فصل هفتم

بارگذاری مرکب

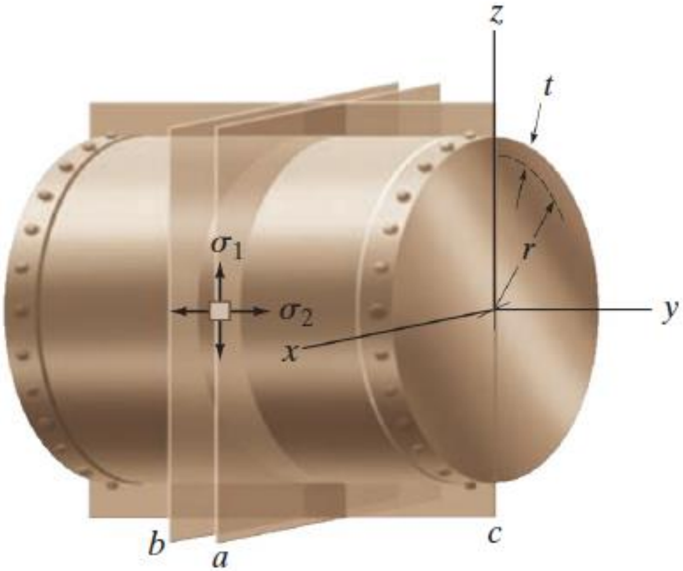
COMBINED LOADINGS

مخازن جدارنازک تحت فشار

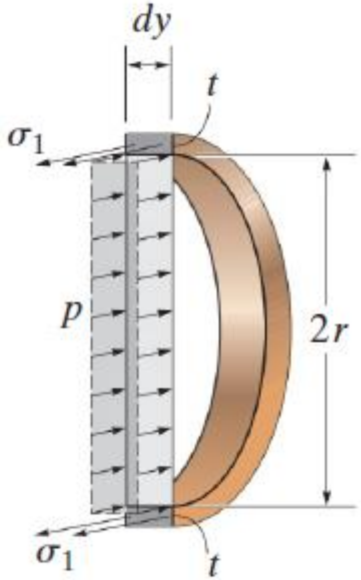
مخازن استوانه ای

Circumferential Stress

تنش پیرامونی



$(r/t \geq 10)$



$$2[\sigma_1(t\,dy)] - p(2r\,dy) = 0$$

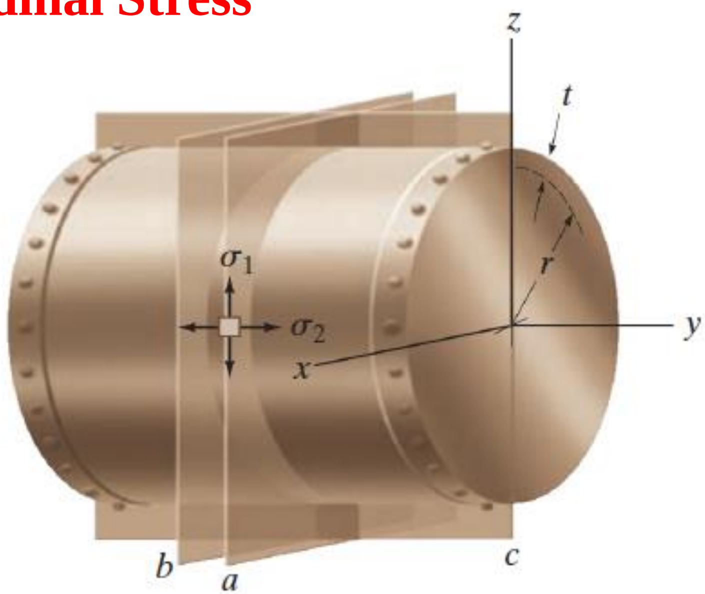
$$\Sigma F_x = 0;$$

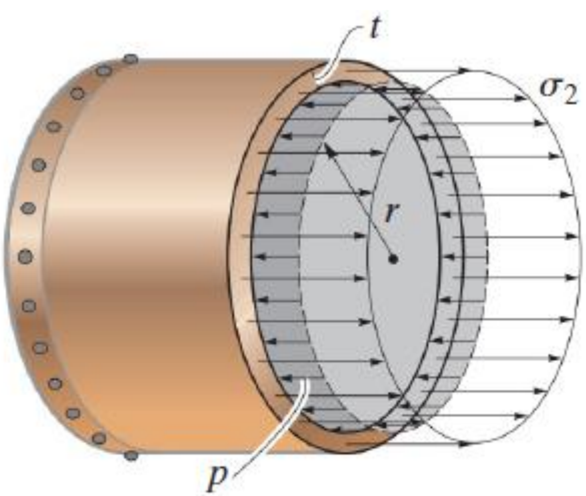
$$\sigma_1 = \frac{pr}{t}$$

مخازن استوانه ای

Longitudinal Stress

تنش طولی





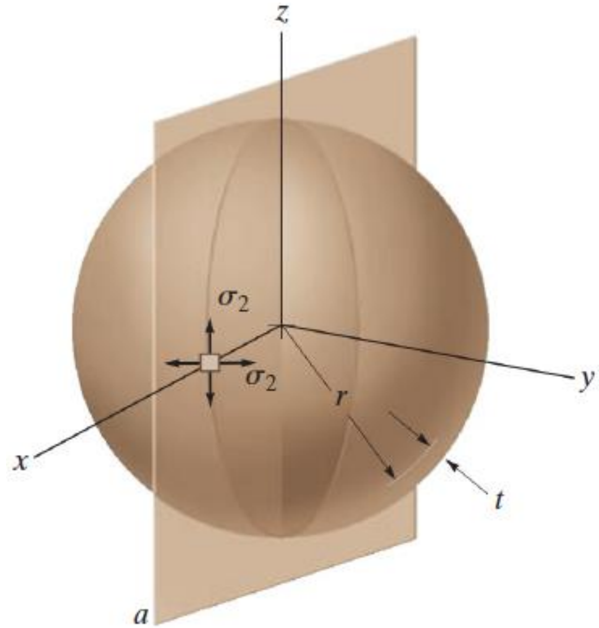
$$\Sigma F_y = 0$$

$$\sigma_2(2\pi rt) - p(\pi r^2) = 0$$

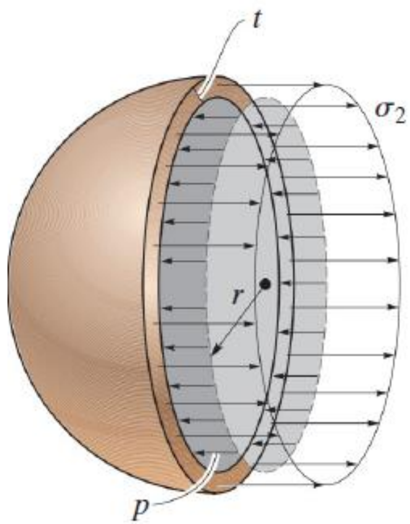
$$\sigma_2 = \frac{pr}{2t}$$

مخازن کروی

$$(r/t \geq 10)$$



$$\Sigma F_y = 0; \qquad \sigma_2(2\pi r t) - p(\pi r^2) = 0$$



$$\sigma_2 = \frac{pr}{2t}$$

Example

A cylindrical pressure vessel has an inner diameter of 4 ft and a thickness of $\frac{1}{2}$ in. Determine the maximum internal pressure it can sustain so that neither its circumferential nor its longitudinal stress component exceeds 20 ksi. Under the same conditions, what is the maximum internal pressure that a similar-size spherical vessel can sustain?

Cylindrical Pressure Vessel.

$$\sigma_1 = \frac{pr}{t}; \qquad 20\ \text{kip/in}^2 = \frac{p(24\ \text{in.})}{\frac{1}{2}\ \text{in.}}$$

$$p = 417\ \text{psi}$$

Spherical Vessel.

$$20\ \text{kip/in}^2 = \frac{p(24\ \text{in.})}{2\left(\frac{1}{2}\ \text{in.}\right)}$$

$$p = 833\ \text{psi}$$

تنش ناشی از بارگذاری مرکب

Normal Force.

- The normal force is related to a uniform normal-stress distribution determined from $\sigma = N/A$.

Shear Force.

- The shear force is related to a shear-stress distribution determined from the shear formula, $\tau = VQ/It$.

