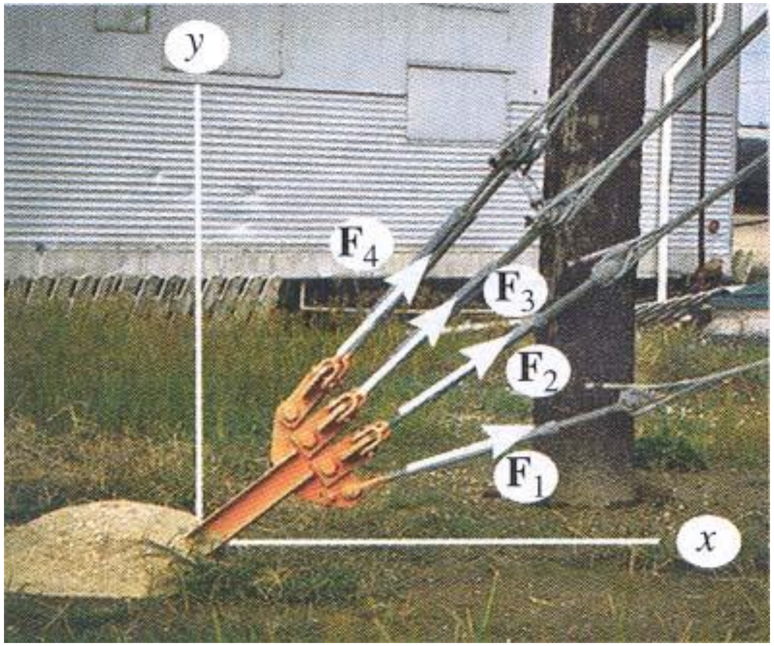
Chapter 2
Force Vectors

APPLICATION OF VECTOR ADDITION



There are four concurrent cable forces acting on the bracket.

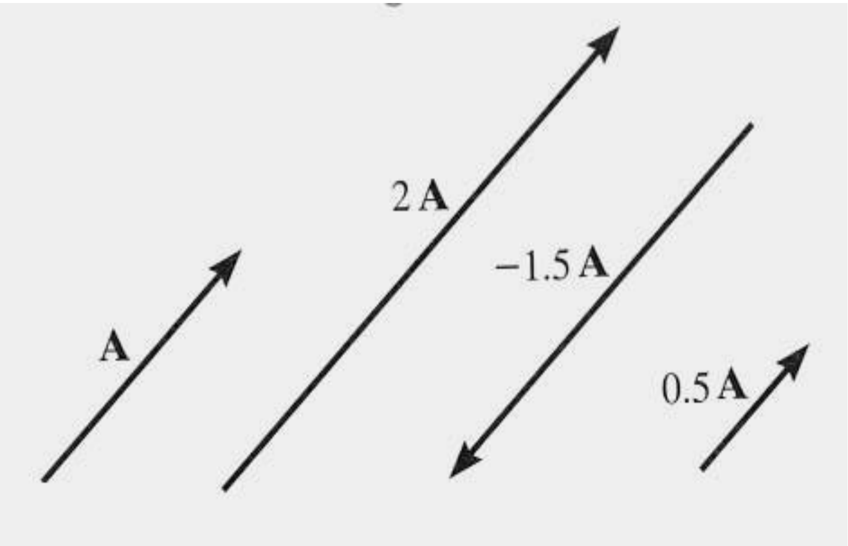
How do you determine the resultant force acting on the bracket ?

SCALARS AND VECTORS

	Scalars	***Vectors***
Examples:	mass, volume	force, velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Parallelogram law
Special Notation:	None	Bold font, a line, an arrow or a “carrot”

In the PowerPoint presentation vector quantity is represented ***Like this*** (in **bold**, *italics*, and black).

VECTOR OPERATIONS

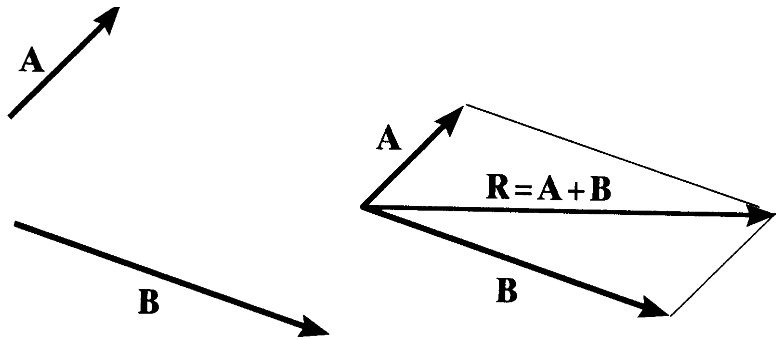


Scalar Multiplication

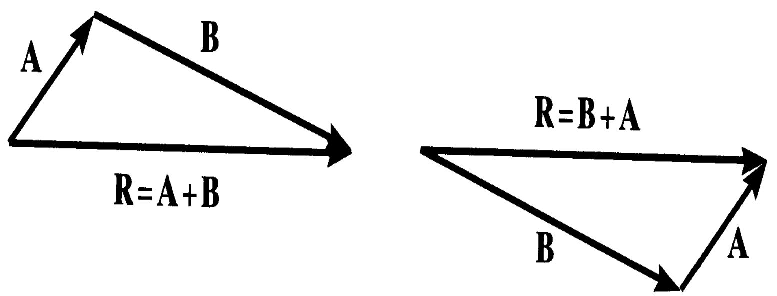
and Division

VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

Parallelogram Law:



Triangle method
(always 'tip to tail'):

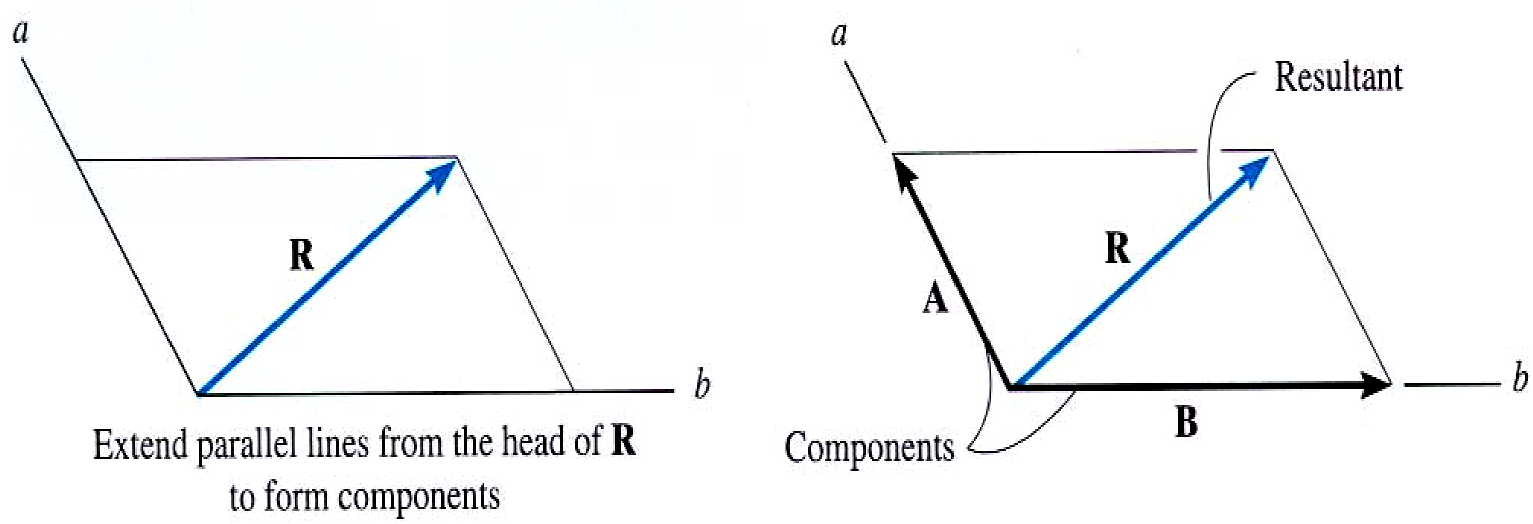


How do you subtract a vector?

How can you add more than two concurrent vectors graphically ?

