CHAPTER 3

Kinetics of Particles: Newton's Second Law

Kinetics of Particles: Newton's Second Law

❑ Introduction

We must analyze all of the forces acting on the wheelchair in order to design a good ramp

High swing velocities can result in large forces on a swing chain or rope, causing it to break.

Kinetics of Particles: Newton's Second Law

❑ Introduction

$$\Sigma \mathbf{F} = m\mathbf{a}$$

- **Newton's Second Law of Motion**
  - If the *resultant force* acting on a particle is not zero, the particle will have an acceleration *proportional to the magnitude of resultant* and in the *direction of the resultant*.
  - Must be expressed with respect to a *Newtonian (or inertial) frame of reference*, i.e., one that is not accelerating or rotating.
  - This form of the equation is for a constant mass system(not be used to solve problems involving the motion of bodies, such as rockets, which gain or lose mass)

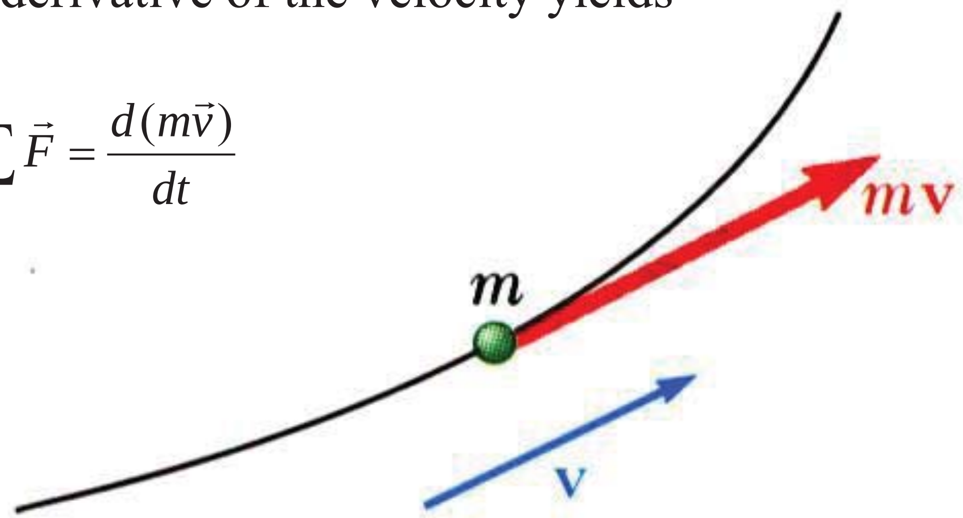
Kinetics of Particles: Newton's Second Law

❑ Linear Momentum of a Particle

- Replacing the acceleration by the derivative of the velocity yields

$$\left.\begin{aligned}\sum \vec{F} &= m\vec{a} \\ \vec{a} &= \frac{d\vec{v}}{dt}\end{aligned}\right\} \Rightarrow \sum \vec{F} = m\frac{d\vec{v}}{dt} \overset{m=cte}{\Rightarrow} \sum \vec{F} = \frac{d(m\vec{v})}{dt}$$

$$\Rightarrow \boxed{\sum \vec{F} = \frac{d(\vec{L})}{dt}}$$



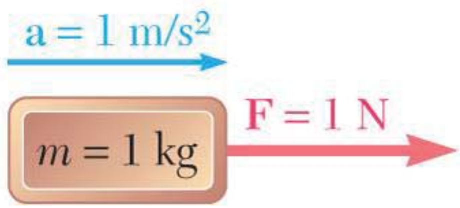
$\vec{L}$ = linear momentum of the particle

- *Linear Momentum Conservation Principle* :

If the resultant force on a particle is zero, the linear momentum of the particle remains constant in both magnitude and direction (Newton's First Law).

Kinetics of Particles: Newton's Second Law

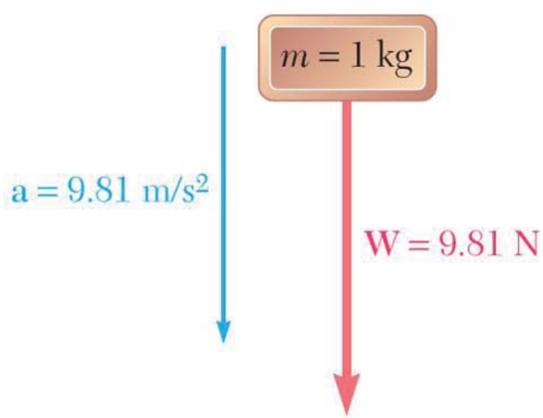
❑ Systems of Units



Length | **Time** | **Mass** | **Force**

Three base units may be chosen arbitrarily

The fourth must be compatible with Newton's 2nd Law.



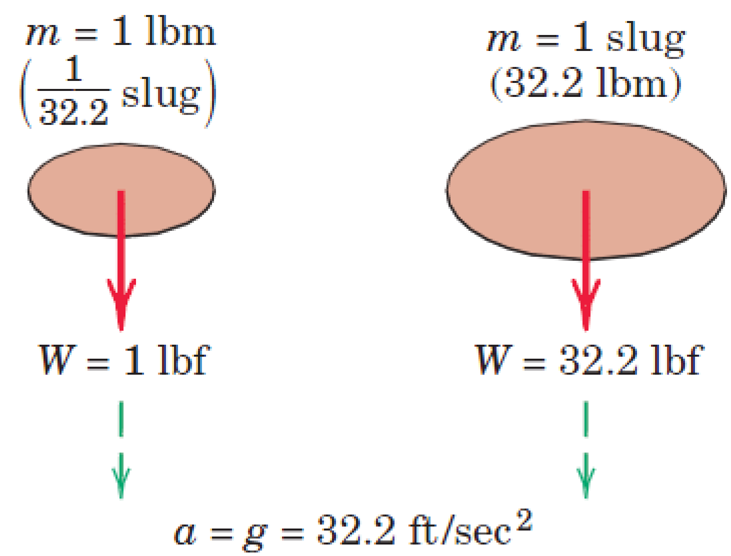
- ***International System of Units (SI Units):*** In this system, the base units are the units of length, mass, and time, and are called, respectively, the meter (m), the kilogram (kg), and the second (s). The unit of force is called the newton (N) and is defined as the force which gives an acceleration of 1 m/s^2 to a mass of 1 kg.

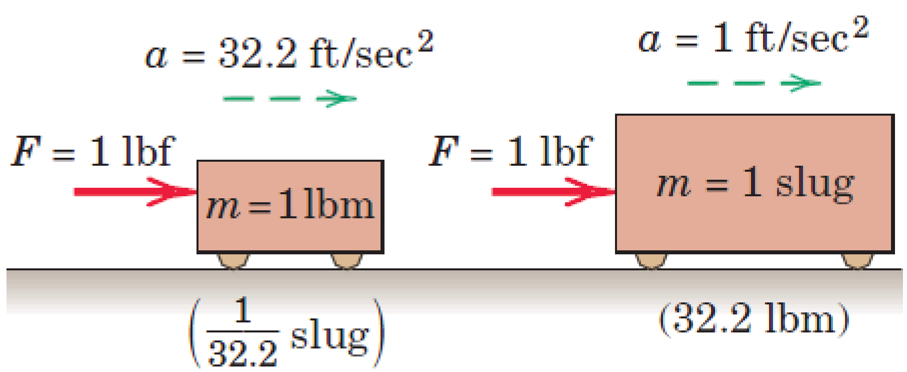
$$1\,\mathrm{N} = (1\,\mathrm{kg})\left(1\,\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 1\,\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$$

Kinetics of Particles: Newton's Second Law

❑ Systems of Units

U.S. Customary Units: The base units are the units of length, force, and time. These units are, respectively, the foot (ft), the pound (lbf), and the second (s). The pound is defined as the weight of a platinum standard, called the standard pound, which is kept at the National Institute of Standards and Technology outside Washington and the mass of which is 0.45359243 kg.





$$1\,\text{slug} = \frac{1\,\text{lbf}}{1\,\text{ft}/\text{s}^2} = 1\,\frac{\text{lbf}\cdot\text{s}^2}{\text{ft}}$$

Kinetics of Particles: Newton's Second Law

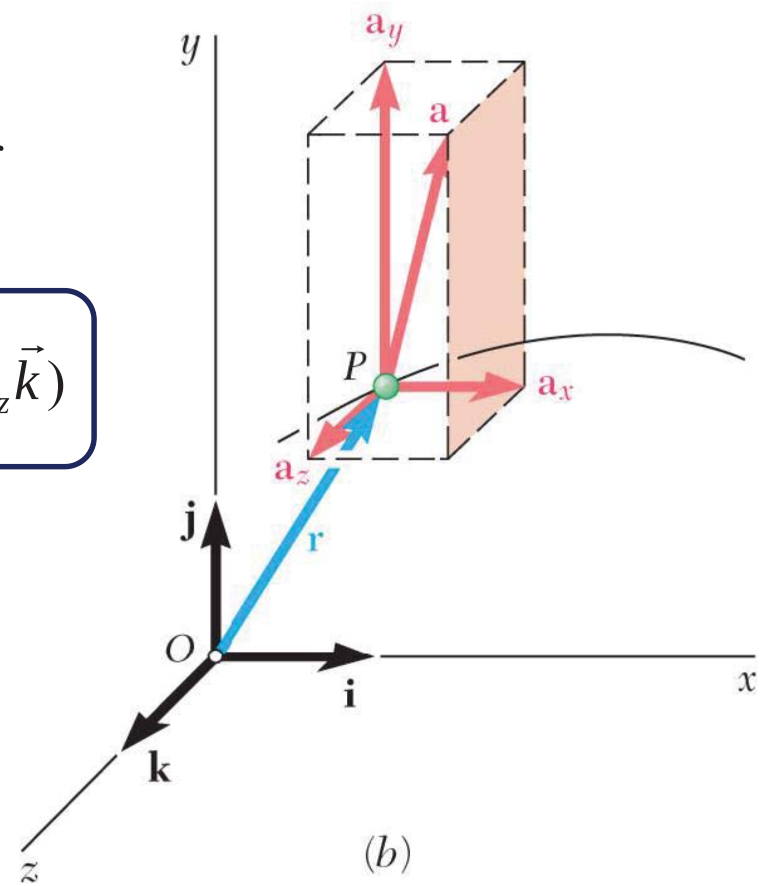
❑ Equations of Motion

- Newton's second law $\sum \vec{F} = m\vec{a}$

- ***Rectangular Components.*** Resolving each force F and the acceleration a into rectangular components

$$\sum \vec{F} = \sum (F_x\vec{i} + F_y\vec{j} + F_z\vec{k}) = m(a_x\vec{i} + a_y\vec{j} + a_z\vec{k})$$

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$
$$\sum F_x = m\ddot{x} \quad \sum F_y = m\ddot{y} \quad \sum F_z = m\ddot{z}$$



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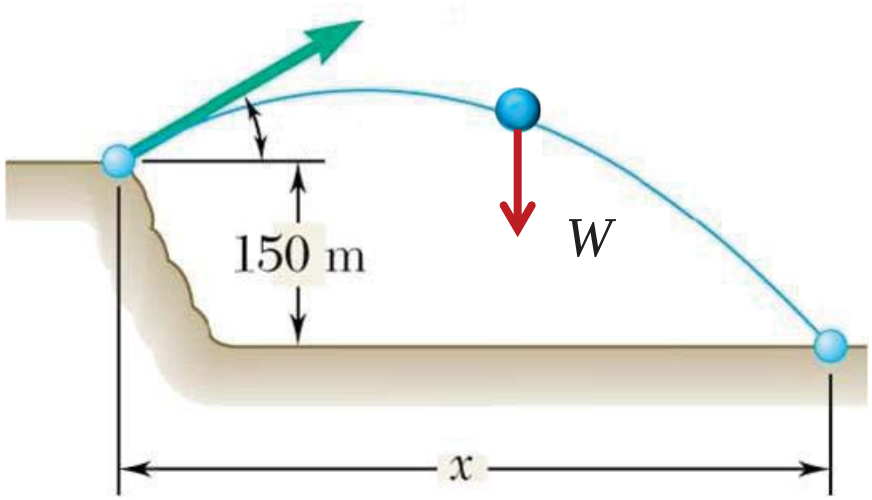
Kinetics of Particles: Newton's Second Law

❑ Equations of Motion

Consider, as an example, the motion of a projectile. If the resistance of the air is neglected, the only force acting on the projectile after it has been fired is its weight W=-Wj. The equations defining the motion of the projectile are therefore

$$m\ddot{x} = 0 \quad m\ddot{y} = -W \quad m\ddot{z} = 0$$



and the components of the acceleration of the projectile are

$$\ddot{x} = 0 \quad \ddot{y} = -\frac{W}{m} = -g \quad \ddot{z} = 0$$

Kinetics of Particles: Newton's Second Law

❑ Dynamic Equilibrium

- Alternate expression of Newton's second law,

$$\sum \vec{F} - m\vec{a} = 0$$

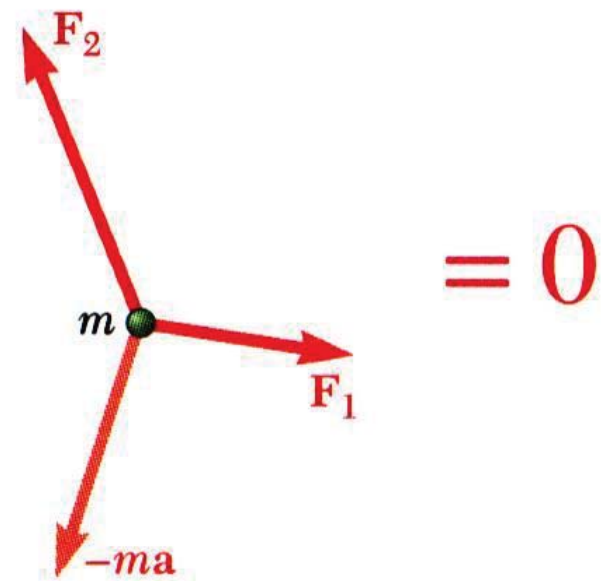
$$-m\vec{a} \equiv \textit{inertial vector}$$

- With the inclusion of the inertial vector, the system of forces acting on the particle is equivalent to zero. The particle is in *dynamic equilibrium*.

- Methods developed for particles in static equilibrium may be applied, e.g., coplanar forces may be represented with a closed vector polygon.

- Inertia vectors are often called ***inertial forces*** as they measure the resistance that particles offer to changes in motion, i.e., changes in speed or direction.