Bending Moment. $\sigma = -My/I$

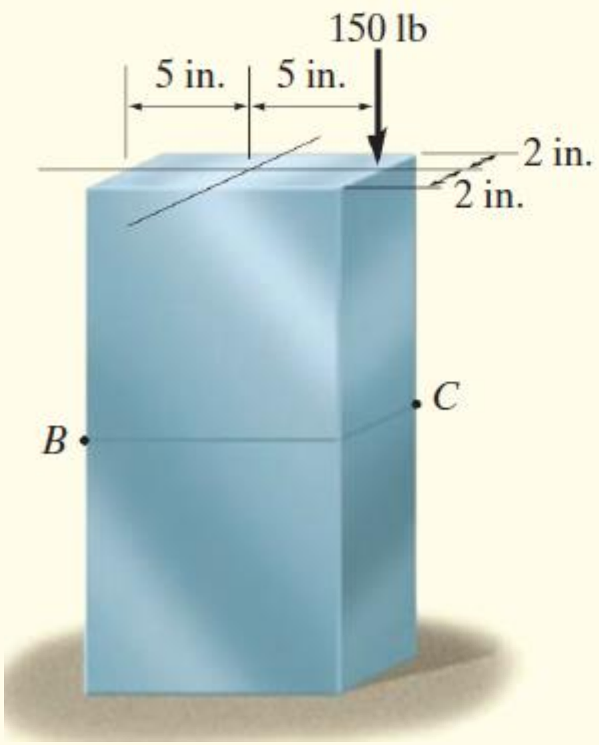
Torsional Moment. $\tau = T\rho/J$.

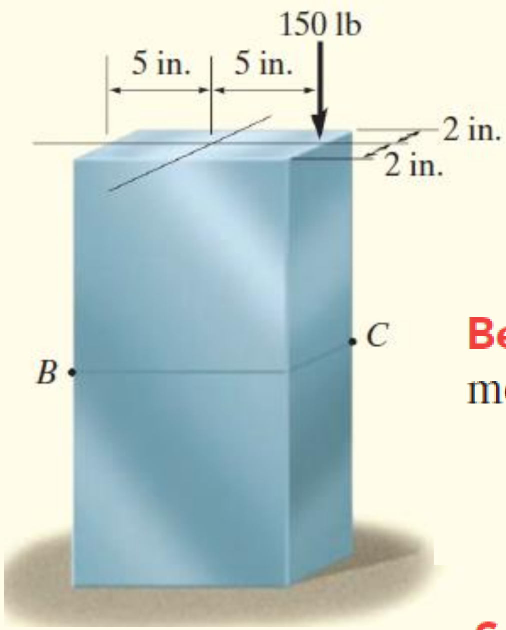
Thin-Walled Pressure Vessels.

$$\sigma_1 = pr/t,$$
$$\sigma_2 = pr/2t.$$

Example

A force of 150 lb is applied to the edge of the member shown in Fig. 8–3*a*. Neglect the weight of the member and determine the state of stress at points *B* and *C*.



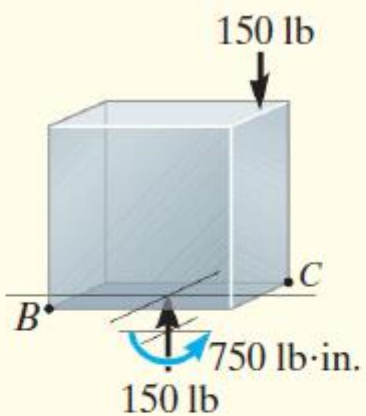


Normal Force. The uniform normal-stress distribution due to the normal force is shown in Fig. 8–3*c*. Here

$$\sigma = \frac{N}{A} = \frac{150\text{ lb}}{(10\text{ in.})(4\text{ in})} = 3.75\text{ psi}$$

Bending Moment. The normal-stress distribution due to the bending moment is shown in Fig. 8–3*d*. The maximum stress is

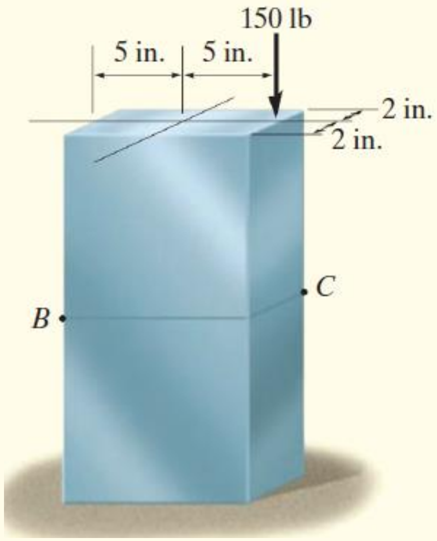
$$\sigma_{\max} = \frac{Mc}{I} = \frac{750\text{ lb}\cdot\text{in.}\,(5\text{ in.})}{\frac{1}{12}(4\text{ in.})\,(10\text{ in.})^3} = 11.25\text{ psi}$$