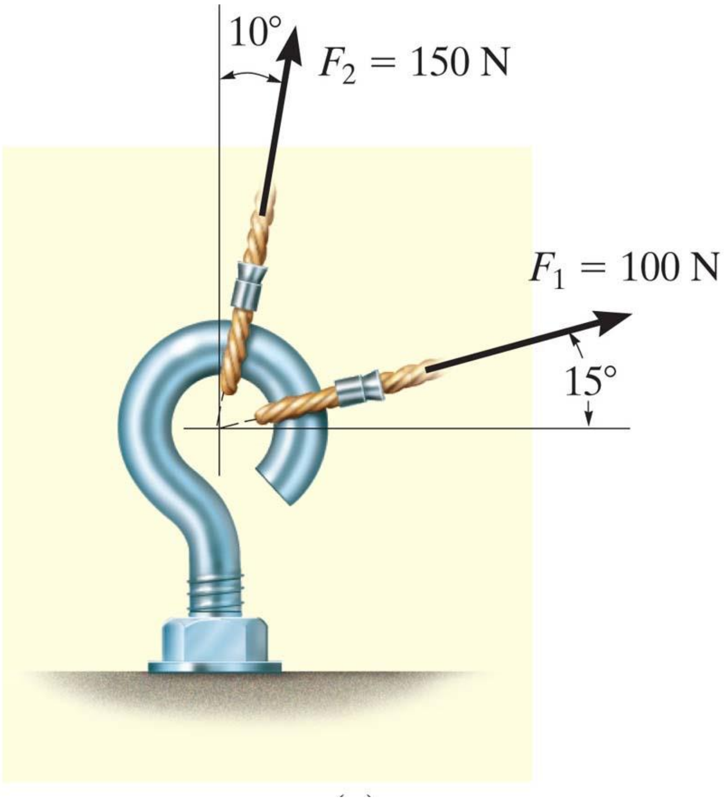
RESOLUTION OF A VECTOR

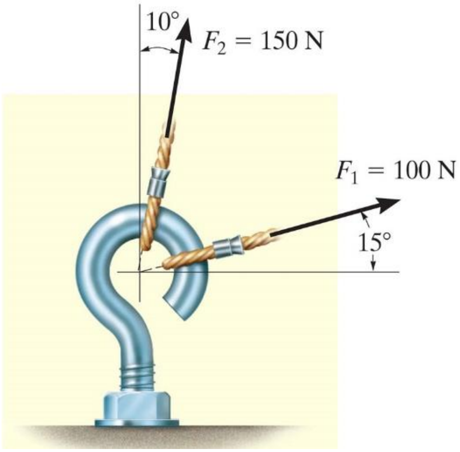
"Resolution" of a vector is breaking up a vector into components. It is kind of like using the parallelogram law in reverse.

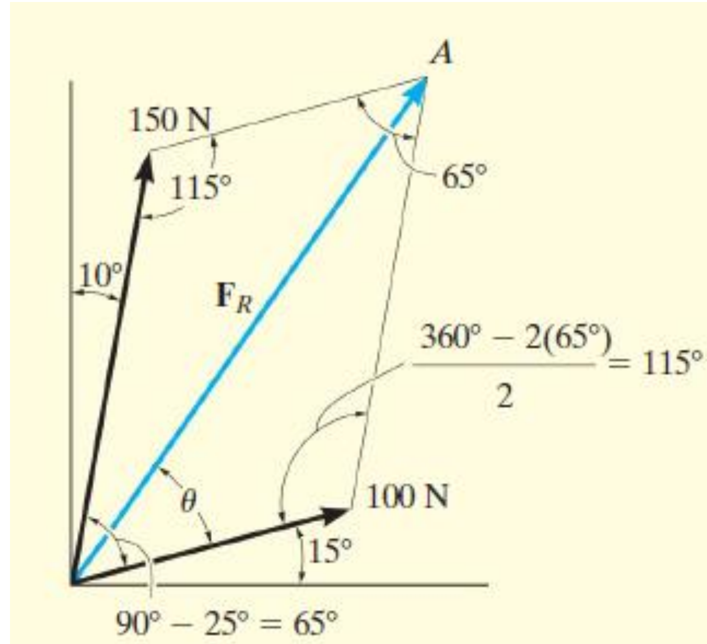


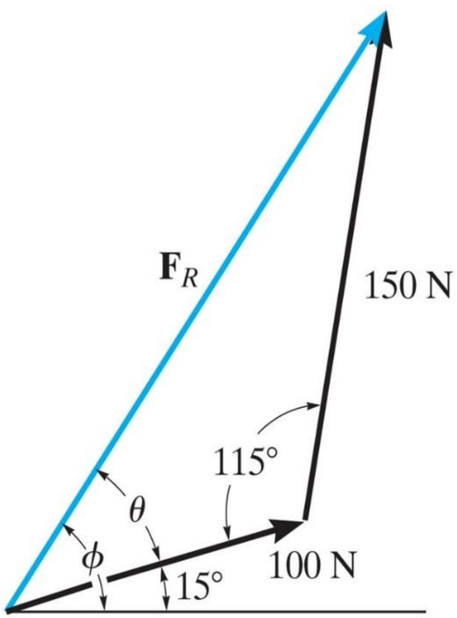
EXAMPLE

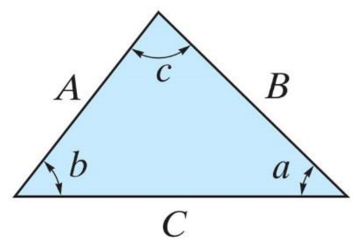
The screw eye is subjected to two forces F_1 and F_2. Determine the magnitude and direction of the resultant force.











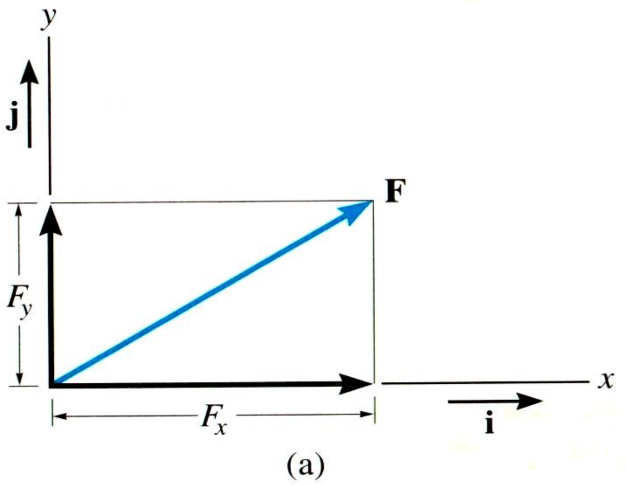
Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

CARTESIAN VECTOR NOTATION



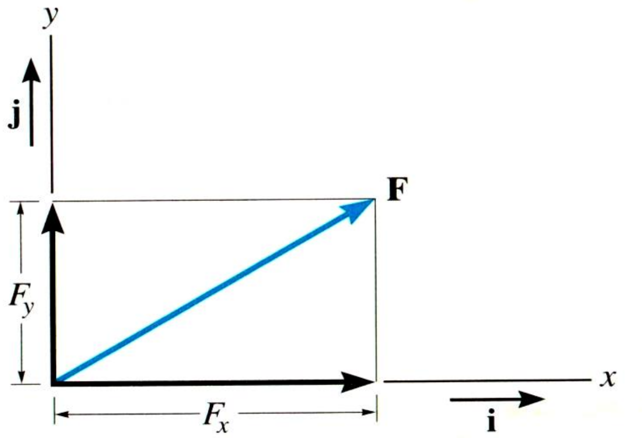
- We ‘resolve’ vectors into components using the x and y axes system.

- Each component of the vector is shown as a magnitude and a direction.