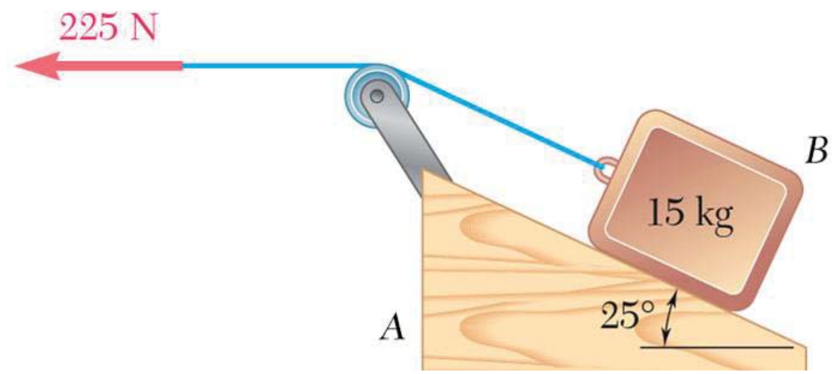
- Inertial forces may be conceptually useful but are not like the contact and gravitational forces found in statics.



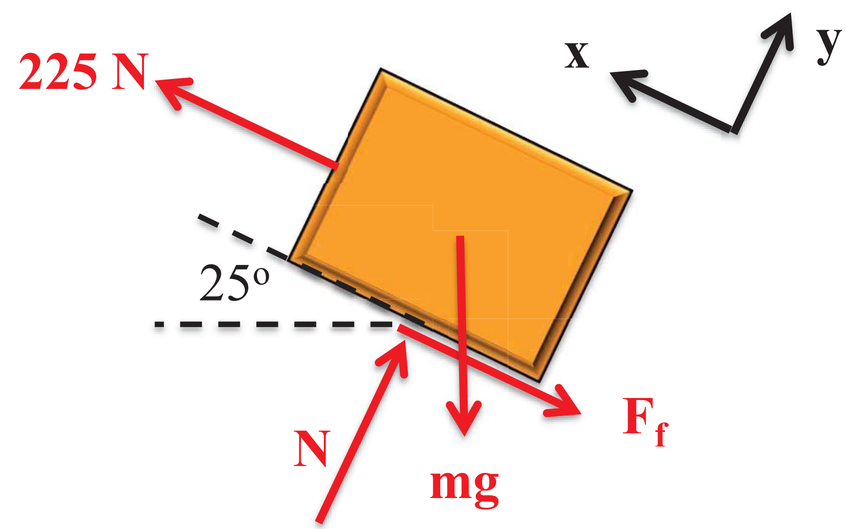
Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

The free body diagram is the same as you have done in statics; we will add the kinetic diagram in our dynamic analysis.



1. Isolate the body of interest (free body)
2. Draw your axis system (e.g., Cartesian, polar, path)
3. Add in applied forces (e.g., weight, 225 N pulling force)
4. Replace supports with forces (e.g., normal force)
5. Draw appropriate dimensions (usually angles for particles)

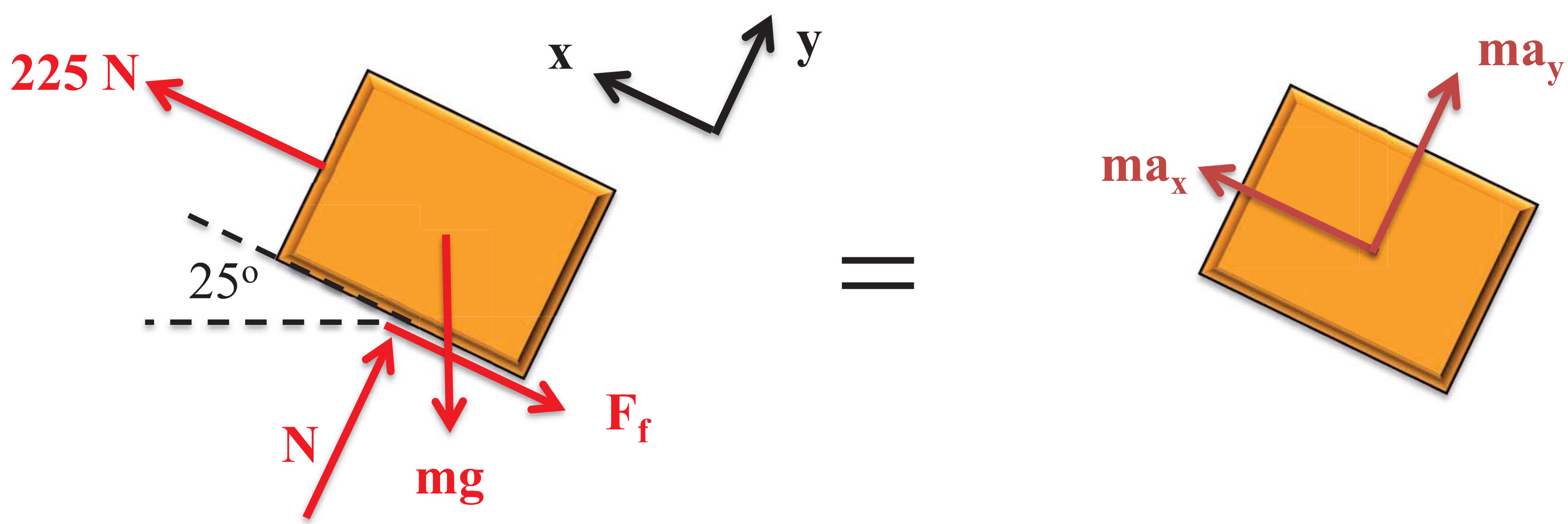


Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

Put the inertial terms for the body of interest on the kinetic diagram.

1. Isolate the body of interest (free body)
2. Draw in the mass times acceleration of the particle; if unknown, do this in the positive direction according to your chosen axes

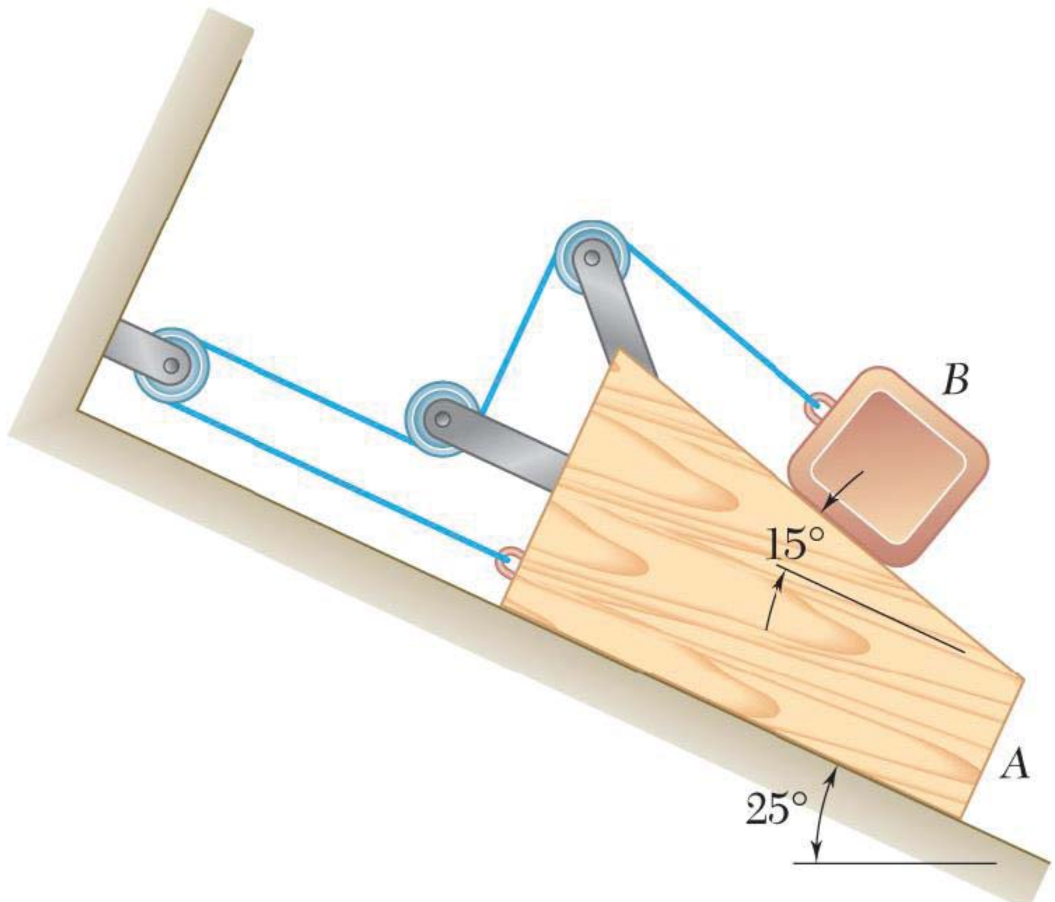


$$\Sigma F = ma$$

Kinetics of Particles: Newton’s Second Law

❑ Free Body Diagrams and Kinetic Diagrams

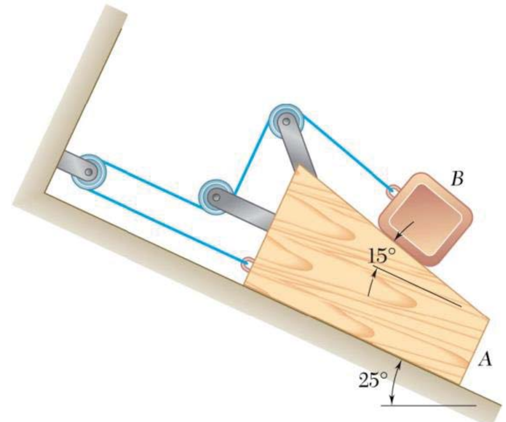
Draw the FBD and KD for block A (note that the massless, frictionless pulleys are attached to block A and should be included in the system).

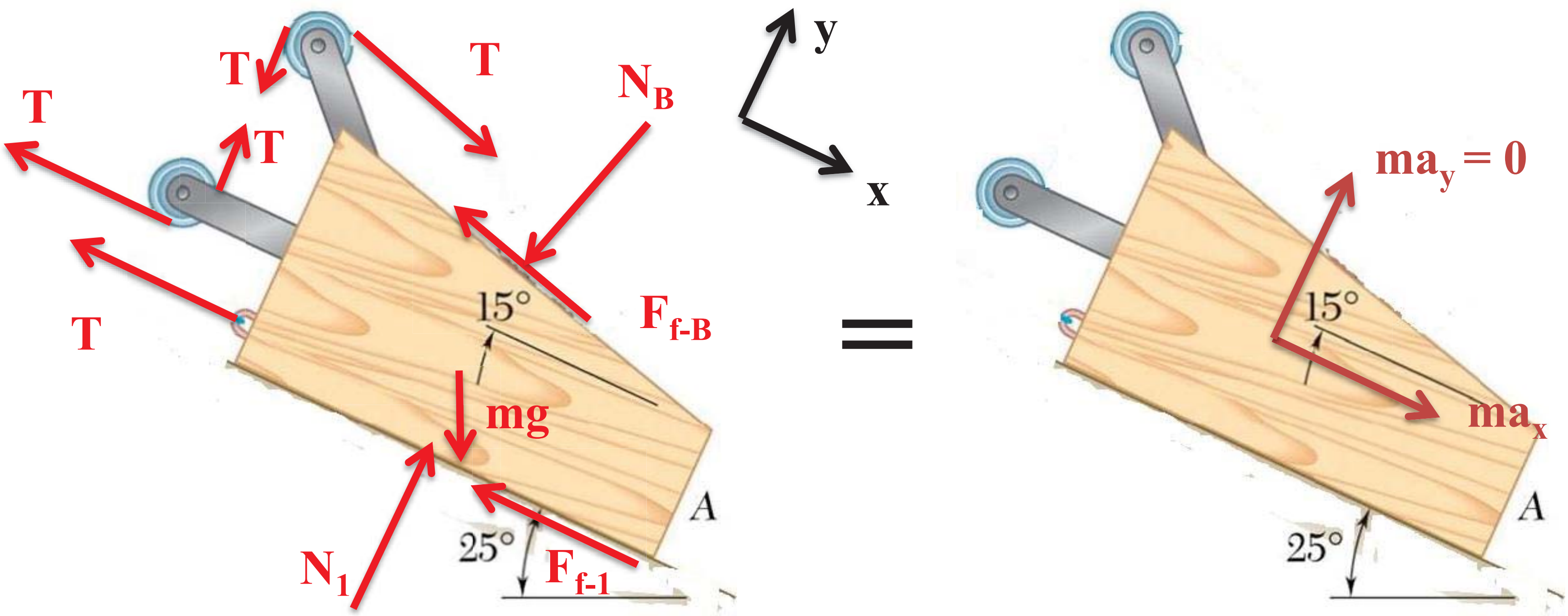


Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

1. Isolate body
2. Axes
3. Applied forces
4. Replace supports with forces
5. Dimensions (already drawn)
6. Kinetic diagram

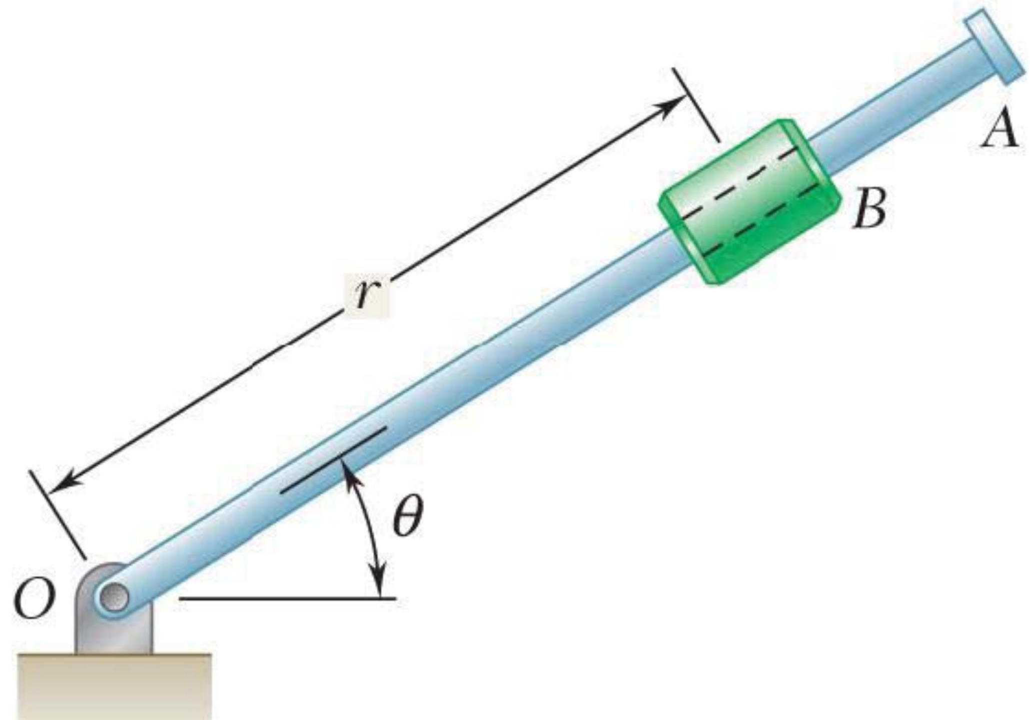




Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

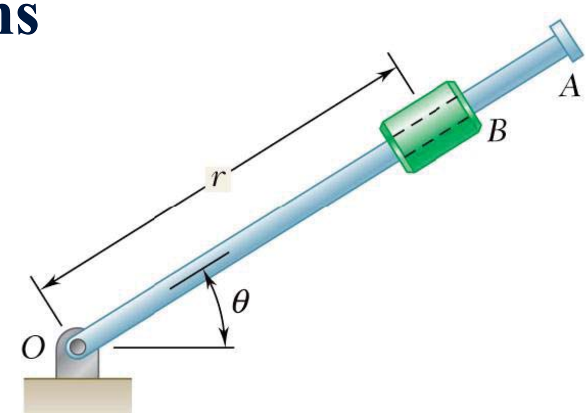
Draw the FBD and KD for the collar B. Assume there is friction acting between the rod and collar, motion is in the vertical plane, and θ is increasing

