Kinematics and Dynamics of Machines

4. Force Analysis of Mechanisms

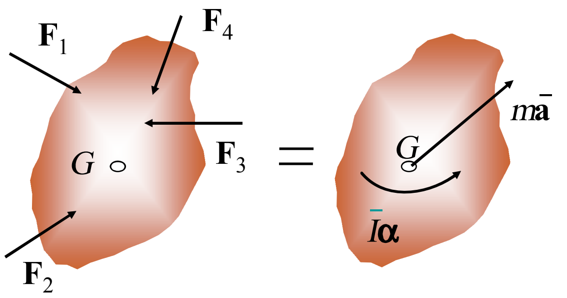
Introduction

- The design of mechanisms and their components requires information about forces acting on these components.
- Some mechanisms are designed to perform a specific kinematic function (like the windshield wiper mechanism), others to transfer energy (like the internal combustion engine).
- However, in any mechanism, identification of forces is needed to determine the proper dimensions of components.

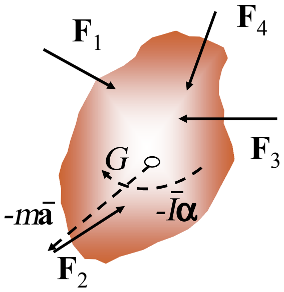
- The objective of the force analysis of mechanisms is to find the *transformation of forces* from the input to the output links.
- This transformation of forces depends on the position of the mechanism; in other words, it is a function of time.
- Thus, it is important to find out how these forces change during one cycle in order to find their maxima.
- One should differentiate between two types of forces: *external* and *internal*. The former are forces that are applied to the links from external sources — driving forces, resistance forces — whereas the latter are forces acting between the joints (they are called *constraint* or *reaction* forces).

Static and Inertial Forces

- In the force analysis of Machines, we frequently refer to two kind of forces: static forces and inertial forces
- Inertial forces or d'Alembert's forces are those equivalent to system inertia
- Static forces refer to all forces except inertia forces.
- *d'Alembert's Principle*: The external forces acting on a rigid body are equivalent to the effective forces of the various particles forming the body.



$$\sum F = m\bar{a}$$
$$\sum M_G = \bar{I}\alpha$$



$$\sum F - m\bar{a} = 0$$
$$\sum M_G - \bar{I}\alpha = 0$$

Free-body Diagram For A Link

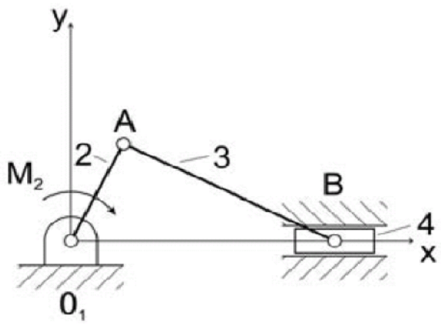
- A diagram of a link with all forces (external and internal) applied to it is called a *free-body diagram*. Under the action of all forces (static and inertial), the link must be in equilibrium.

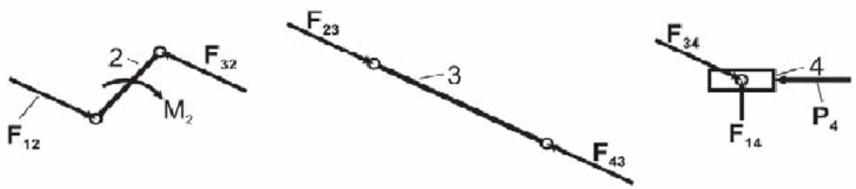
$$\sum_{i=0}^{n} F_i = 0, \qquad \sum_{j=0}^{m} M_j = 0$$

- This requirement results in relationships between the known and unknown forces for a single link.

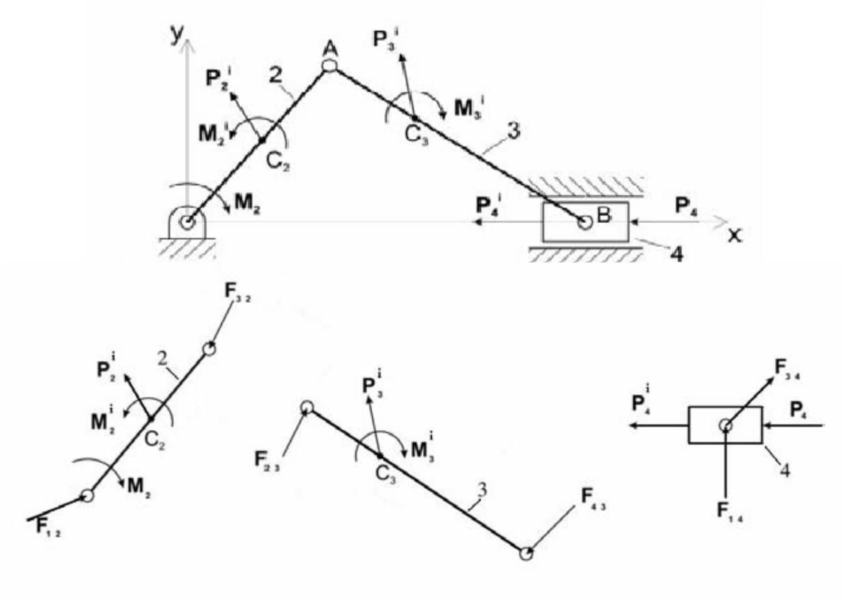
- The internal forces originate in joints since joints constrain the relative motion between the connected links.
- In the case of a revolute joint, in general, both the magnitude and the direction of the constraint force are unknown, whereas in the case of a prismatic joint only the magnitude of the constraint force is unknown.

Example 1. Negligible inertia forces





Example 2. Considerable inertia forces



Impact Center

- If the mass of a combined pendulum is concentrated in a point so that its oscillation period remain the same, the new point is named *Impact center*.
- If an impact force is implied to impact center, the reaction of pendulum axes will be zero
- Location of impact center depends upon location of axes point with respect to the center of gravity

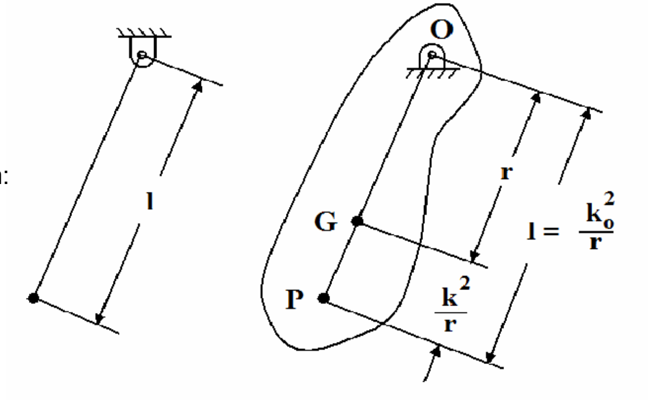
Locating Impact Center

For a Lamped Mass Pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

For a distributed Mass Pendulum:

$$T = 2\pi\sqrt{\frac{I_O}{m g r}} = 2\pi\sqrt{\frac{m k_O^2}{m g r}} = 2\pi\sqrt{\frac{k_O^2}{g r}}$$



$$(I_O = I + m r^2 \quad or \quad k_O^2 = k^2 + r^2)$$

Axes Reaction

In case of zero axes reaction, the sole acting forces are inertia forces which can be replaced with a single force at distance h from centroide:

$$h = \frac{M^i}{F^i} = \frac{\bar{I}\alpha}{m\bar{a}} = \frac{mk^2\alpha}{mr\alpha} == \frac{k^2}{r}$$