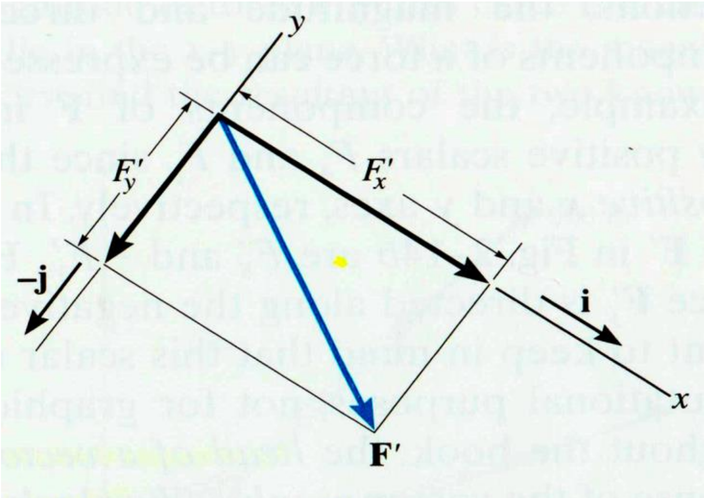
- The directions are based on the x and y axes. We use the “unit vectors” ***i*** and ***j*** to designate the x and y axes.

For example,

$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j}$ ***or*** $\boldsymbol{F'} = F'_x\,\boldsymbol{i} + F'_y\,\boldsymbol{j}$



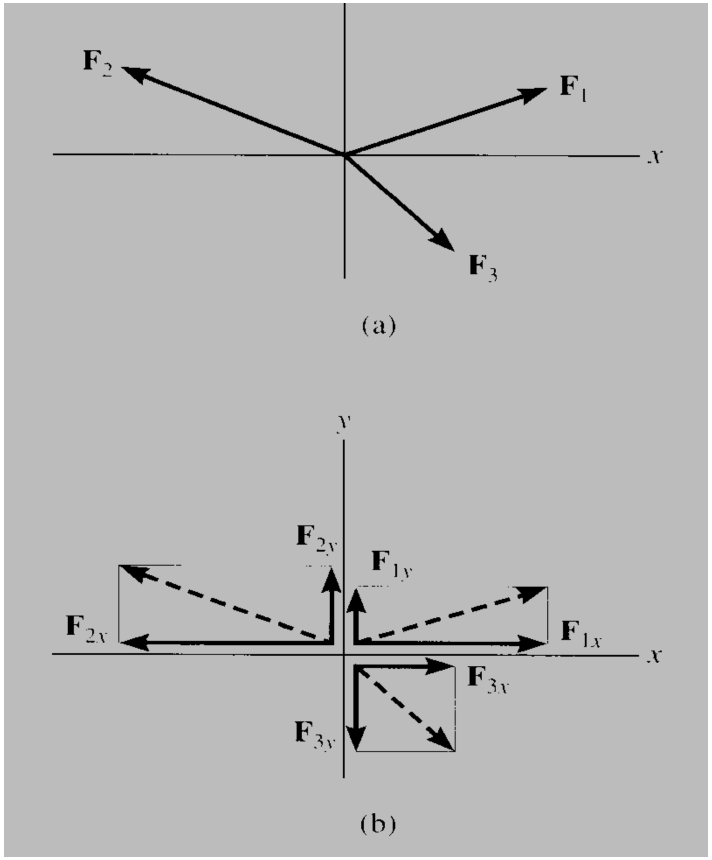
(a)



(b)

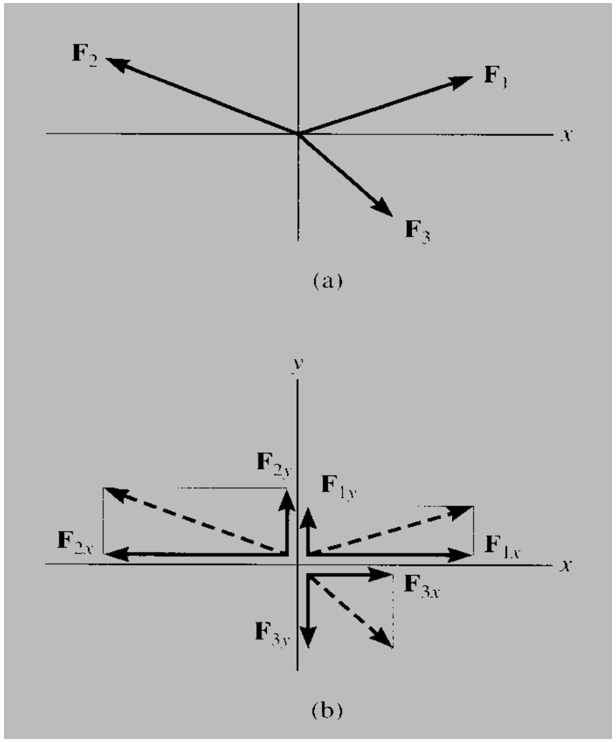
The x and y axes are always perpendicular to each other. Together,they can be directed at any inclination.

ADDITION OF SEVERAL VECTORS



- Step 1 is to resolve each force into its components
- Step 2 is to add all the x components together and add all the y components together. These two totals become the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

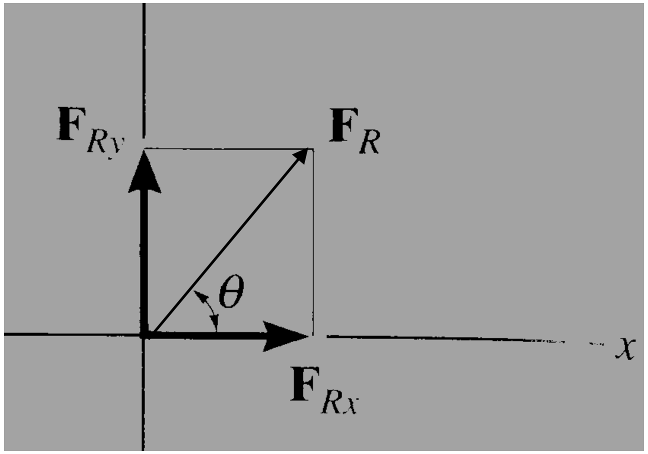
Example of this process,



(a)
(b)

$$
\begin{aligned}
\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\
&= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\
&= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\
&= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}
\end{aligned}
$$

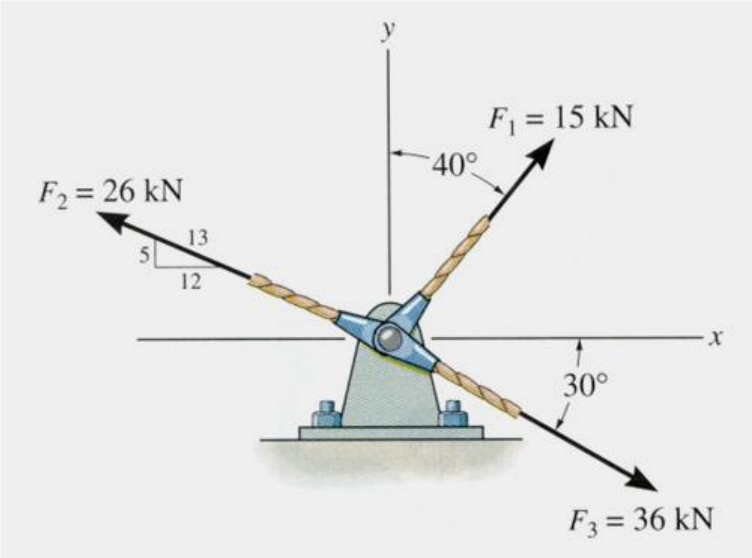
You can also represent a 2-D vector with a magnitude and angle.



$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

EXAMPLE



Given: Three concurrent forces acting on a bracket.

Find: The magnitude and angle of the resultant force.

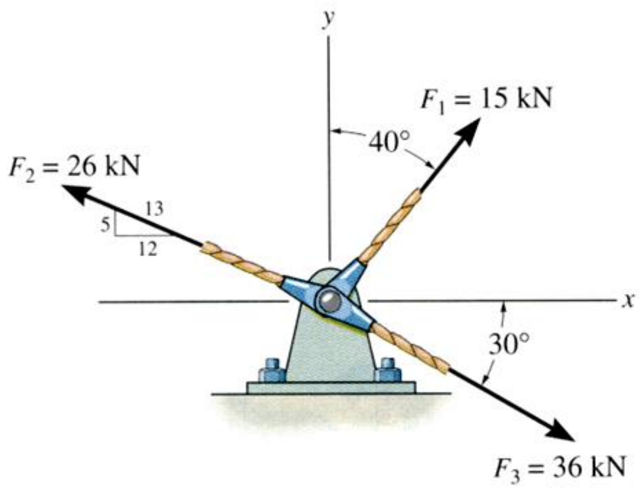
Plan:

a) Resolve the forces in their x-y components.