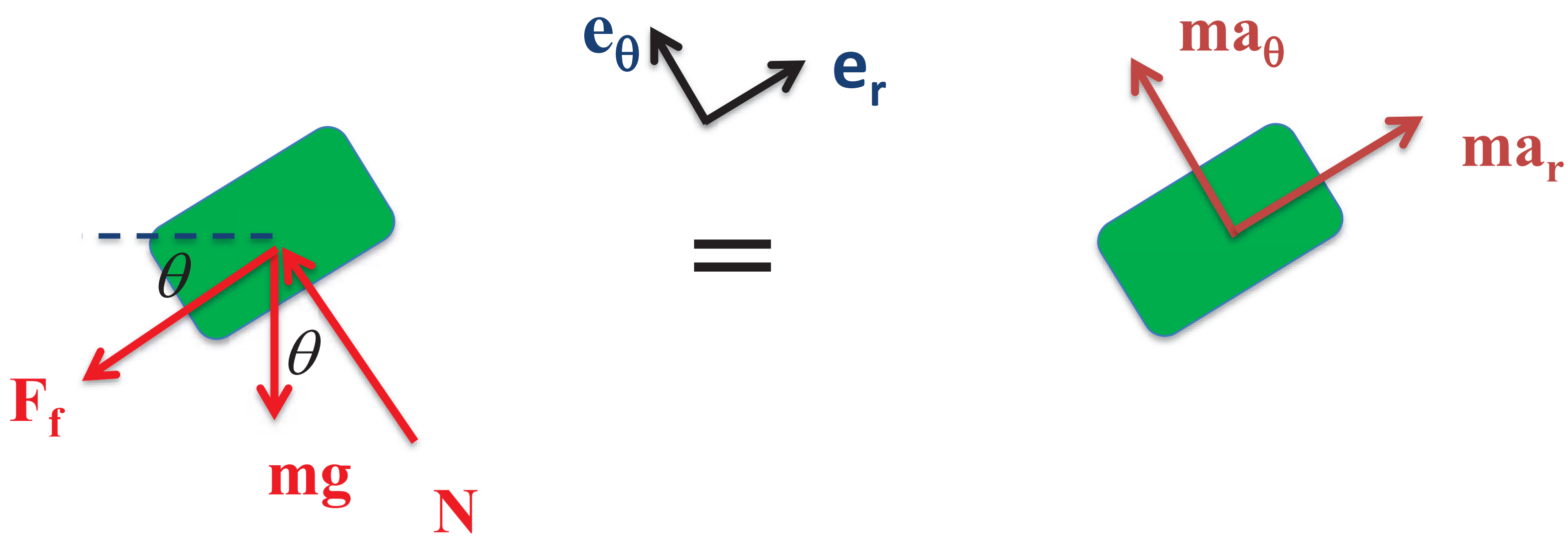


Kinetics of Particles: Newton's Second Law

❑ Free Body Diagrams and Kinetic Diagrams

1. Isolate body
2. Axes
3. Applied forces
4. Replace supports with forces
5. Dimensions
6. Kinetic diagram

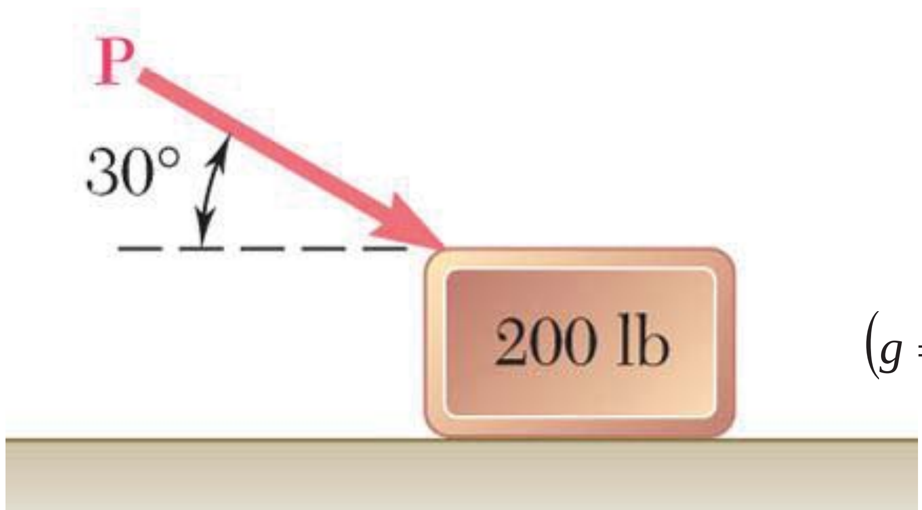




Kinetics of Particles: Newton's Second Law

❑ Sample Problem 01

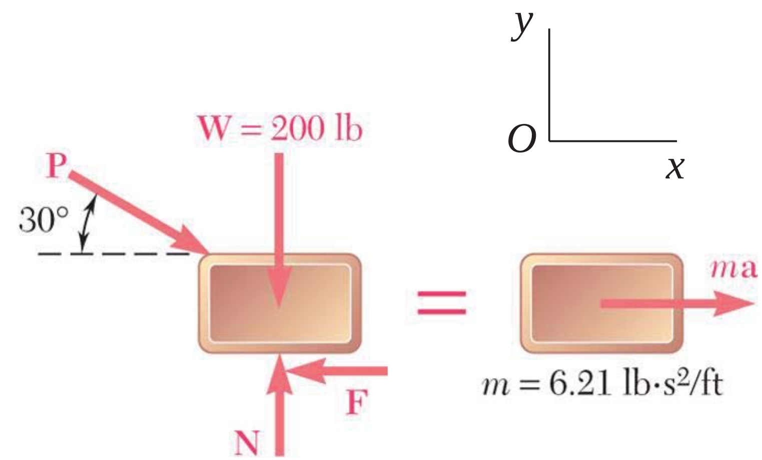
A 200-lb block rests on a horizontal plane. Find the magnitude of the force ***P*** required to give the block an acceleration of 10 ft/s^2 to the right. The coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$.



$\left(g = 32.2\ \text{ft/s}^2\right)$

Kinetics of Particles: Newton’s Second Law

- **Sample Problem 01**



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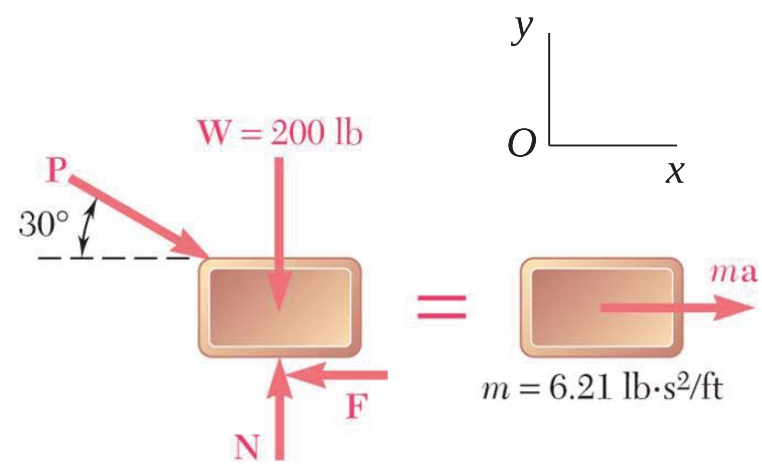
❑ Sample Problem 01

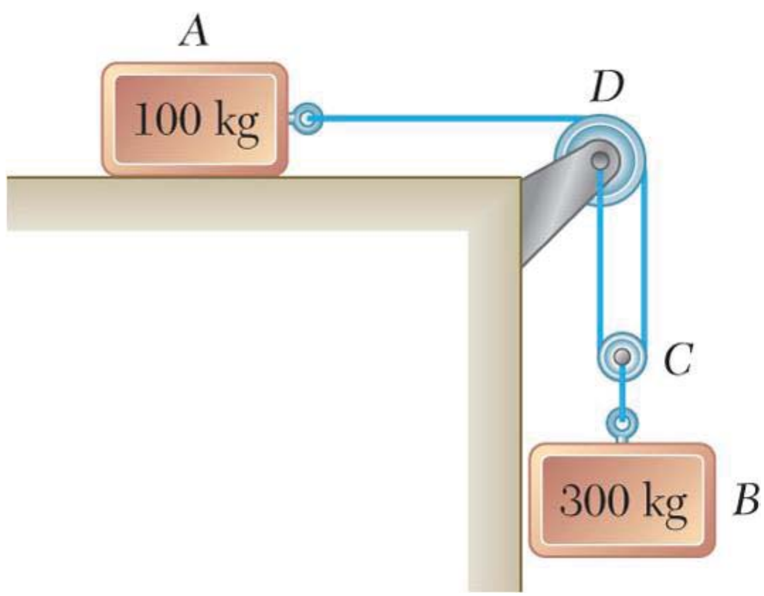
SOLUTION:



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 02

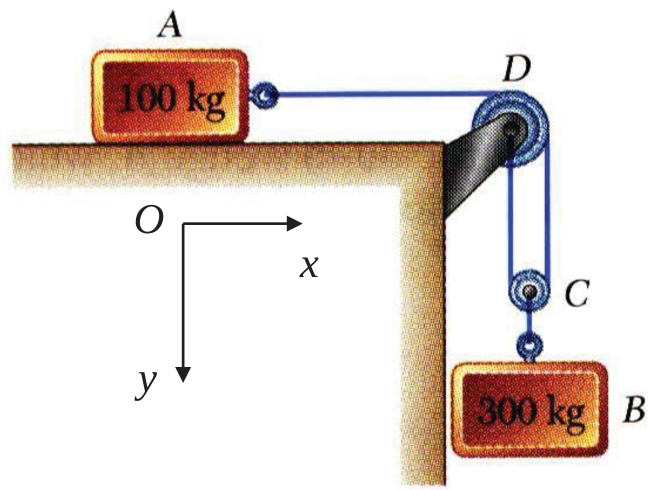
The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in the cord.

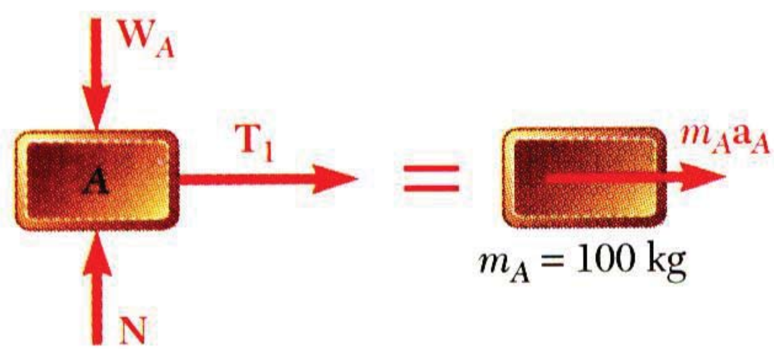


Kinetics of Particles: Newton's Second Law

❑ Sample Problem 02

SOLUTION:

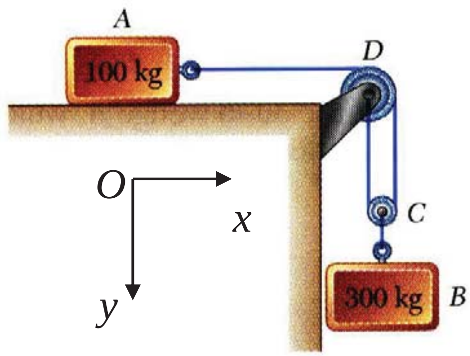


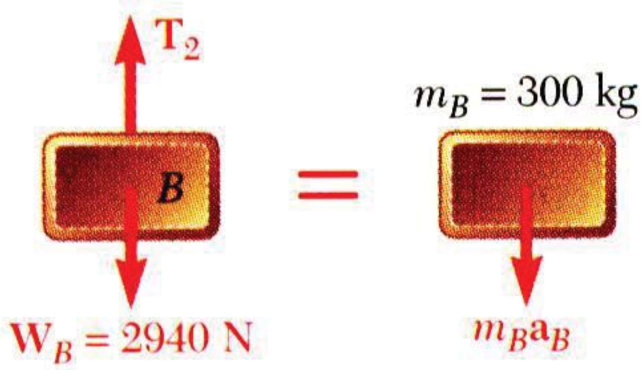


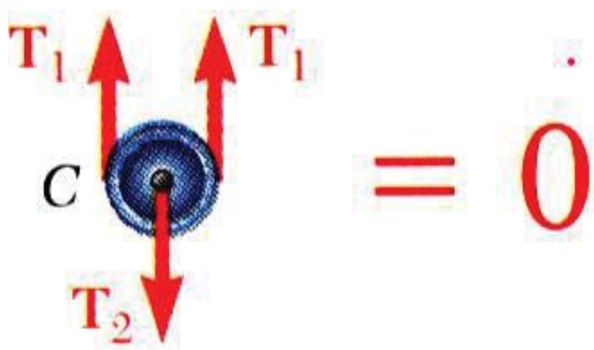
Kinetics of Particles: Newton's Second Law

❑ Sample Problem 02

SOLUTION:





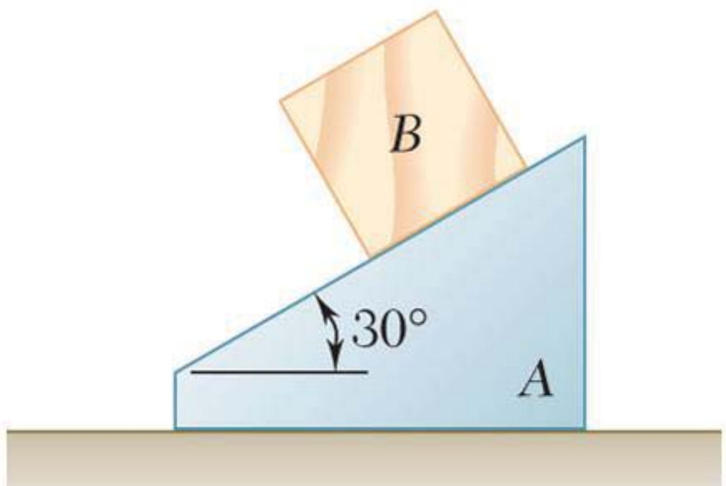


Kinetics of Particles: Newton's Second Law

❑ Sample Problem 03

The 12-lb block *B* starts from rest and slides on the 30-lb wedge *A*, which is supported by a horizontal surface.