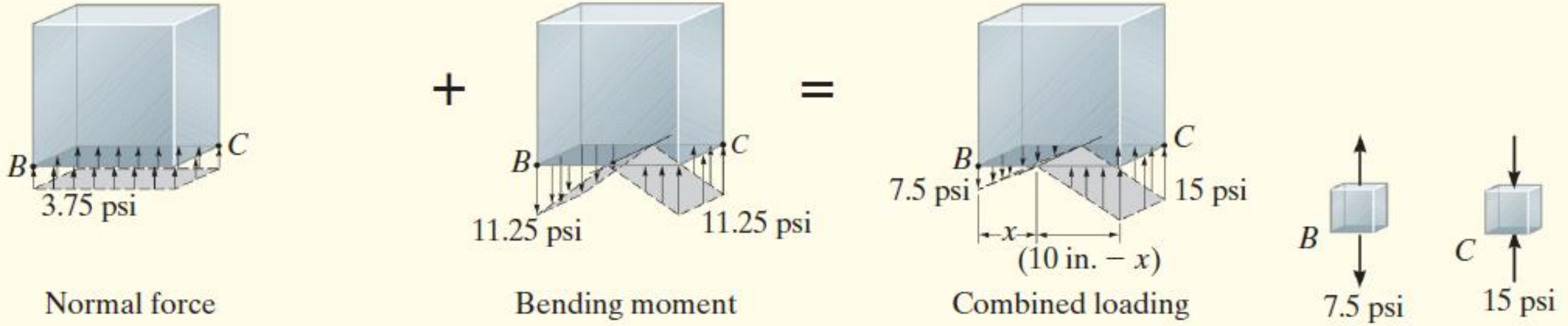


Superposition. Algebraically adding the stresses at *B* and *C*, we get

$$\sigma_B = -\frac{N}{A} + \frac{Mc}{I} = -3.75\text{ psi} + 11.25\text{ psi} = 7.5\text{ psi} \quad \text{(tension)}$$ *Ans.*

$$\sigma_C = -\frac{N}{A} - \frac{Mc}{I} = -3.75\text{ psi} - 11.25\text{ psi} = -15\text{ psi (compression)}$$ *Ans.*