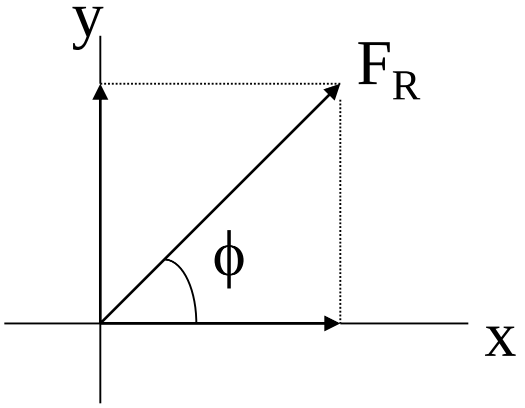
b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

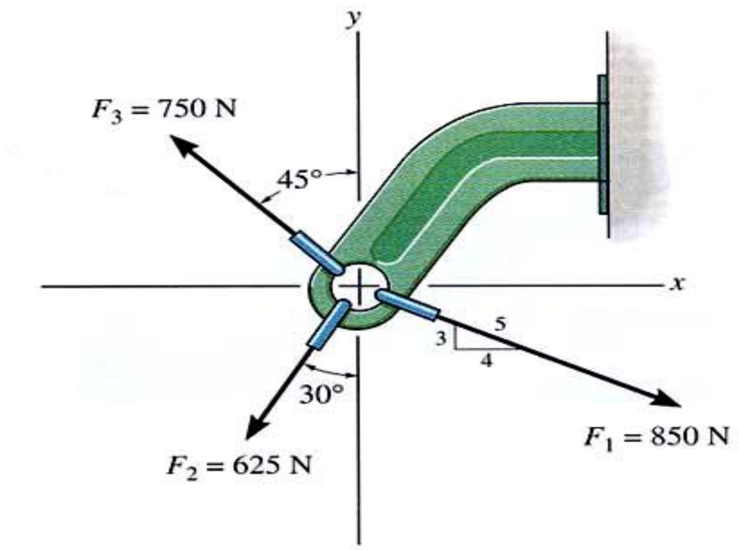
EXAMPLE (continued)



EXAMPLE
(continued)



EXAMPLE



Given: Three concurrent forces acting on a bracket

Find: The magnitude and angle of the resultant force.

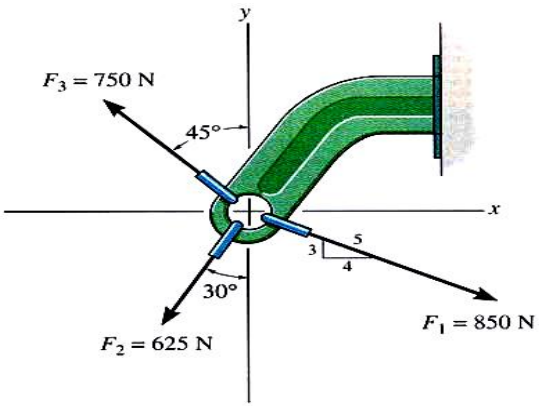
Plan:

a) Resolve the forces in their x-y components.

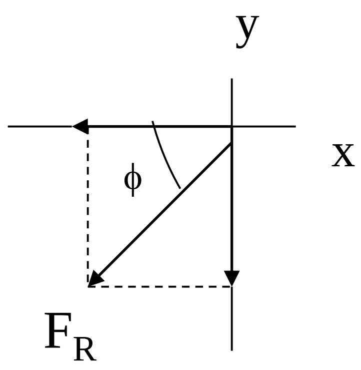
b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

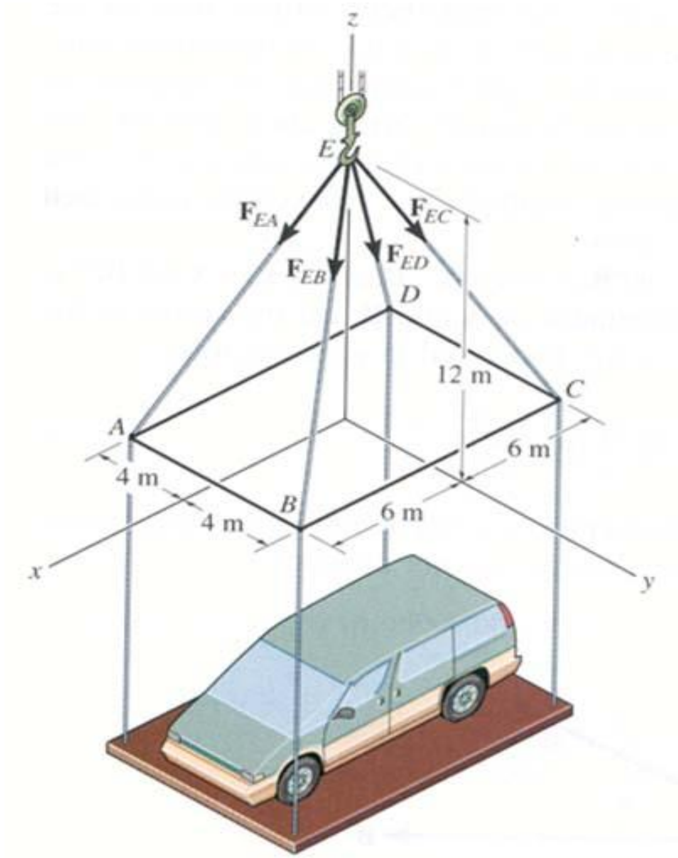
EXAMPLE



EXAMPLE



APPLICATIONS

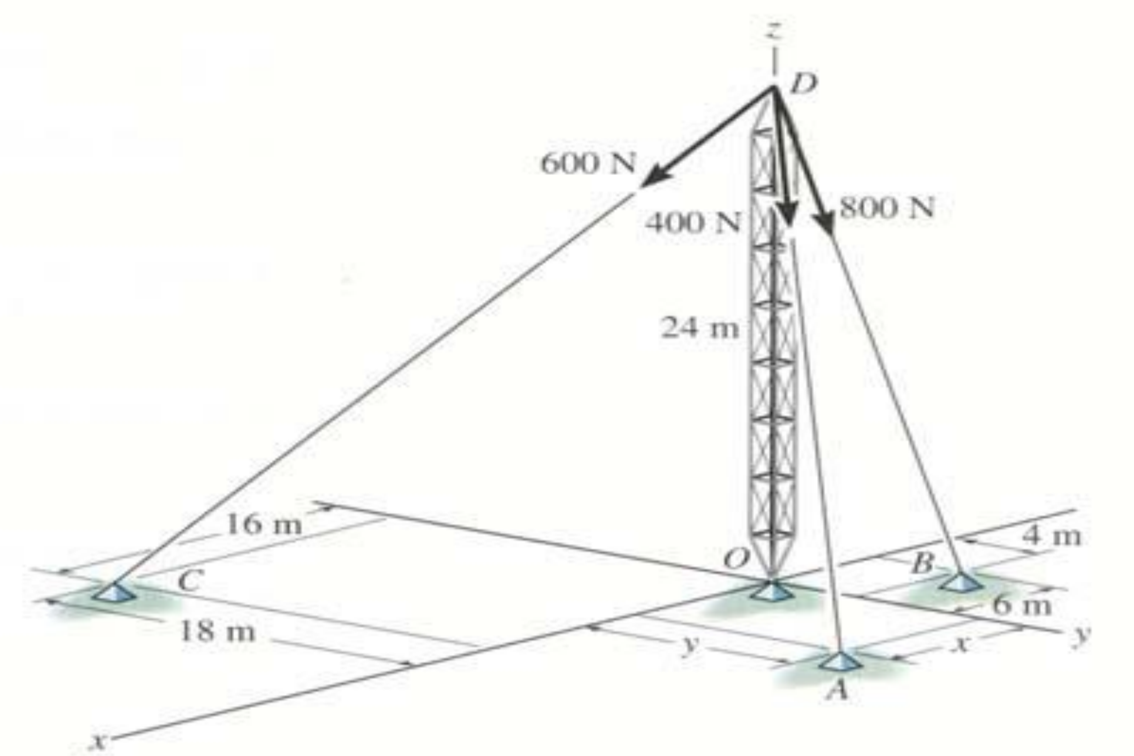


Many problems in real-life involve 3-Dimensional Space.