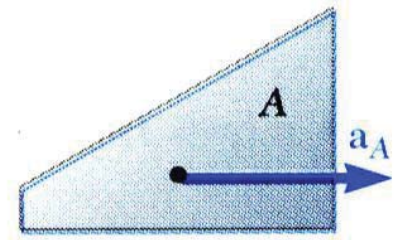
Neglecting friction, determine *(a)* the acceleration of the wedge A, and *(b)* the acceleration of the block B relative to the wedge A.

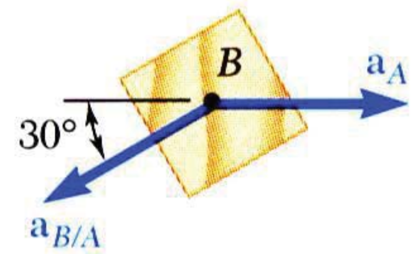


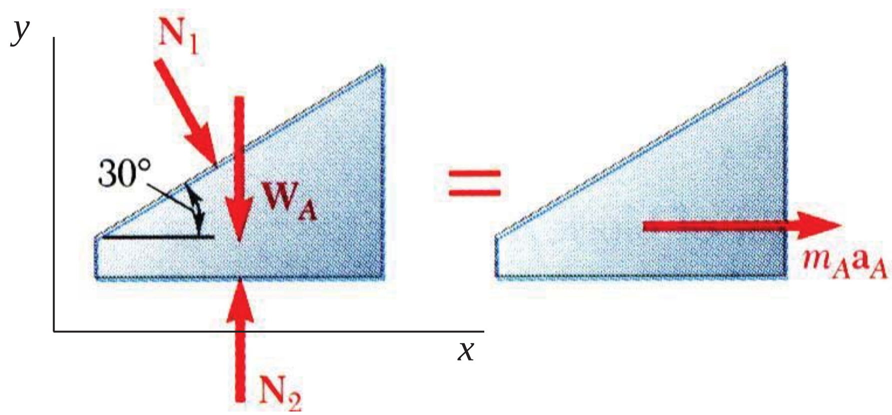
Kinetics of Particles: Newton's Second Law

❑ Sample Problem 03

SOLUTION:







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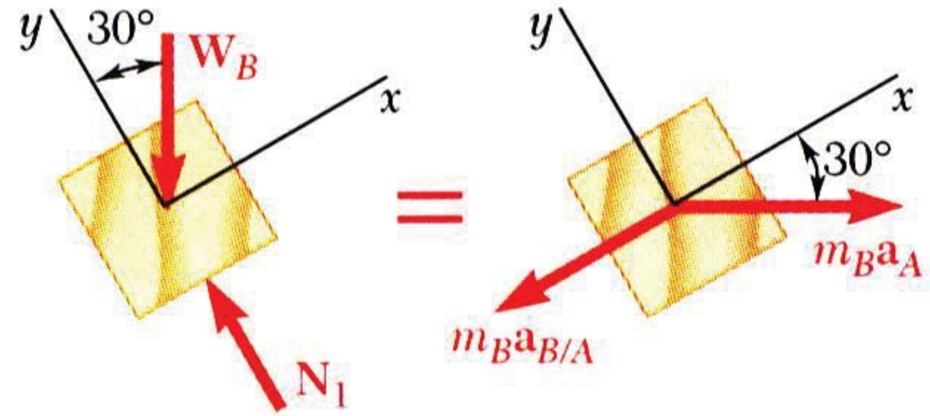
Kinetics of Particles: Newton's Second Law

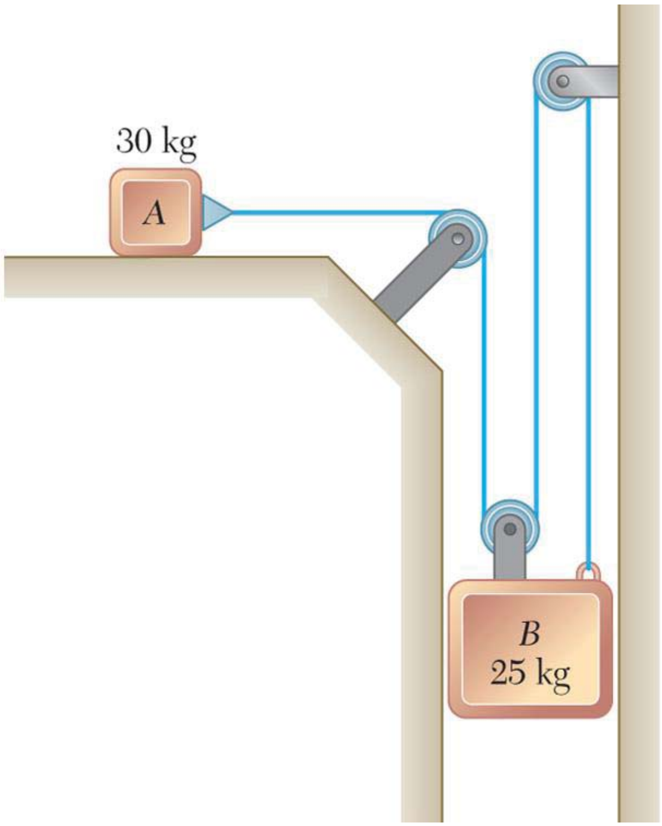
- **Sample Problem 03**



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 04

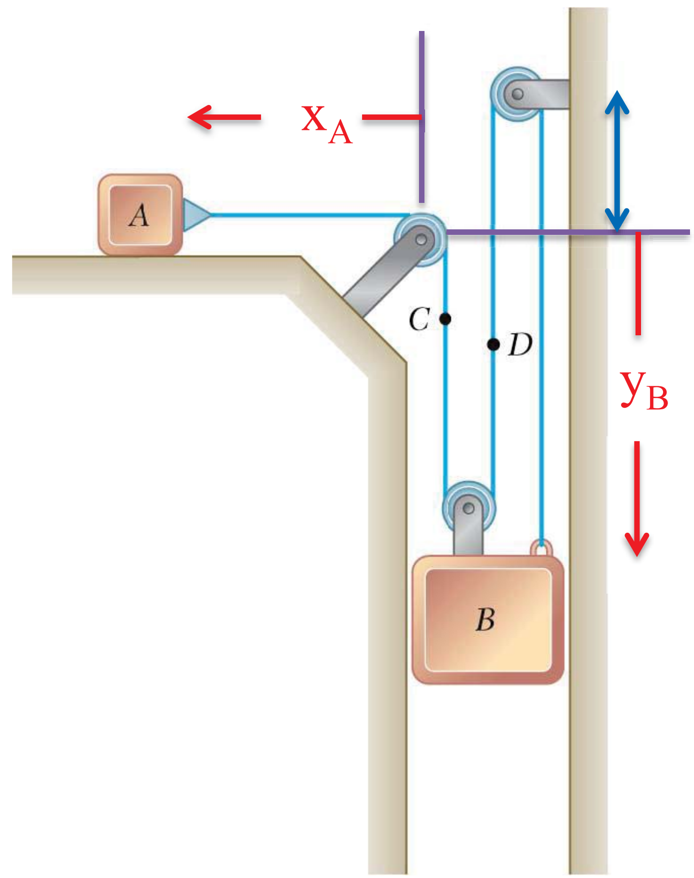
The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the horizontal surface, determine (a) the acceleration of each block, (b) the tension in the cable.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 04

SOLUTION:

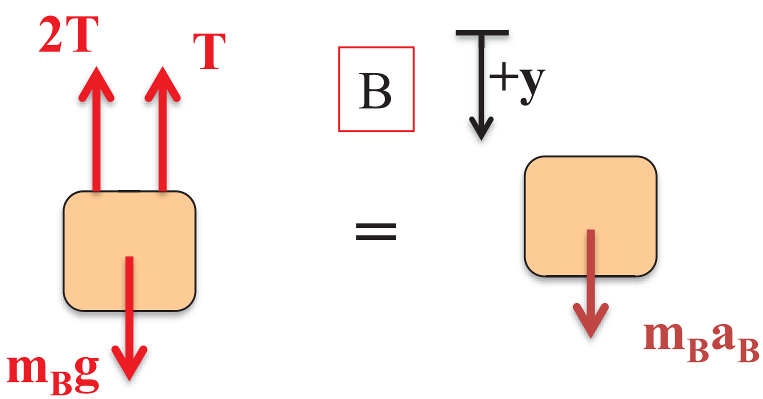


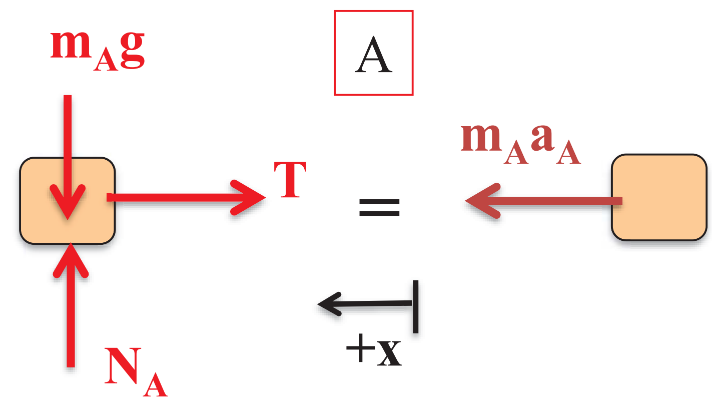
Kinetics of Particles: Newton's Second Law

❑ Sample Problem 04

SOLUTION:

- Draw the FBD and KD for each block





Kinetics of Particles: Newton's Second Law

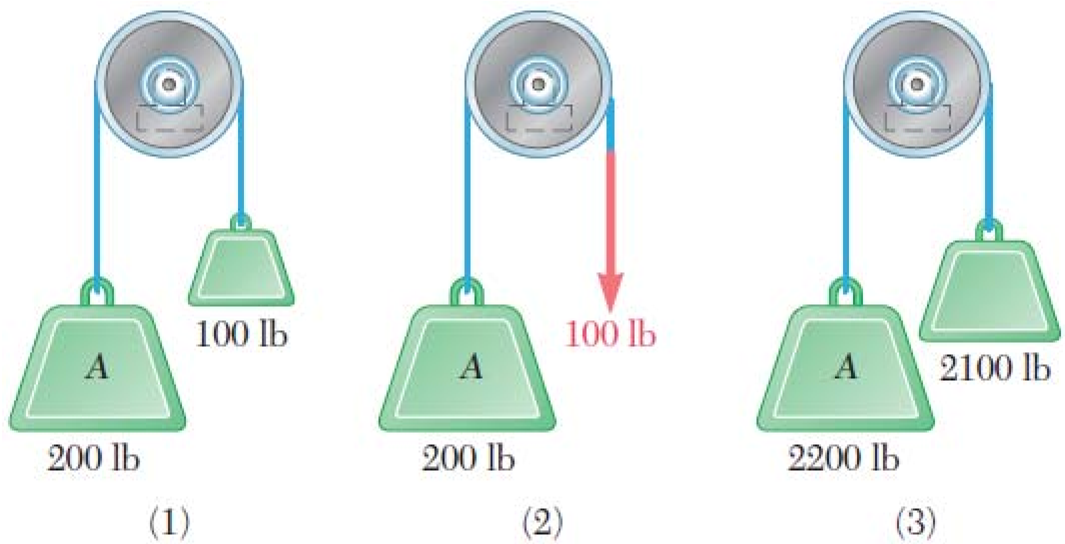
❑ Sample Problem 04

SOLUTION:

Kinetics of Particles: Newton's Second Law

❑ Sample Problem 05

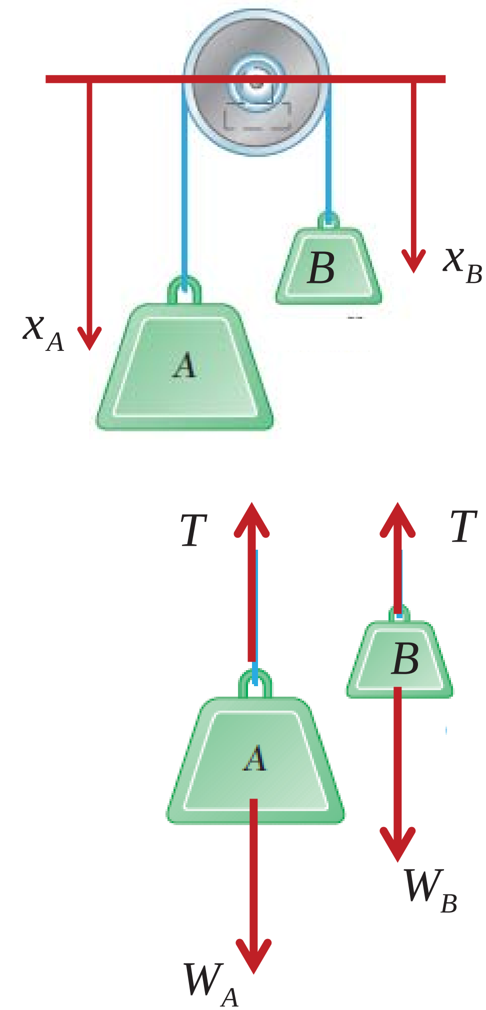
Each of the systems shown is initially at rest. Neglecting axle friction and the masses of the pulleys, determine for each system (a) the acceleration of block A, (b) the velocity of block A after it has moved through 10 ft, (c) the time required for block A to reach a velocity of 20 ft/s.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 05

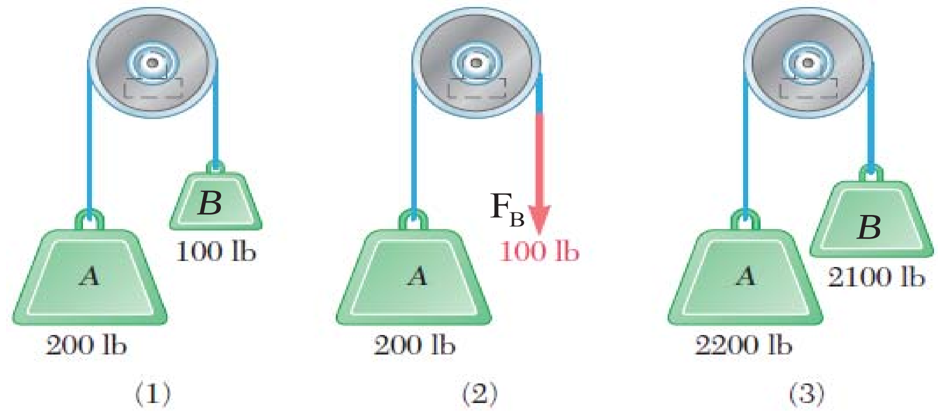
SOLUTION:



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 05

SOLUTION:



Kinetics of Particles: Newton's Second Law

❑ Kinetics: Normal and Tangential Coordinates

Aircraft and roller coasters can both experience large normal forces during turns.