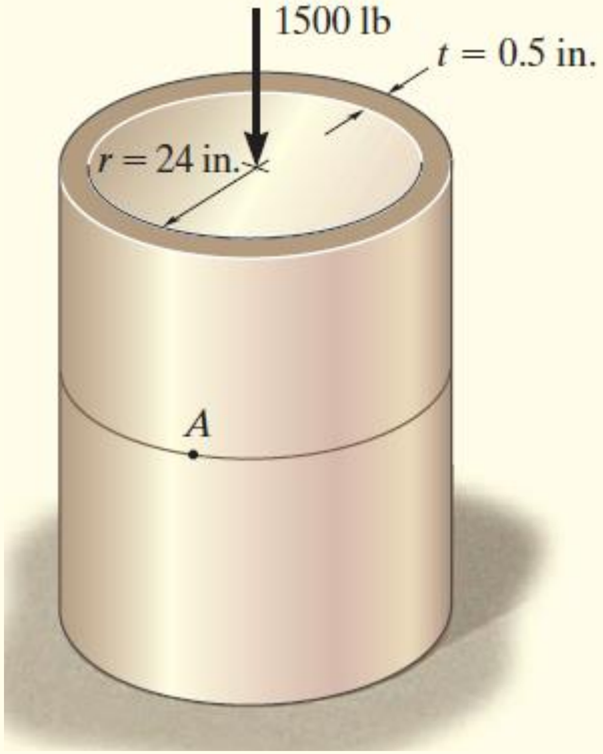


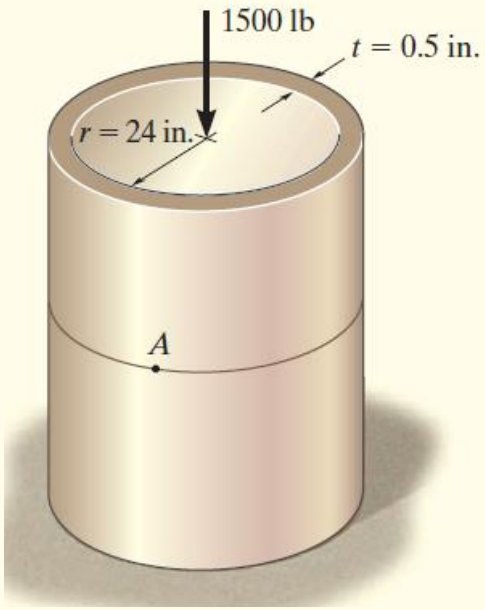


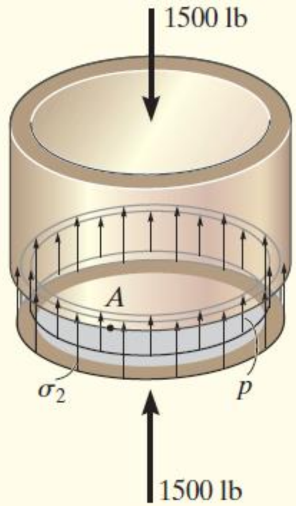
$$\frac{7.5\text{ psi}}{x} = \frac{15\text{ psi}}{(10\text{ in.} - x)}; \quad x = 3.33\text{ in.}$$

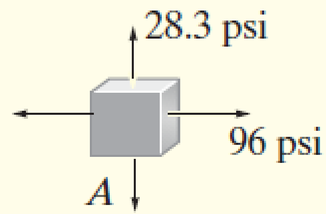
Example

The gas tank in Fig. 8–4*a* has an inner radius of 24 in. and a thickness of 0.5 in. If it supports the 1500-lb load at its top, and the gas pressure within it is 2 lb/in^2, determine the state of stress at point *A*.









Circumferential Stress.

$$\sigma_1 = \frac{pr}{t} = \frac{2\ \text{lb/in}^2\ (24\ \text{in.})}{0.5\ \text{in.}} = 96\ \text{psi}$$

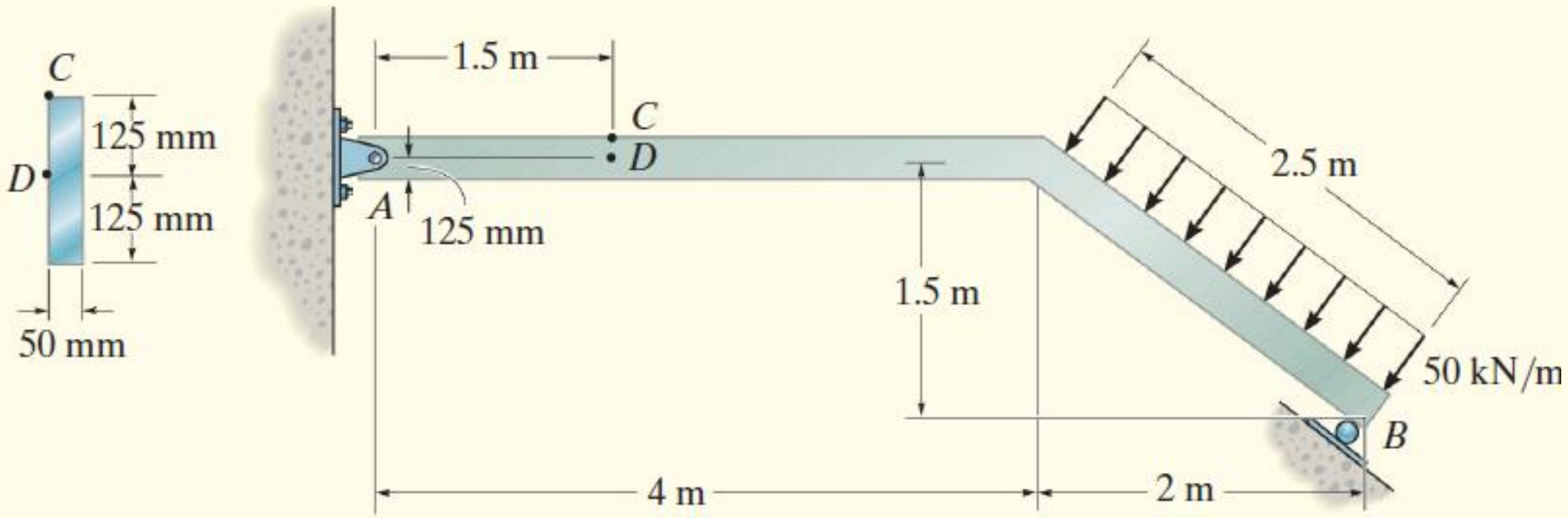
Longitudinal Stress.

$$\sigma_2 = -\frac{N}{A} + \frac{pr}{2t} = -\frac{1500\ \text{lb}}{\pi[(24.5\ \text{in.})^2 - (24\ \text{in.})^2]} + \frac{2\ \text{lb/in}^2\ (24\ \text{in.})}{2\ (0.5\ \text{in.})}$$