How will you represent each of the cable forces in Cartesian vector form?

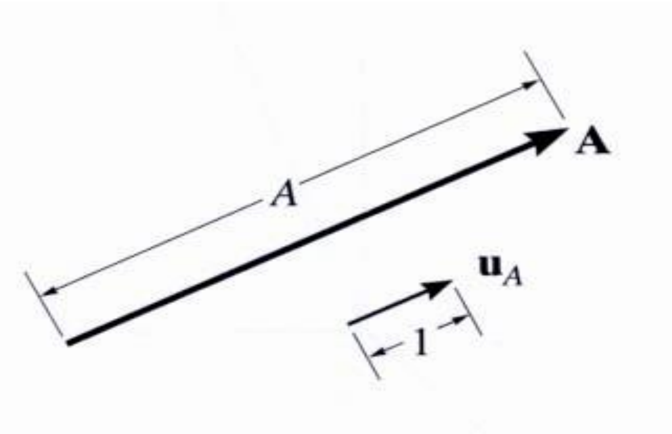
APPLICATIONS
(continued)

Given the forces in the cables, how will you determine the resultant force acting at D, the top of the tower?



A UNIT VECTOR

For a vector ***A*** with a magnitude of A, an unit vector is defined as $\boldsymbol{U}_A$ **=** ***A*** / A .

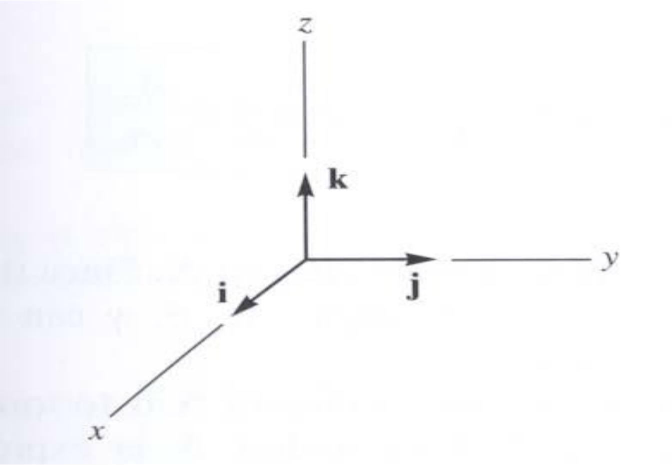


Characteristics of a unit vector**:**

a) Its magnitude is 1.

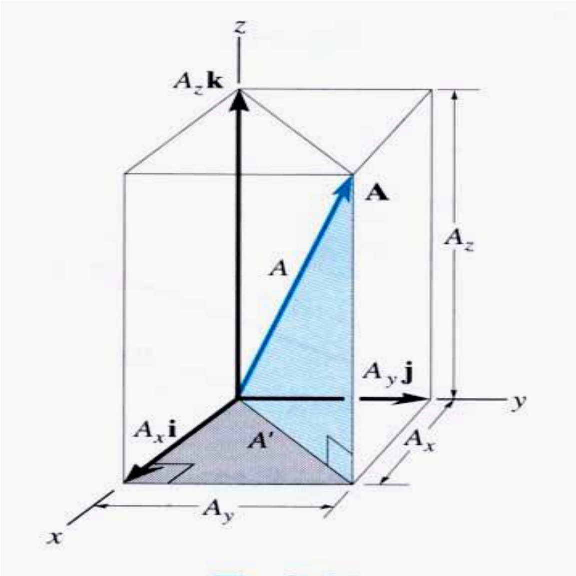
b) It is dimensionless.

c) It points in the same direction as the original vector (***A***).



The unit vectors in the Cartesian axis system are ***i***, ***j***, and ***k***. They are unit vectors along the positive x, y, and z axes respectively.

3-D CARTESIAN VECTOR TERMINOLOGY



Consider a box with sides A_X, A_Y, and A_Z meters long.

The vector ***A*** can be defined as

$\boldsymbol{A}$ **=** **(**$A_X\,\boldsymbol{i} + A_Y\,\boldsymbol{j} + A_Z\,\boldsymbol{k}$**)** m

The projection of the vector ***A*** in the x-y plane is A′. The magnitude of this projection, A′, is found by using the same approach as a 2-D vector: $A' = (A_X^2 + A_Y^2)^{1/2}$.

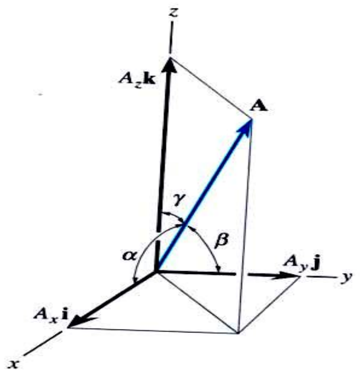
The magnitude of the position vector ***A*** can now be obtained as

$$A = ((A')^2 + A_Z^2)^{½} = (A_X^2 + A_Y^2 + A_Z^2)^{½}$$

3-D CARTESIAN VECTOR TERMINOLOGY (continued)

The direction or orientation of vector ***A*** is defined by the angles α, β, and γ.

These angles are measured between the vector and the positive X, Y and Z axes, respectively. Their range of values are from 0° to 180°



Using trigonometry, "direction cosines" are found using the formulas

$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

These angles are not independent. They must satisfy the following equation.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

or written another way, $u_A = \cos\alpha\, \boldsymbol{i} + \cos\beta\, \boldsymbol{j} + \cos\gamma\, \boldsymbol{k}$.

ADDITION/SUBTRACTION OF VECTORS

Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

For example, if

$$\boldsymbol{A} = A_X \, \boldsymbol{i} + A_Y \, \boldsymbol{j} + A_Z \, \boldsymbol{k} \quad \text{and}$$

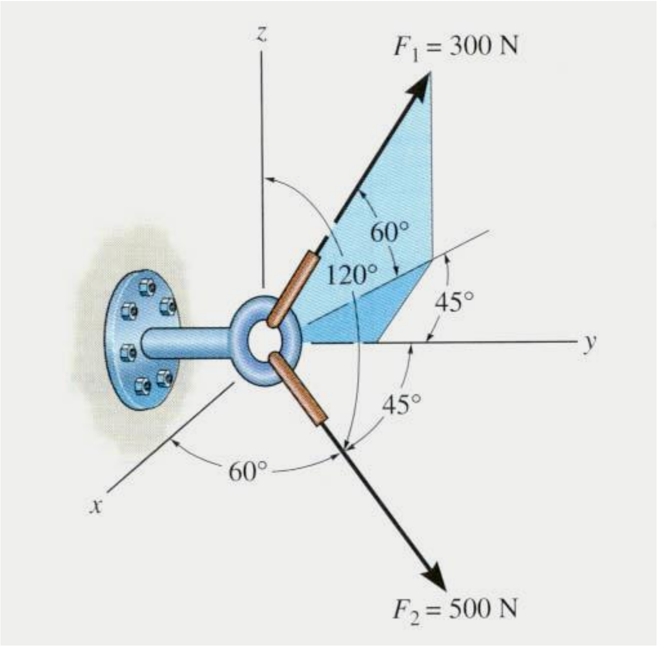
$$\boldsymbol{B} = B_X \, \boldsymbol{i} + B_Y \, \boldsymbol{j} + B_Z \, \boldsymbol{k}, \quad \text{then}$$

$$\boldsymbol{A} + \boldsymbol{B} = (A_X + B_X)\, \boldsymbol{i} + (A_Y + B_Y)\, \boldsymbol{j} + (A_Z + B_Z)\, \boldsymbol{k}$$

or

$$\boldsymbol{A} - \boldsymbol{B} = (A_X - B_X)\, \boldsymbol{i} + (A_Y - B_Y)\, \boldsymbol{j} + (A_Z - B_Z)\, \boldsymbol{k}.$$

GROUP PROBLEM SOLVING



Given: The screw eye is subjected to two forces.

Find: The magnitude and the coordinate direction angles of the resultant force.