Kinetics of Particles: Newton's Second Law

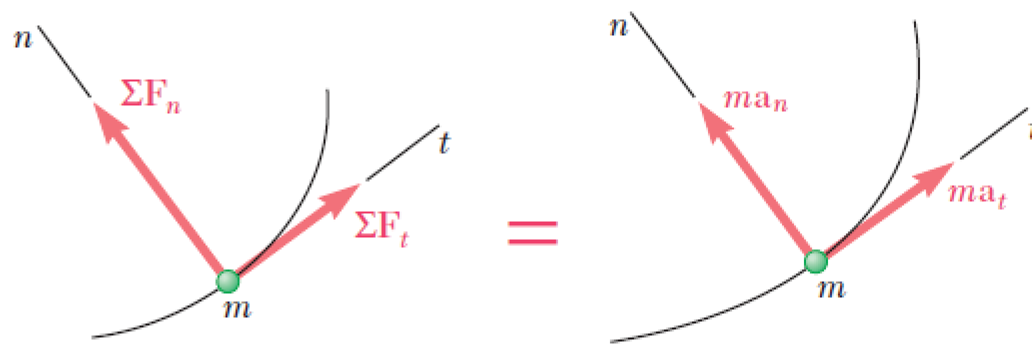
❑ Equations of Motion

- ***Tangential and Normal Components.*** Resolving the forces and the acceleration of the particle into components along the tangent to the path (in the direction of motion) and the normal (toward the inside of the path)

$$\sum \vec{F} = \sum (F_t \vec{e}_t + F_n \vec{e}_n) = m(a_t \vec{e}_t + a_n \vec{e}_n)$$

$$\sum F_t = ma_t \qquad \sum F_n = ma_n$$

$$\sum F_t = m\frac{dv}{dt} \qquad \sum F_n = m\frac{v^2}{\rho}$$



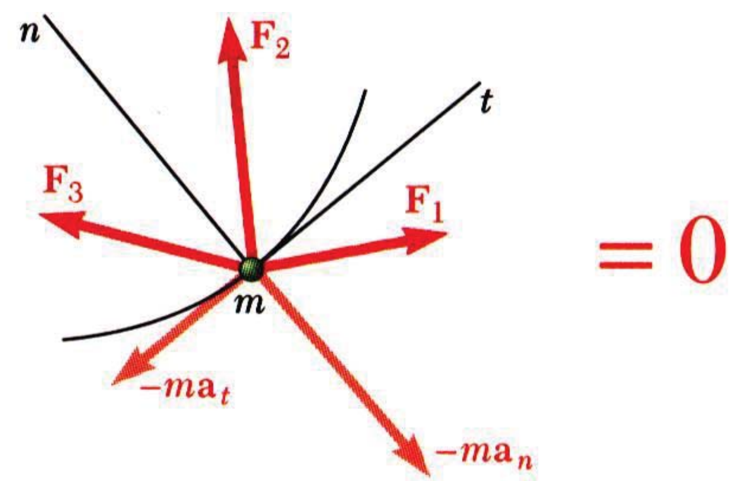
Kinetics of Particles: Newton's Second Law

❑ Dynamic Equilibrium

$$\sum \vec{F} - m\vec{a} = 0$$

$$-m\vec{a} \equiv inertial\ vector$$

The tangential component of the inertia vector provides a measure of the resistance the particle offers to a change in speed, while its normal component (also called centrifugal force) represents the tendency of the particle to leave its curved path.



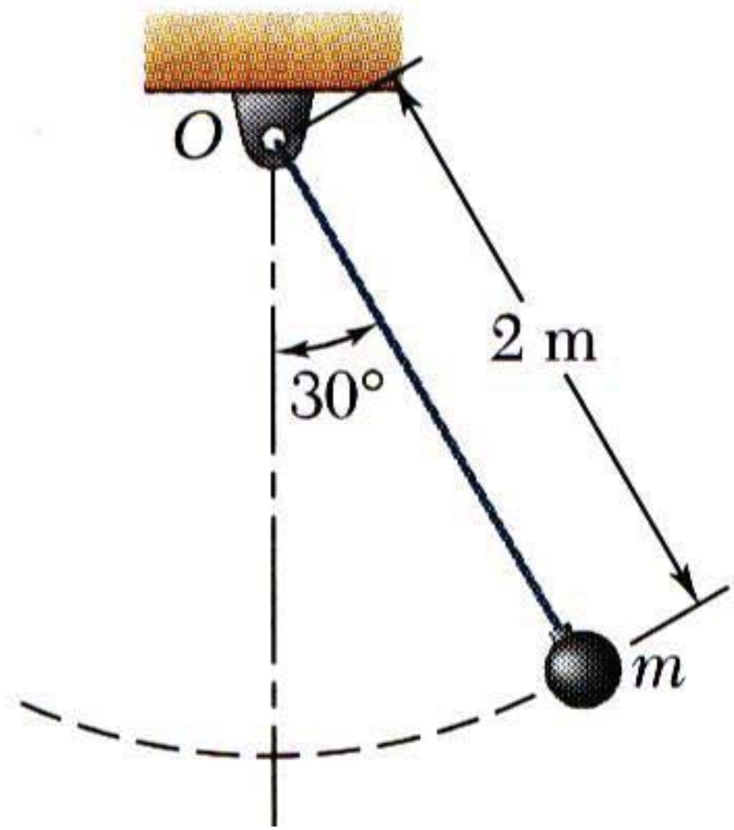
We should note that either of these two components may be zero under special conditions:

(1) if the particle starts from rest, its initial velocity is zero and the normal component of the inertia vector is zero at t = 0;

(2) if the particle moves at constant speed along its path, the tangential component of the inertia vector is zero and only its normal component needs to be considered.

❑ Sample Problem 06

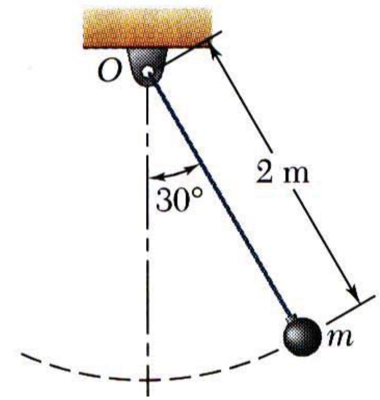
The bob of a 2-m pendulum describes an arc of a circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown, find the velocity and acceleration of the bob in that position.

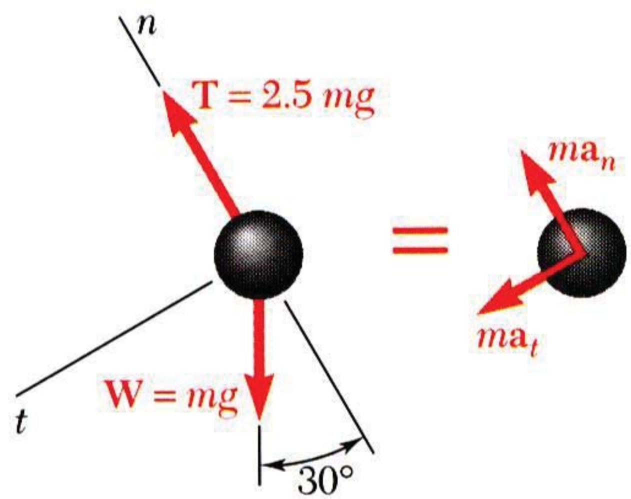


Kinetics of Particles: Newton's Second Law

❑ Sample Problem 06

SOLUTION:





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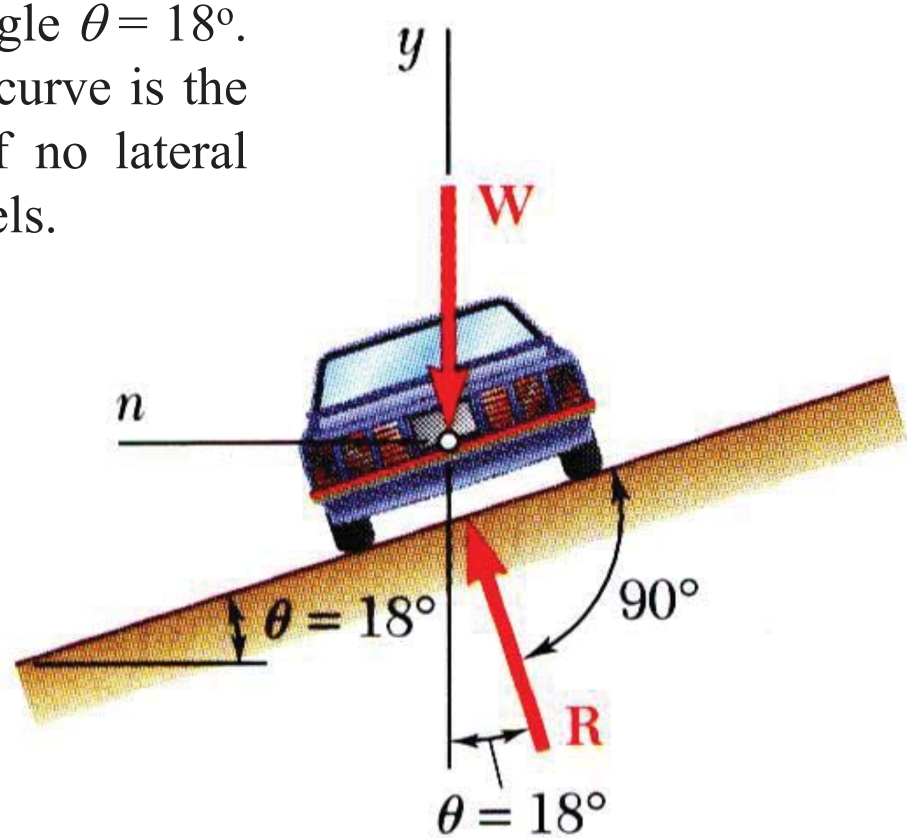
❑ Sample Problem 07

Determine the rated speed of a highway curve of radius $\rho = 400$ ft banked through an angle $\theta = 18^{o}$. The rated speed of a banked highway curve is the speed at which a car should travel if no lateral friction force is to be exerted at its wheels.



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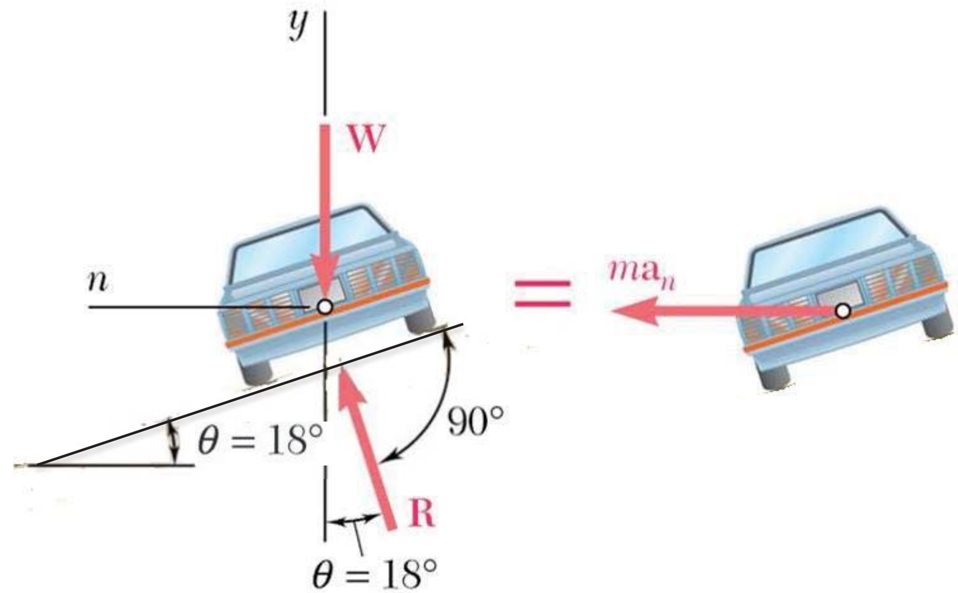
- **Sample Problem 07**

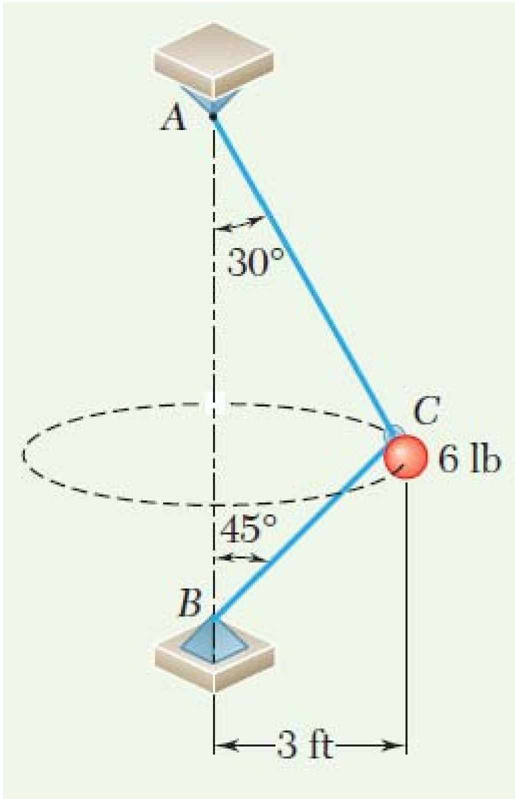
SOLUTION:



Kinetics of Particles: Newton's Second Law

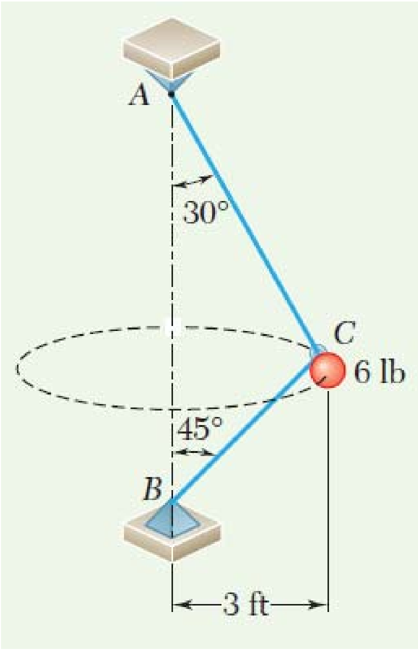
❑ Sample Problem

Two wires *AC* and *BC* are tied at *C* to a sphere that revolves at the constant speed *v* in the horizontal circle shown. Knowing that the wires will break if their tension exceeds 15 lb, determine the range of values of *v* for which both wires remain taut and the wires do not break



Kinetics of Particles: Newton's Second Law

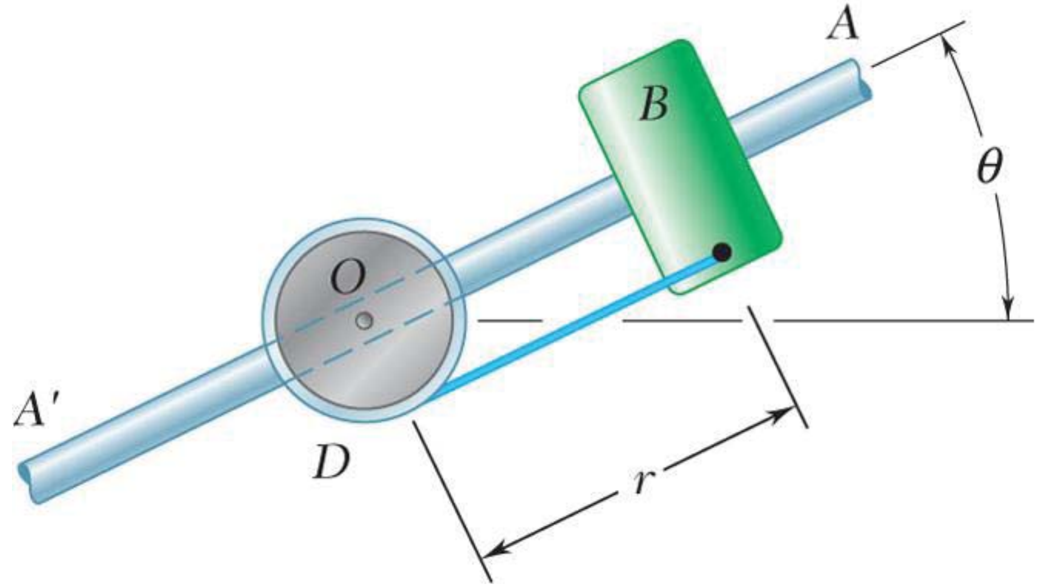
❑ Sample Problem



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 08

The 3-kg collar *B* rests on the frictionless arm *AA′*. The collar is held in place by the rope attached to drum *D* and rotates about *O* in a horizontal plane. The linear velocity of the collar B is increasing according to $v = 0.2\, t^2$ where *v* is in m/s and t is in sec. Find the tension in the rope and the force of the bar on the collar after 5 seconds if $r = 0.4$ m.