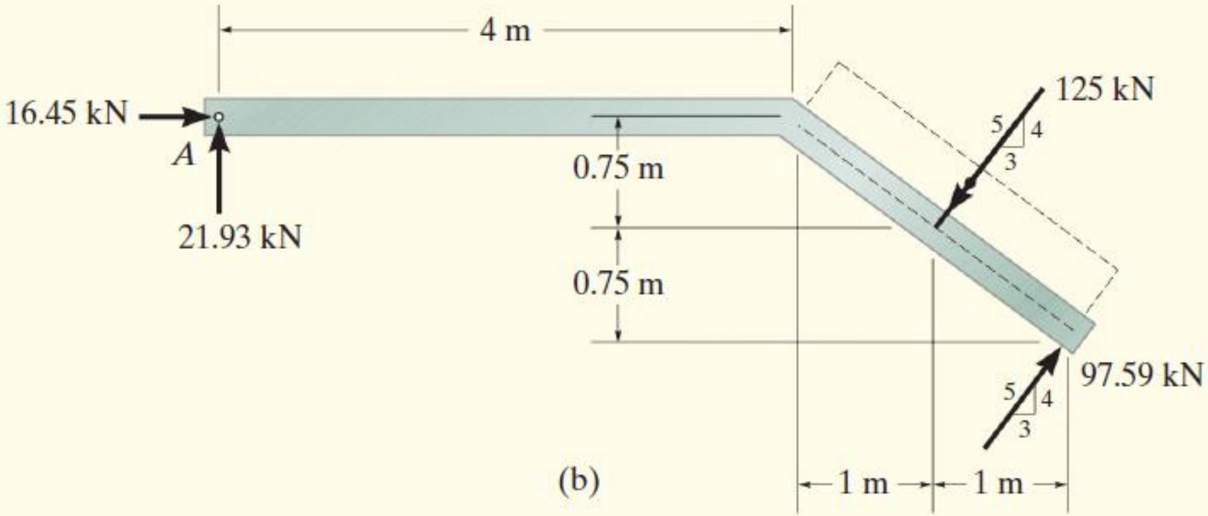
$$= 28.3\ \text{psi}$$

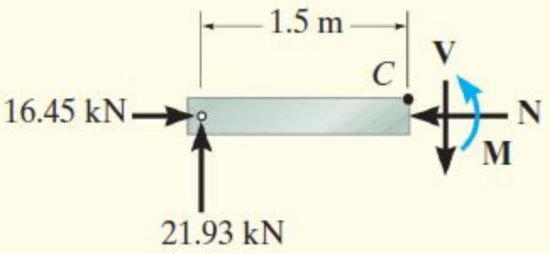
Example

The member shown in Fig. 8–5*a* has a rectangular cross section. Determine the state of stress that the loading produces at point *C* and point *D*.



(a)



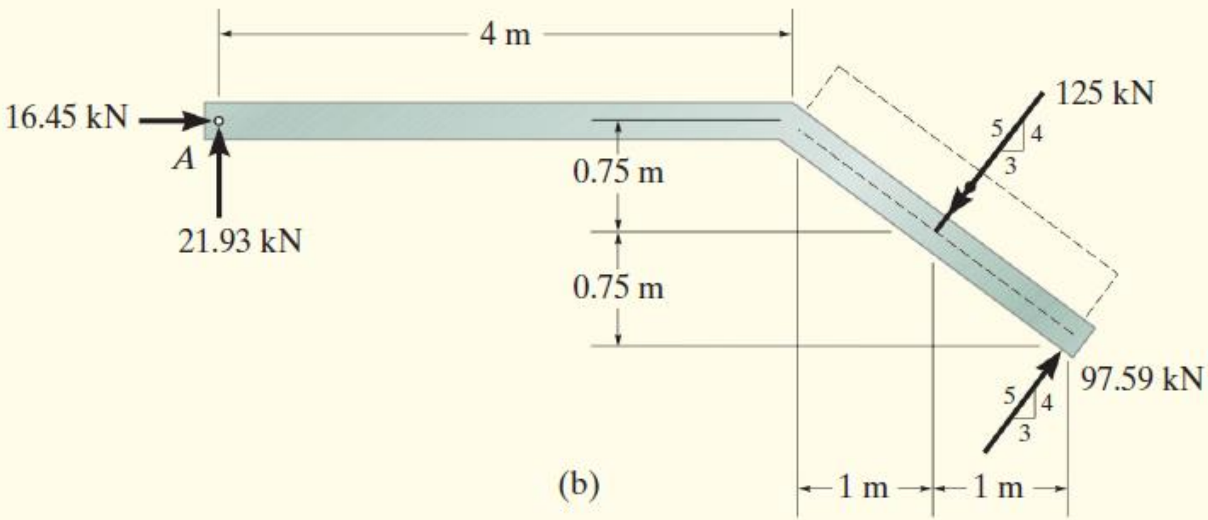


$N = 16.45 \text{ kN} \qquad V = 21.93 \text{ kN} \qquad M = 32.89 \text{ kN}\cdot\text{m}$