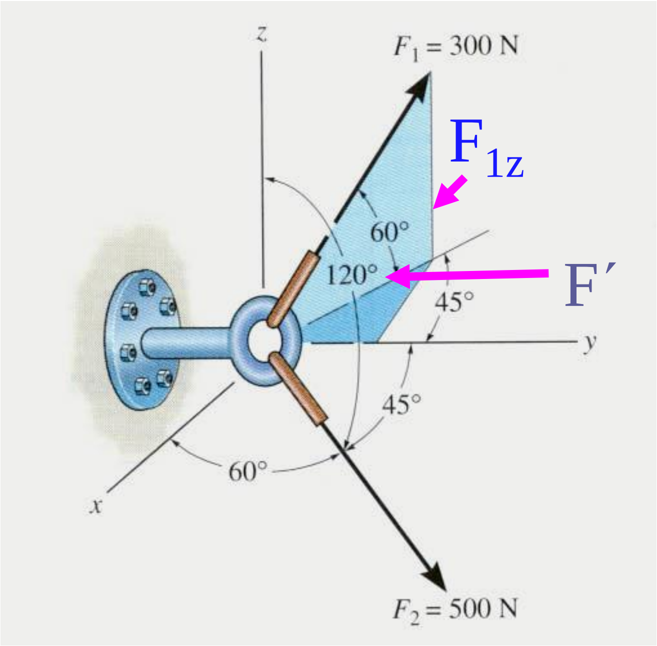
Plan:

1) Using the geometry and trigonometry, write $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ in the Cartesian vector form.

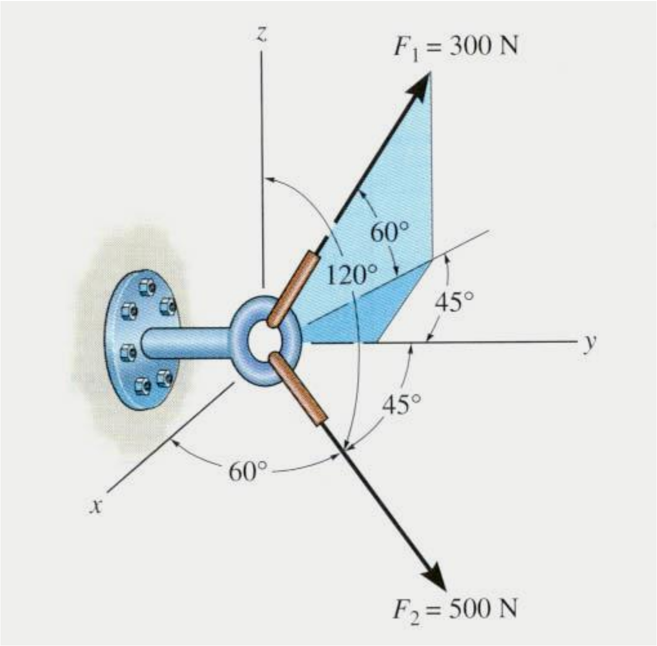
2) Add $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ to get $\boldsymbol{F_R}$.

3) Determine the magnitude and α, β, γ .

GROUP PROBLEM SOLVING (continued)



GROUP PROBLEM SOLVING (continued)



POSITION VECTORS & FORCE VECTORS

<u>Section III Objectives</u>:

Students will be able to :

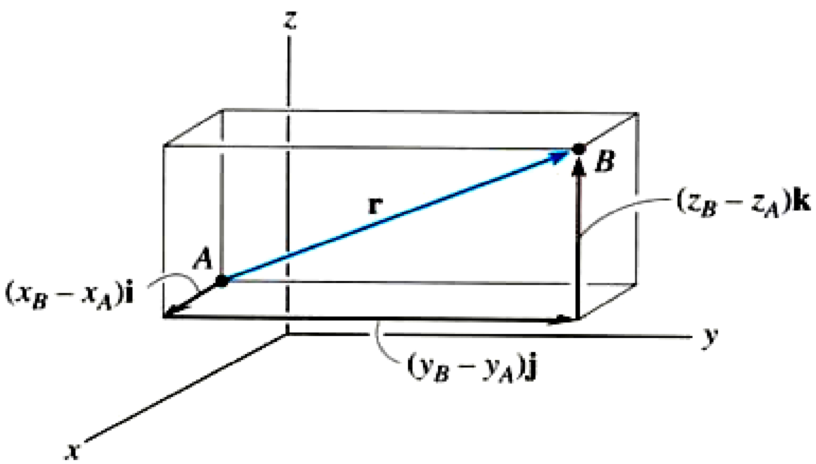
a) Represent a position vector in Cartesian coordinate form, from given geometry.

b) Represent a force vector directed along a line.

POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.

Consider two points, A & B, in 3-D space. Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.



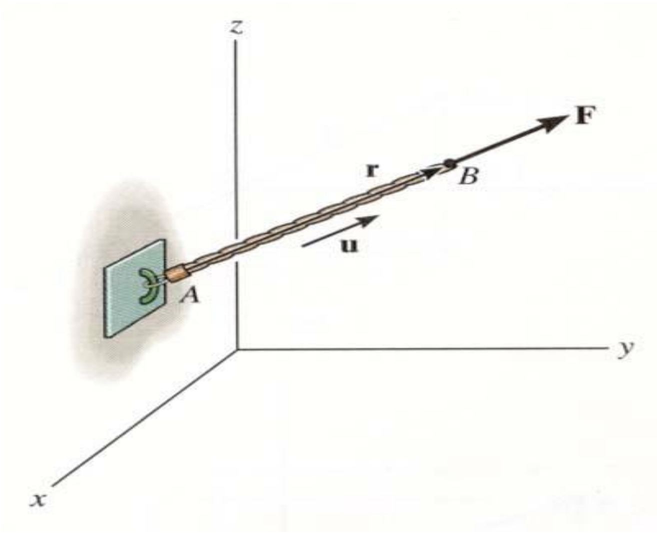
The position vector directed from A to B, $\boldsymbol{r}_{AB}$, is defined as

$$\boldsymbol{r}_{AB} = \{(X_B - X_A)\boldsymbol{i} + (Y_B - Y_A)\boldsymbol{j} + (Z_B - Z_A)\boldsymbol{k}\}\text{m}$$

Please note that B is the ending point and A is the starting point.

So ALWAYS subtract the "tail" coordinates from the "tip" coordinates!

FORCE VECTOR DIRECTED ALONG A LINE



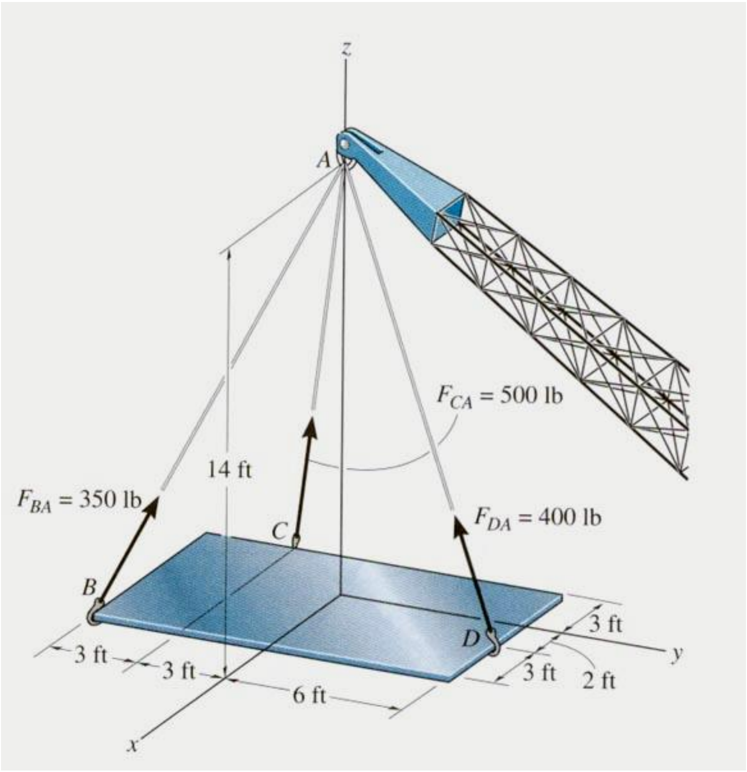
If a force is directed along a line, then we can represent the force vector in Cartesian Coordinates by using a unit vector and the force magnitude. So we need to:

a) Find the position vector, $\boldsymbol{r}_{AB}$, along two points on that line.

b) Find the unit vector describing the line's direction, $\boldsymbol{u}_{AB} = (\boldsymbol{r}_{AB}/r_{AB})$.

c) Multiply the unit vector by the magnitude of the force, $\boldsymbol{F} = F\ \boldsymbol{u}_{AB}$.