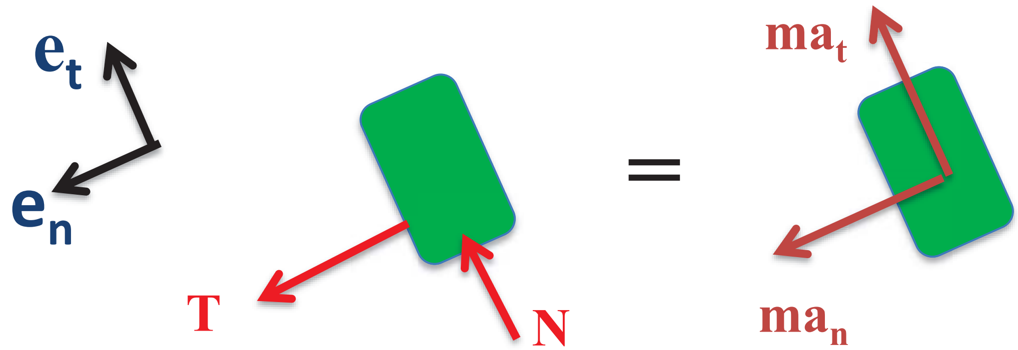
Kinetics of Particles: Newton's Second Law

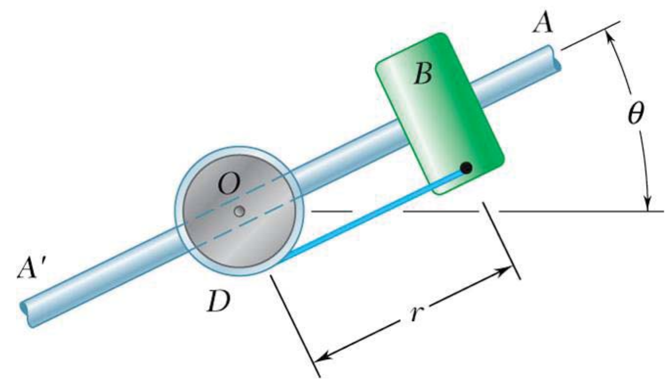
❑ Sample Problem 08

- Given: $v = 0.2\,t^2$, $r = 0.4$ m
- Find: T and N at $t = 5$ sec

SOLUTION:

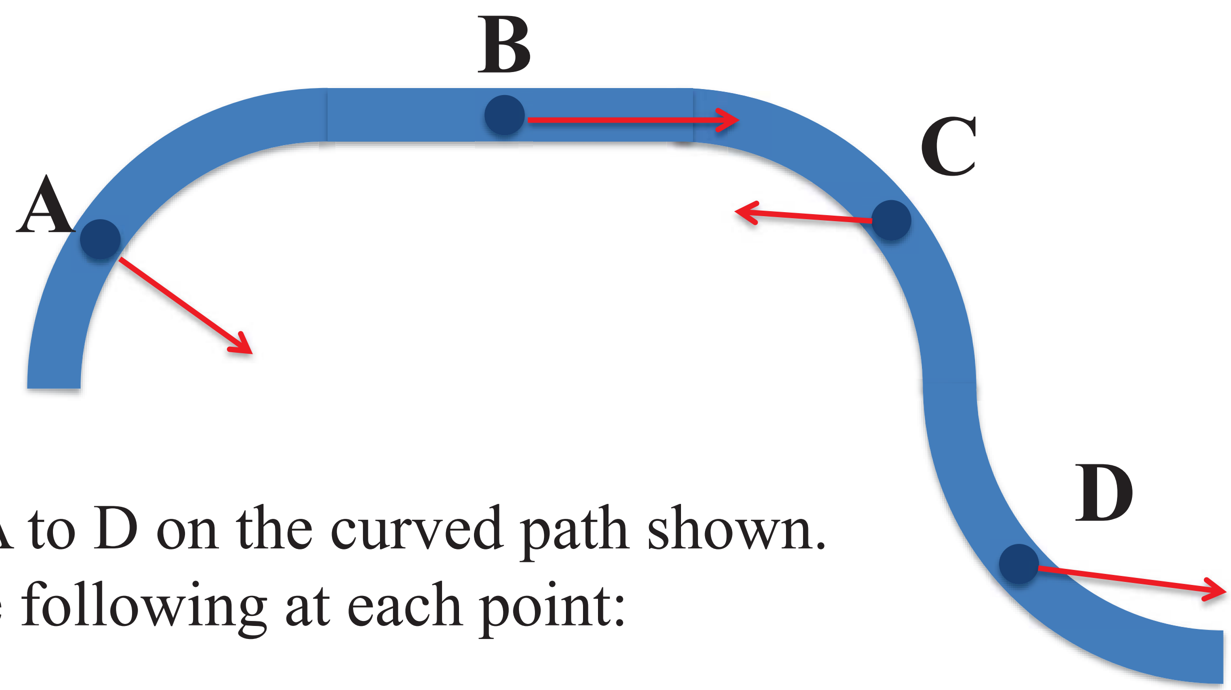
Draw the FBD and KD of the collar





Kinetics of Particles: Newton's Second Law

❑ Concept Question



A car is driving from A to D on the curved path shown.
The driver is doing the following at each point:

A – going at a constant speed
C – stepping on the brake

B – stepping on the accelerator
D – stepping on the accelerator

Draw the approximate direction of the car's acceleration at each point.

Kinetics of Particles: Newton's Second Law

❑ Kinetics: Radial and Transverse Coordinates

Hydraulic actuators and extending robotic arms are often analyzed using radial and transverse coordinates.

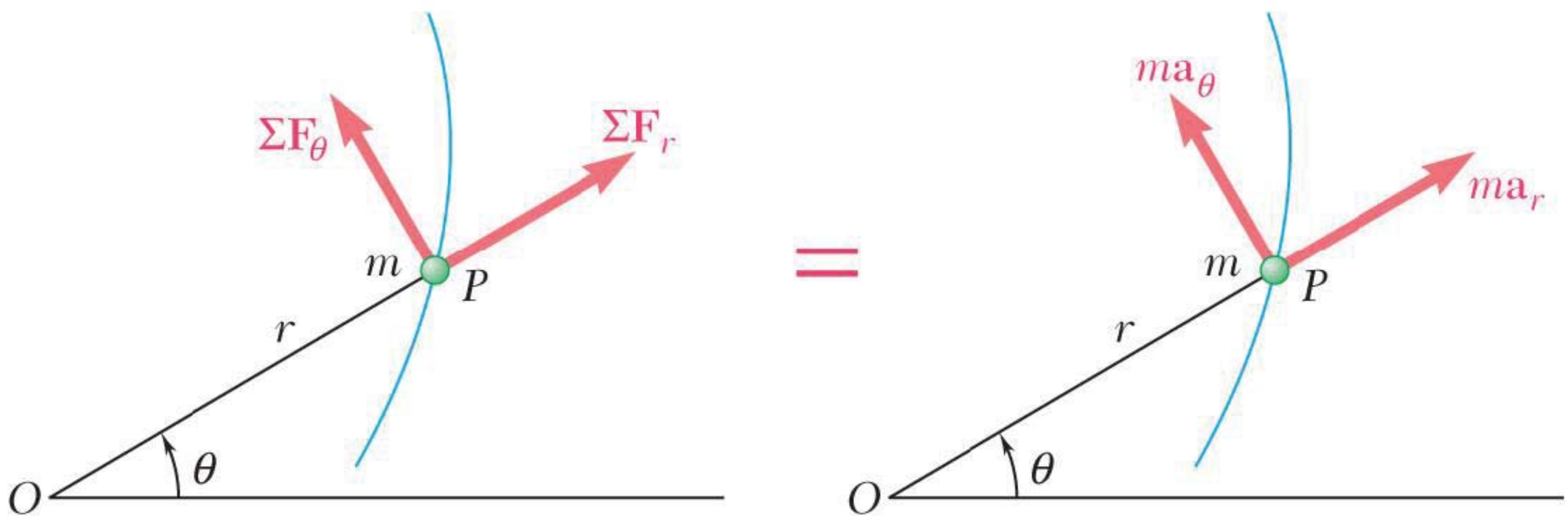
Kinetics of Particles: Newton's Second Law

❑ Equations of Motion in Radial & Transverse Components

- Consider particle at r and θ, in polar coordinates,

$$\left.\begin{aligned}\sum F_r &= ma_r \\ a_r &= (\ddot{r} - r\dot{\theta}^2)\end{aligned}\right\} \Rightarrow \boxed{\sum F_r = m(\ddot{r} - r\dot{\theta}^2)}$$

$$\left.\begin{aligned}\sum F_\theta &= ma_\theta \\ a_\theta &= (r\ddot{\theta} + 2\dot{r}\dot{\theta})\end{aligned}\right\} \Rightarrow \boxed{\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})}$$



Kinetics of Particles: Newton's Second Law

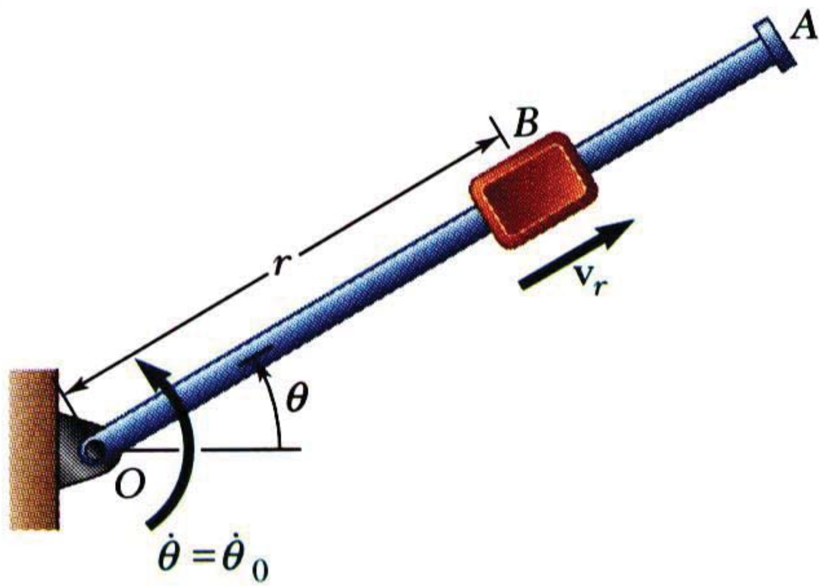
❑ Sample Problem 09

A block B of mass m can slide freely on a frictionless arm OA which rotates in a horizontal plane at a constant $\dot{\theta}_0$ rate.

Knowing that B is released at a distance r_0 from O, express as a function of r

a) the component v_r of the velocity of B along OA, and

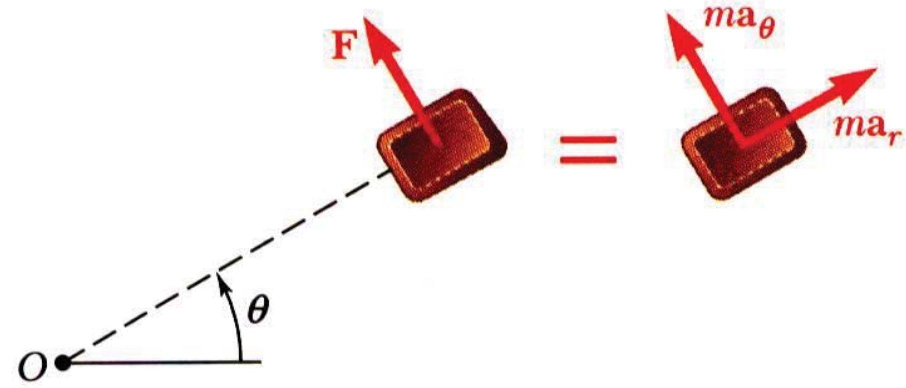
b) the magnitude of the force exerted on B by the arm OA.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 09

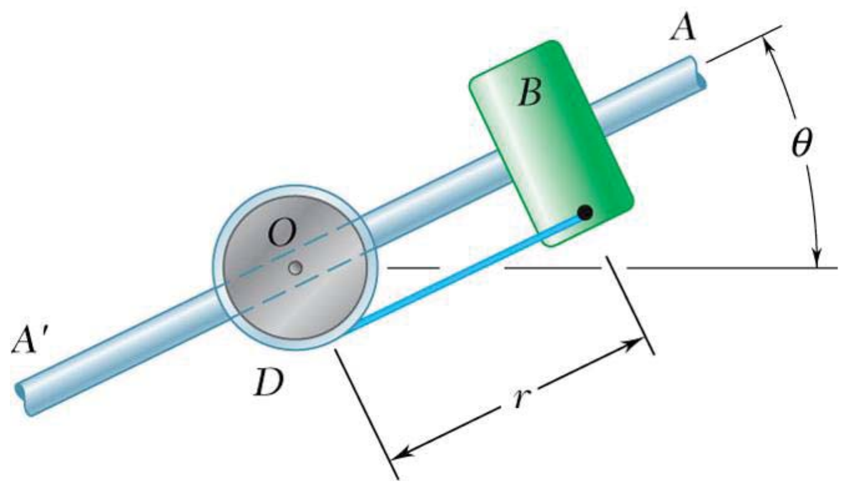
SOLUTION:



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 10

The 3-kg collar *B* slides on the frictionless arm *AA′*. The arm is attached to drum *D* and rotates about *O* in a horizontal plane at the rate $\dot{\theta} = 0.75t$ where $\dot{\theta}$ and *t* are expressed in rad/s and seconds, respectively. As the arm-drum assembly rotates, a mechanism within the drum releases the cord so that the collar moves outward from *O* with a constant speed of 0.5 m/s. Knowing that at $t = 0$, $r = 0$, determine the time at which the tension in the cord is equal to the magnitude of the horizontal force exerted on *B* by arm *AA′*.