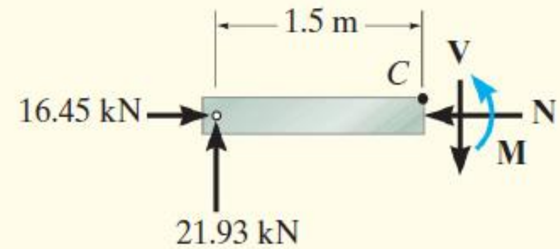
Stress Components at C.

Normal Force.

$$\sigma_C = \frac{N}{A} = \frac{16.45(10^3)\text{ N}}{(0.050\text{ m})\,(0.250\text{ m})} = 1.32\text{ MPa}$$



(b)

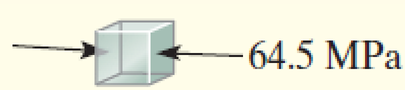


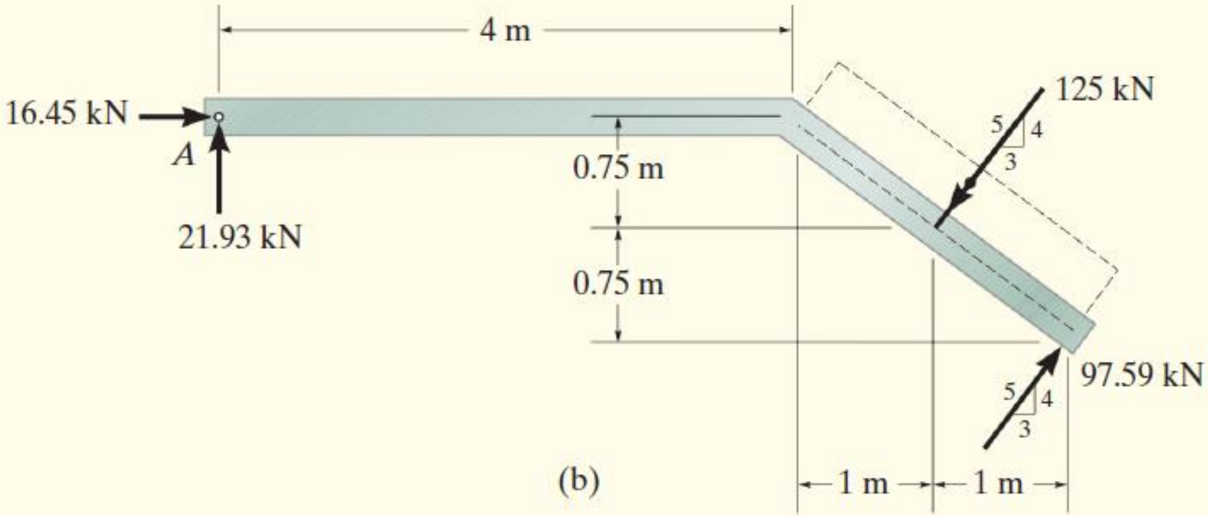
Bending Moment.

$$\sigma_C = \frac{Mc}{I} = \frac{(32.89(10^3)\ \text{N}\cdot\text{m})(0.125\ \text{m})}{\left[\frac{1}{12}(0.050\ \text{m})(0.250\ \text{m})^3\right]} = 63.16\ \text{MPa}$$

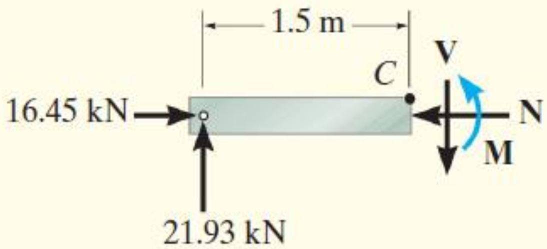
Superposition. There is no shear-stress component. Adding the normal stresses gives a compressive stress at *C* having a value of

$$\sigma_C = 1.32\ \text{MPa} + 63.16\ \text{MPa} = 64.5\ \text{MPa}$$ *Ans.*





(b)



Stress Components at *D*.