EXAMPLE



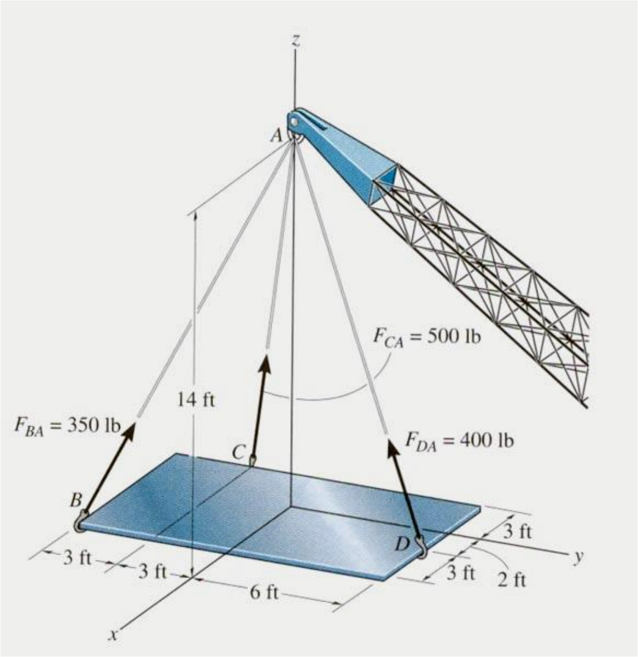
Given: 400 lb force along the cable DA.

Find: The force $\boldsymbol{F}_{DA}$ in the Cartesian vector form.

<u>Plan</u>:

- Find the position vector $\boldsymbol{r}_{DA}$ and the unit vector $\boldsymbol{u}_{DA}$.
- Obtain the force vector as $\boldsymbol{F}_{DA}$ = 400 lb $\boldsymbol{u}_{DA}$.

EXAMPLE (continued)



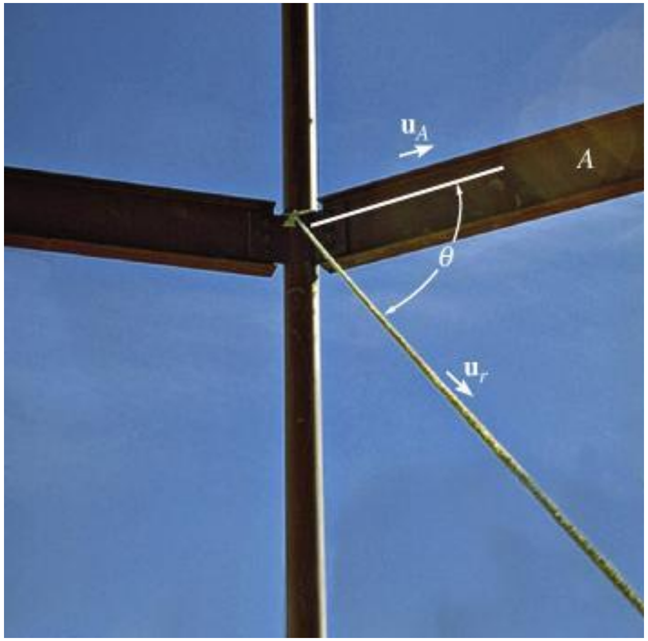
DOT PRODUCT

Section IV's Objective:

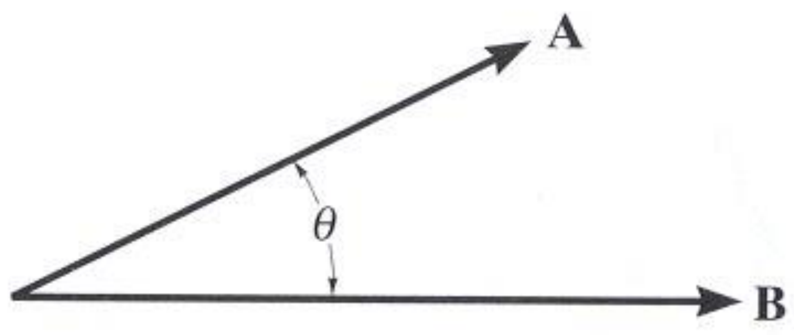
Students will be able to use the dot product to:

a) determine an angle between two vectors, and,

b) determine the projection of a vector along a specified line.



DEFINITION



The dot product of vectors ***A*** and ***B*** is defined as ***A•B*** = A B cos θ.

Angle θ is the smallest angle between the two vectors and is always in a range of 0^{o} to 180^{o}.

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors.

DOT PRODUCT DEFINITON
(continued)

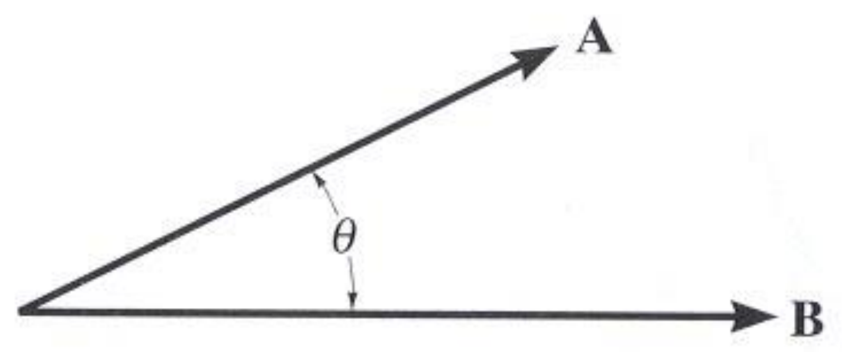
Examples: $\boldsymbol{i} \bullet \boldsymbol{j} = 0$

$\boldsymbol{i} \bullet \boldsymbol{i} = 1$

$$\boldsymbol{A} \bullet \boldsymbol{B} = (A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}) \bullet (B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



For the given two vectors in the Cartesian form, one can find the angle by

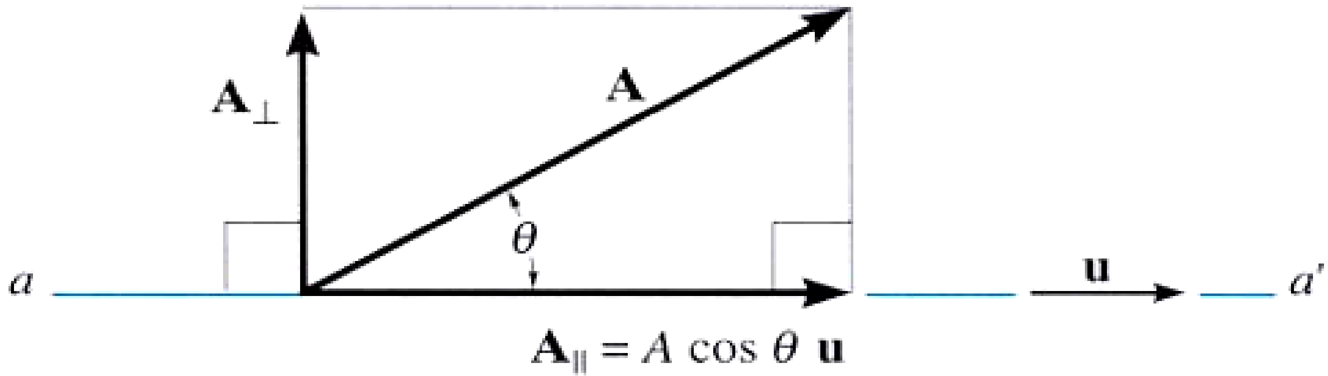
a) Finding the dot product, $\boldsymbol{A} \bullet \boldsymbol{B} = (A_x B_x + A_y B_y + A_z B_z)$,

b) Finding the magnitudes (A & B) of the vectors ***A*** & ***B***, and

c) Using the definition of dot product and solving for θ, i.e.,

$$\theta = \cos^{-1} [(\boldsymbol{A} \bullet \boldsymbol{B})/(A\,B)], \text{ where } 0^\circ \leq \theta \leq 180^\circ.$$