Kinetics of Particles: Newton's Second Law

❑ Sample Problem 10

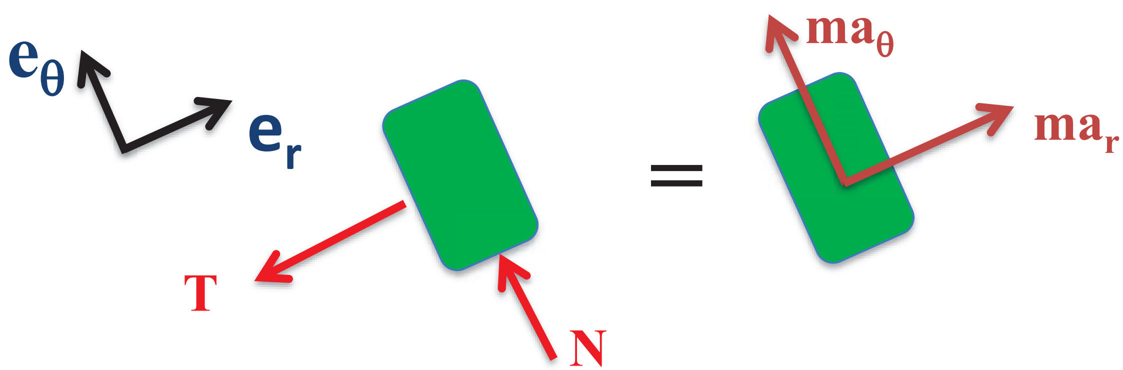
- Given: $\dot{\theta} = 0.75t \quad r_{(0)} = 0$

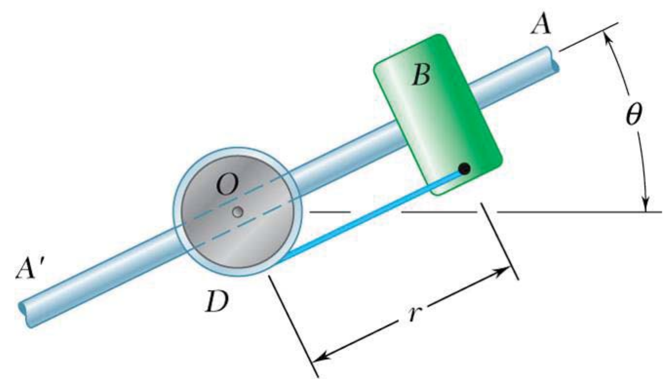
  $\dot{r} = 0.5\,(m/s)$

- Find: time when T = N

SOLUTION:

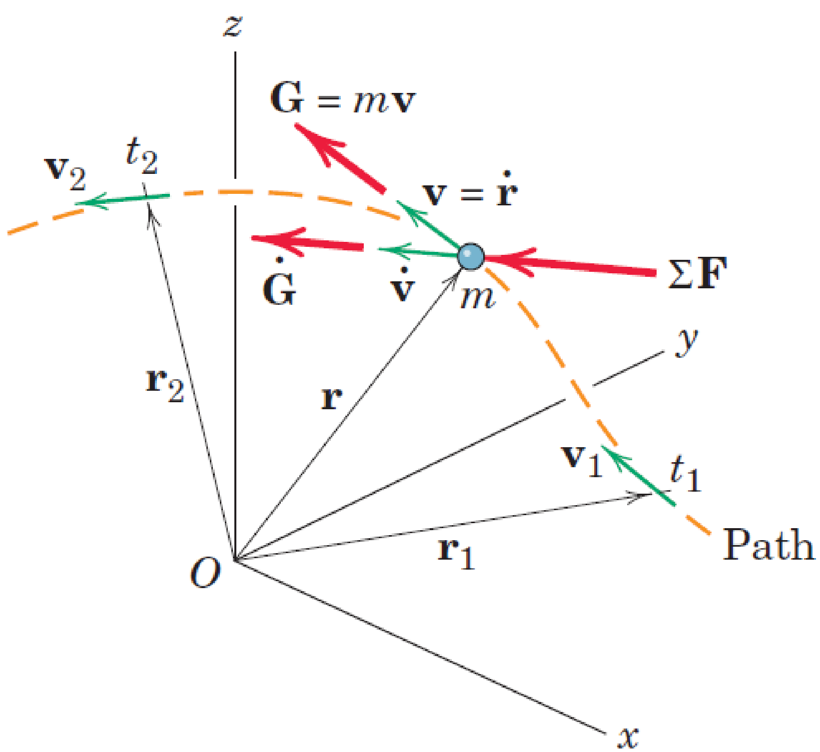
Draw the FBD and KD of the collar





Kinetics of Particles: Impulse and Momentum

❑ Principle of Linear Impulse and Momentum for a System of Particles



$$\Sigma \mathbf{F} = m\dot{\mathbf{v}} = \frac{d}{dt}(m\mathbf{v}) \qquad \text{or} \qquad \boxed{\Sigma \mathbf{F} = \dot{\mathbf{G}}}$$

linear momentum
$\mathbf{G} = m\mathbf{v}$

The resultant of all forces acting on a particle equals its time rate of change of linear momentum.

we recognize that, in addition to the equality of the magnitudes of $\sum \boldsymbol{F}$ and $\dot{\boldsymbol{G}}$, the direction of the resultant force coincides with the direction of the rate of change in linear momentum, which is the direction of the rate of change in velocity.

Kinetics of Particles: Impulse and Momentum

Principle of Linear Impulse and Momentum for a System of Particles

We now write the three scalar components as

$$\Sigma F_x = \dot{G}_x \qquad \Sigma F_y = \dot{G}_y \qquad \Sigma F_z = \dot{G}_z$$

The Linear Impulse-Momentum Principle

$$\boxed{\Sigma \mathbf{F} = \dot{\mathbf{G}}} \quad \Rightarrow \quad \Sigma \mathbf{F}\, dt = d\mathbf{G} \quad \Rightarrow \quad \overbrace{\int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt}^{\text{linear impulse}} = \mathbf{G}_2 - \mathbf{G}_1 = \Delta \mathbf{G}$$

$$\boxed{\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2}$$

The product of force and time is defined as the *linear impulse* of the force, and mentioned equation states that:
the total linear impulse on m equals the corresponding change in linear momentum of m.

Kinetics of Particles: Impulse and Momentum

Principle of Linear Impulse and Momentum for a System of Particles

$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$$

$$m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x\, dt = m(v_2)_x$$

$$m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y\, dt = m(v_2)_y$$

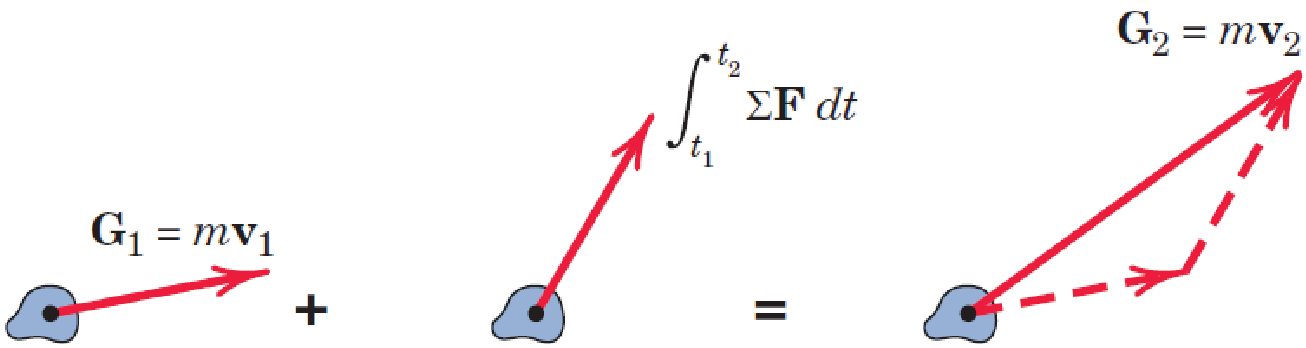
$$m(v_1)_z + \int_{t_1}^{t_2} \Sigma F_z\, dt = m(v_2)_z$$

These three scalar impulse-momentum equations are completely independent

Kinetics of Particles: Impulse and Momentum

Principle of Linear Impulse and Momentum for a System of Particles

$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = \mathbf{G}_2$$



Kinetics of Particles: Impulse and Momentum

Conservation of Linear Momentum for a System of Particles

$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2$$

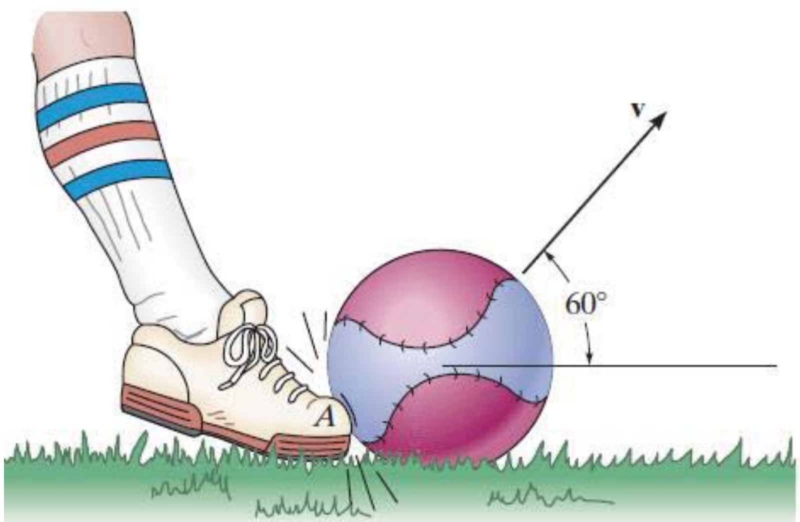
When the sum of the *external impulses* acting on a system of particles is *zero*, equation reduces to a simplified form, namely

$$\Sigma m_i (\mathbf{v}_i)_1 = \Sigma m_i (\mathbf{v}_i)_2$$

This equation is referred to as the *conservation of linear momentum*. It states that the total linear momentum for a system of particles remains constant during the time period t_1 to t_2.

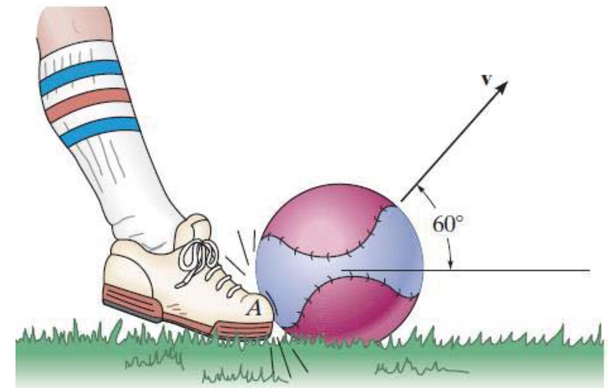
❑ Sample Problem 11

A man kicks the 150-g ball such that it leaves the ground at an angle of 60° and strikes the ground at the same elevation a distance of 12 m away. Determine the impulse of his foot on the ball at *A*. Neglect the impulse caused by the ball's weight while it's being kicked.



Kinetics of Particles: Impulse and Momentum

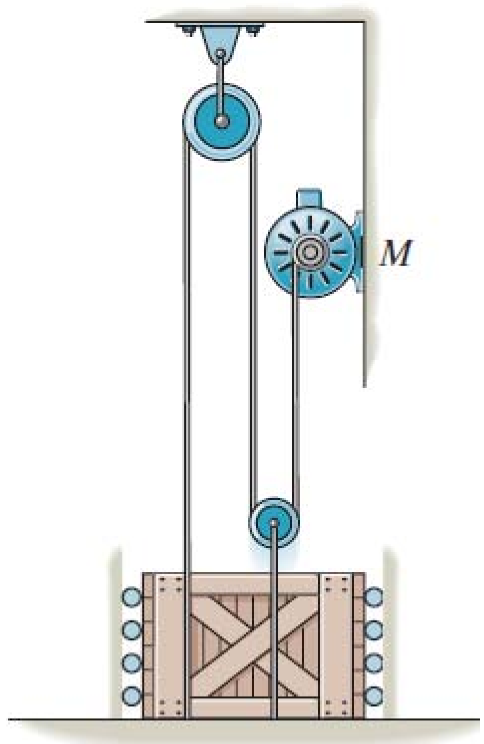
- **Sample Problem 11**



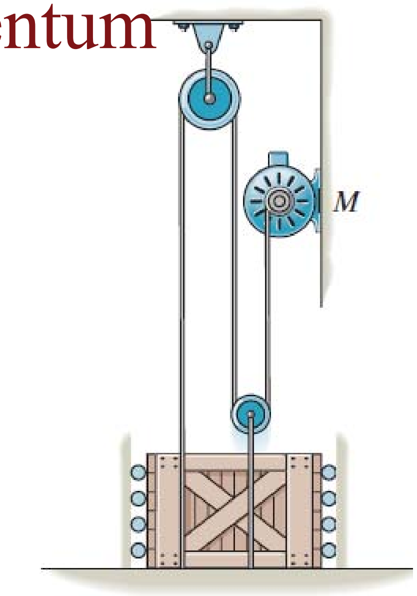
Kinetics of Particles: Impulse and Momentum

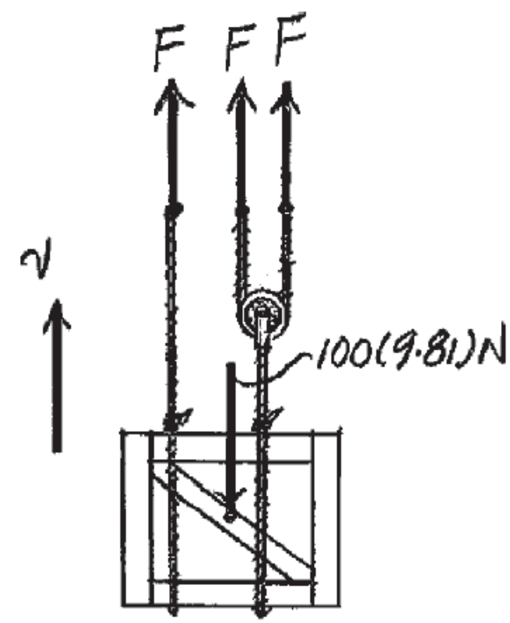
❑ Sample Problem 12

The motor, M, pulls on the cable with a force $F = (10t^2 + 300)$ N, where t is in seconds. If the 100 kg crate is originally at rest at $t = 0$, determine its speed when $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



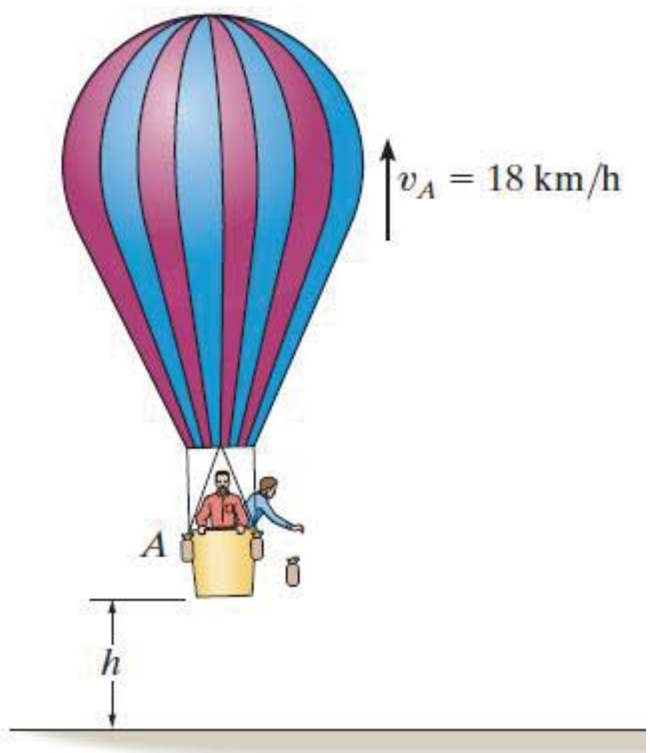
- **Sample Problem 12**





Sample Problem 13

The balloon has a total mass of 400 kg including the passengers and ballast. The balloon is rising at a constant velocity of 18 km/h when $h = 10$ m. If the man drops the 40-kg sand bag, determine the velocity of the balloon when the bag strikes the ground. Neglect air resistance.