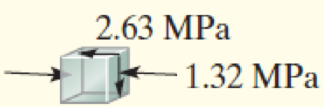
Normal Force. This is the same as at $C, \sigma_D = 1.32$ MPa, Fig. 8–5*d*.

Shear Force. Since D is at the neutral axis, and the cross section is rectangular, we can use the special form of the shear formula, Fig. 8–5*e*.

$$\tau_D = 1.5\frac{V}{A} = 1.5\frac{21.93(10^3)\text{ N}}{(0.25\text{ m})(0.05\text{ m})} = 2.63\text{ MPa} \qquad \textit{Ans.}$$

Bending Moment. Here D is on the neutral axis and so $\sigma_D = 0$.

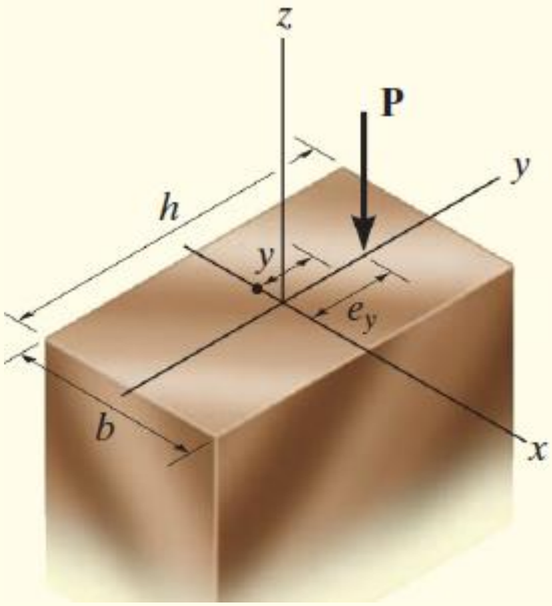
Superposition. The resultant stress on the element is shown in Fig. 8–5*h*.

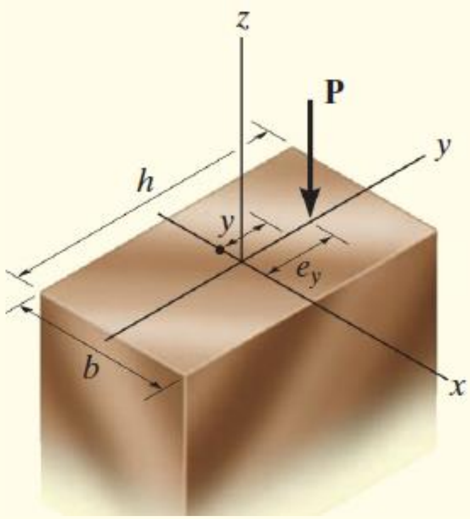


(h)

Example

A rectangular block has a negligible weight and is subjected to a vertical force **P**, Fig. 8–8*a*. (*a*) Determine the range of values for the eccentricity e_y of the load along the *y* axis, so that it does not cause any tensile stress in the block. (*b*) Specify the region on the cross section where **P** may be applied without causing tensile stress.



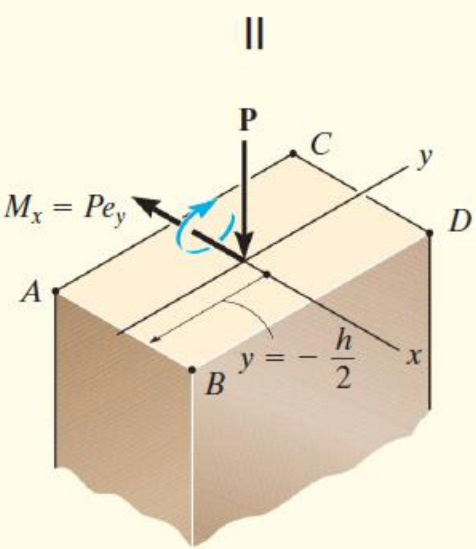


$$\sigma = -\frac{P}{A} - \frac{(Pe_y)y}{I_x} = -\frac{P}{A}\left(1 + \frac{Ae_y y}{I_x}\right)$$

smallest compressive stress will occur along edge AB,

$$\sigma_{\min} = -\frac{P}{A}\left(1 - \frac{Ae_y h}{2I_x}\right)$$

$$1 > \frac{Ae_y h}{2I_x}$$



Since $A = bh$ and $I_x = \frac{1}{12}bh^3$, then

$$1 > \frac{6e_y}{h} \quad \text{or} \quad e_y < \frac{1}{6}h$$