DETERMINING THE PROJECTION OF A VECTOR



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, $\boldsymbol{U}_{\boldsymbol{aa}'}$ along line aa′

2. Find the scalar projection of ***A*** along line aa′ by

$$A_{\parallel} = \boldsymbol{A} \bullet \boldsymbol{U} = A_x U_x + A_y U_y + A_z U_z$$

DETERMINING THE PROJECTION OF A VECTOR (continued)

3. If needed, the projection can be written as a vector, $\boldsymbol{A}_{||}$, by using the unit vector $\boldsymbol{U}_{aa'}$ and the magnitude found in step 2.

$$\boldsymbol{A}_{||} = A_{||}\,\boldsymbol{U}_{aa'}$$

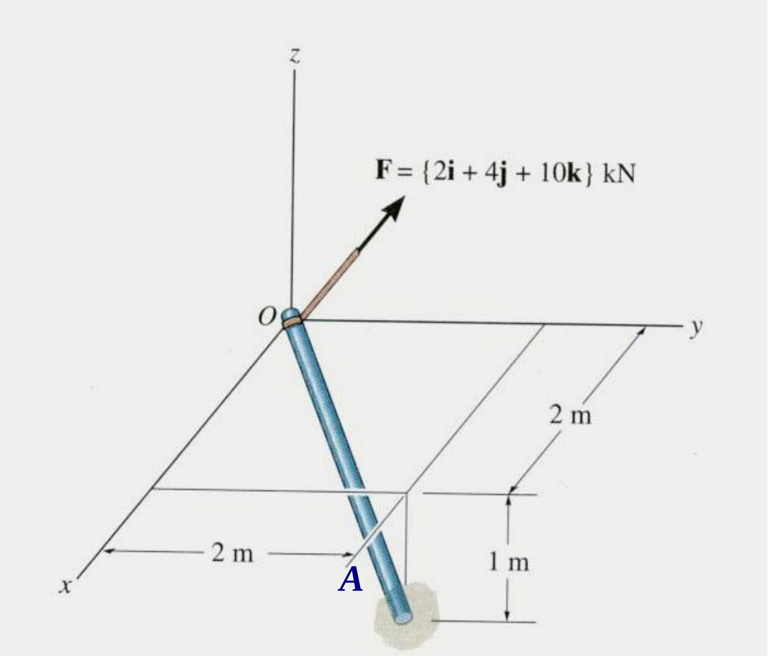
4. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_{\perp} = (A^2 - A_{||}^2)^{½} \text{ and}$$

$$\boldsymbol{A}_{\perp} = \boldsymbol{A} - \boldsymbol{A}_{||}$$

(rearranging the vector sum of $\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{||}$)

EXAMPLE



Given: The force acting on the pole

Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole OA.

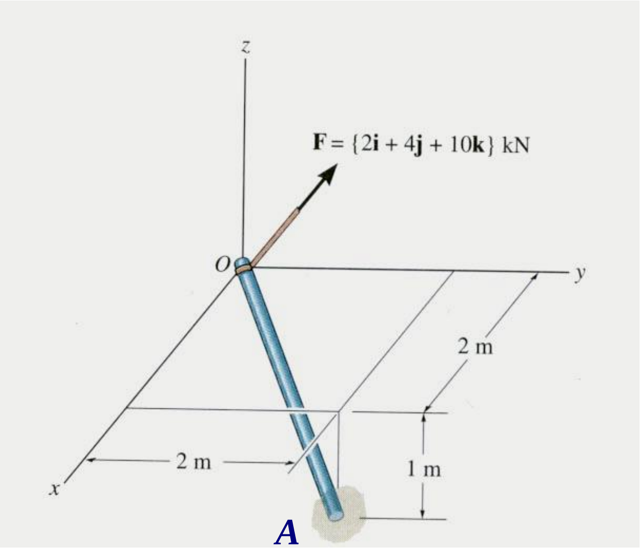
<u>Plan</u>:

1. Get $\boldsymbol{r_{OA}}$

2. $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{OA}})/(F\ \ r_{OA})\}$

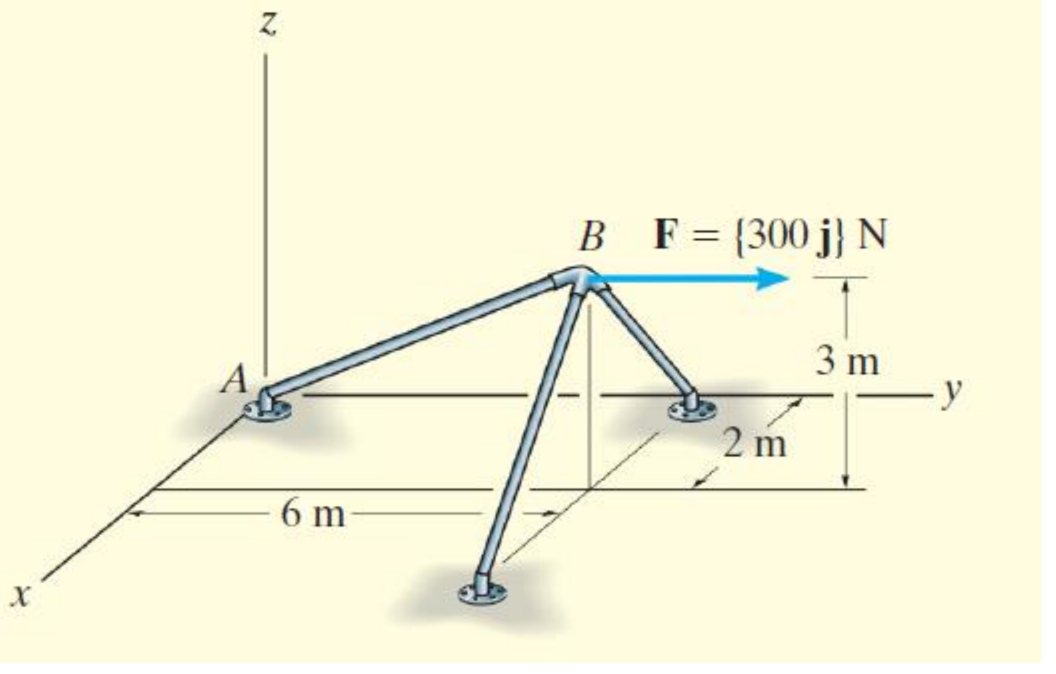
3. $F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u_{OA}}$ or $F \cos \theta$

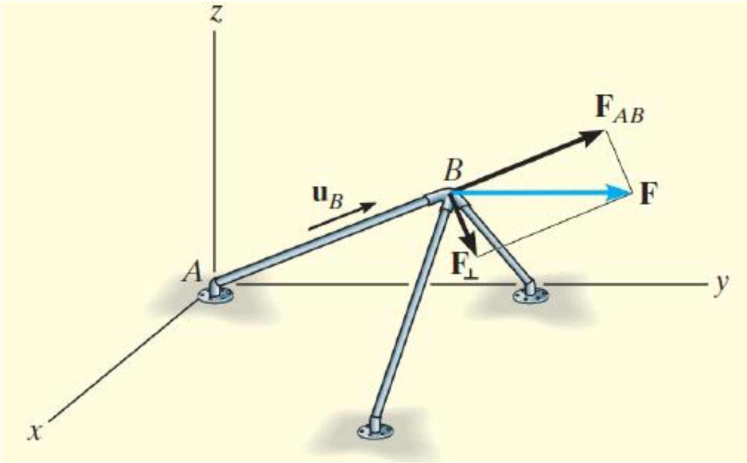
EXAMPLE
(continued)



Example

The frame shown in Fig. 2–43*a* is subjected to a horizontal force $\mathbf{F} = \{300\mathbf{j}\}$ N. Determine the magnitudes of the components of this force parallel and perpendicular to member AB.




z
$\mathbf{F}_{AB}$
B
$\mathbf{u}_B$
F
$\mathbf{F}_\perp$
A
y
x

Example