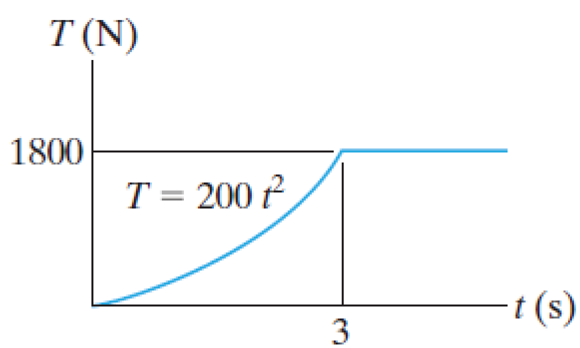


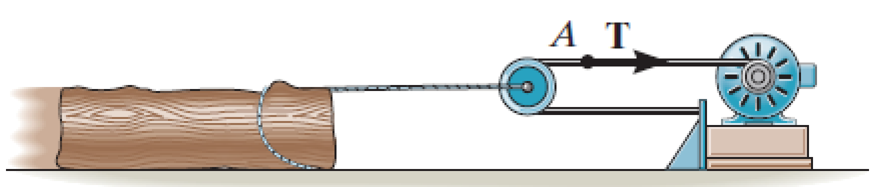
- **Sample Problem 13**

Kinetics of Particles: Impulse and Momentum

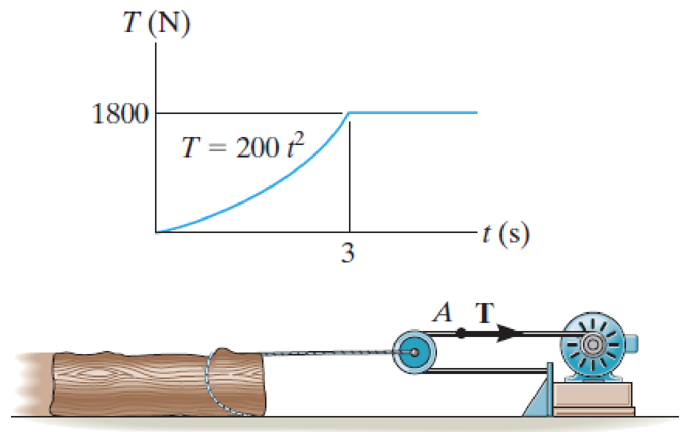
❑ Sample Problem 14

The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.



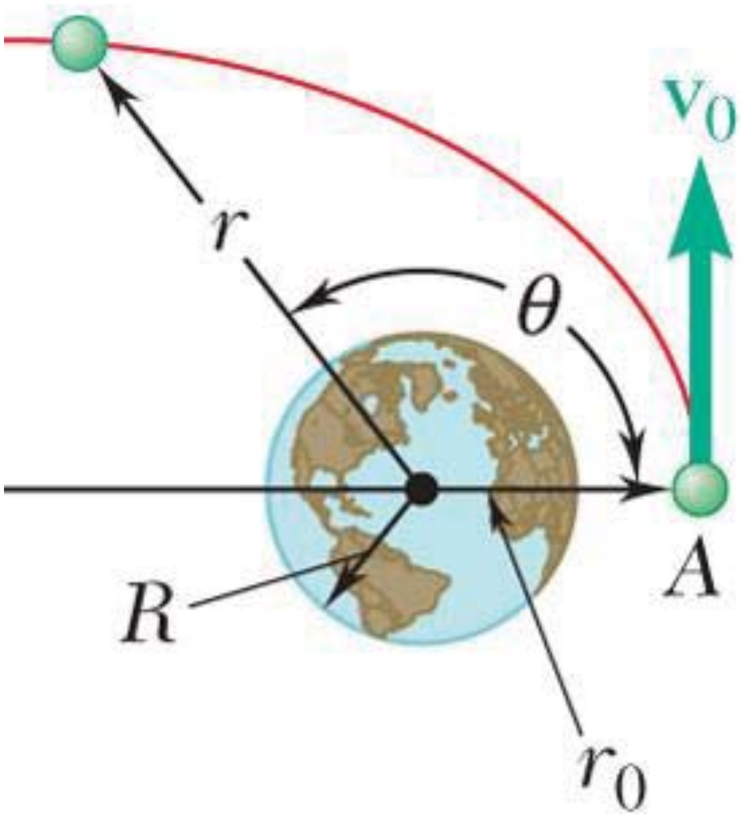


- **Sample Problem 14**



❑ Angular Momentum of a Particle

Satellite orbits are analyzed using conservation of angular momentum.

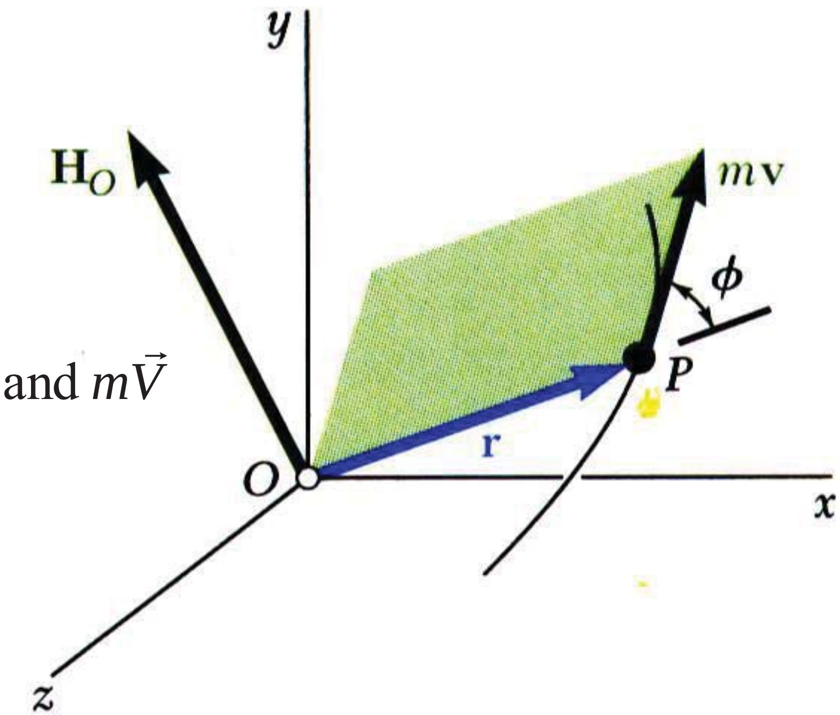


Kinetics of Particles: Impulse and Momentum

❑ Angular Momentum of a Particle

- The moment about O of the vector mv is called the ***moment of momentum***, or the ***angular momentum***, of the particle about O at that instant and is denoted by **Ho**.

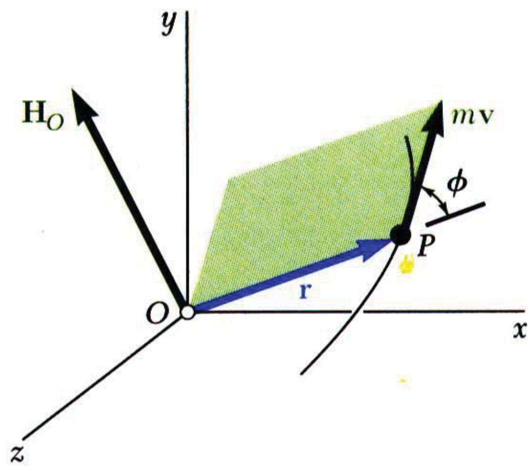
$$\vec{H}o = \vec{r} \times m\vec{v}$$



- $\vec{H}_O$ is ***perpendicular*** to plane containing $\vec{r}$ and $m\vec{V}$

$$Ho = r\, m\, v \sin\phi \quad (kg \cdot m^2 / s)$$

❑ Angular Momentum of a Particle



The components of **Ho** in ***Cartesian coordinates***

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$m\vec{v} = mv_x\vec{i} + mv_y\vec{j} + mv_z\vec{k}$$

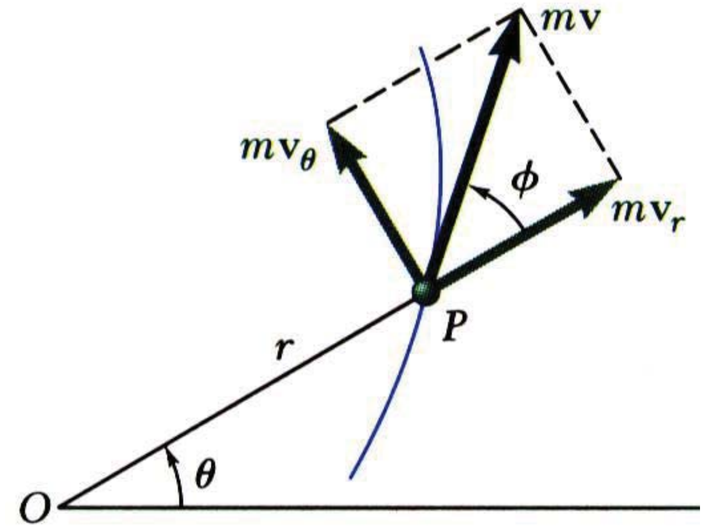
$$\Rightarrow \quad Ho = r \times mv = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix} \Rightarrow \boxed{Ho = H_x\vec{i} + H_y\vec{j} + H_z\vec{k}} \Rightarrow \boxed{\begin{cases} H_x = m(yv_z - zv_y) \\ H_y = m(zv_x - xv_z) \\ H_z = m(xv_y - yv_x) \end{cases}}$$

In the case of a particle moving in the ***xy plane***

$$\begin{matrix} z = 0 \\ v_z = 0 \end{matrix} \Rightarrow \boxed{\begin{matrix} H_x = H_y = 0 \\ H_O = H_z = m(xv_y - yv_x) \end{matrix}}$$

Kinetics of Particles: Impulse and Momentum

❑ Angular Momentum of a Particle



The components of **Ho** in ***polar coordinates***

$$Ho = r\,m\,v\sin\phi = r\,m\,v_\theta$$

$$v_\theta = r\,\dot{\theta} \quad \Rightarrow \quad \boxed{Ho = mr^2\dot{\theta}}$$

• Derivative of angular momentum with respect to time,

$$\vec{H}o = \vec{r}\times m\vec{v} \quad \Rightarrow \quad \frac{d(\vec{H}o)}{dt} = \dot{\vec{H}}_O = \dot{\vec{r}}\times m\vec{V} + \vec{r}\times m\dot{\vec{V}} = \underbrace{\vec{V}\times m\vec{V}}_{0} + \vec{r}\times m\vec{a}$$

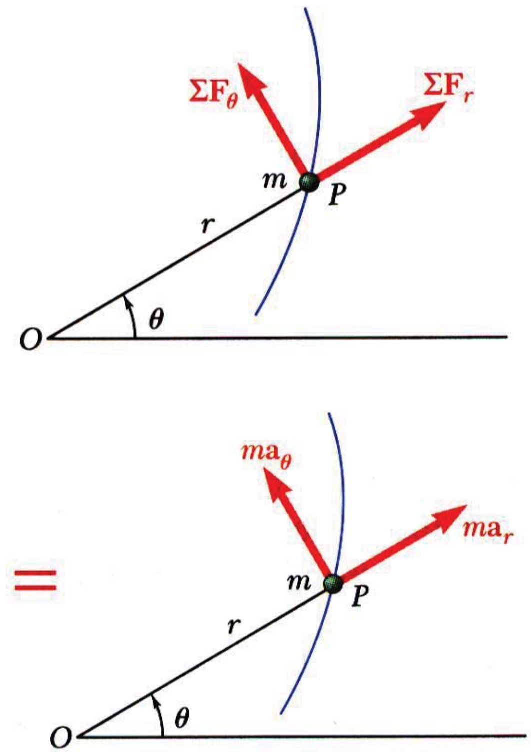
$$\Rightarrow \quad \dot{\vec{H}}_O = \vec{r}\times m\vec{a} = \vec{r}\times\sum\vec{F} \quad \Rightarrow \quad \boxed{\dot{\vec{H}}_O = \sum\vec{M}_O}$$

• It follows from Newton's second law that ***the sum of the moments about O of the forces acting on the particle is equal to the rate of change of the angular momentum of the particle about O.***

Kinetics of Particles: Impulse and Momentum

❑ Equations of Motion in Radial & Transverse Components

- Consider particle at r and θ, in polar coordinates,



- Consider particle at r and θ, in polar coordinates,

$$\sum F_r = m(\ddot{r} - r\dot{\theta}^2) \qquad \sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \quad (*)$$

- Equation (*) could have been derived from

$$\sum \vec{M}_O = \dot{\vec{H}}_O \quad \Rightarrow \quad r\sum F_\theta = \frac{d}{dt}(mr^2\dot{\theta}) = m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta})$$

$$\Rightarrow \sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Kinetics of Particles: Impulse and Momentum

❑ Principle of Angular Impulse and Momentum

$$\sum \vec{M}_O = \dot{\vec{H}}_O$$

$$\Sigma \mathbf{M}_O \, dt = d\mathbf{H}_O$$

$$\Sigma \int_{t_1}^{t_2} \mathbf{M}_O \, dt = (\mathbf{H}_O)_2 - (\mathbf{H}_O)_1$$

$$(\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O \, dt = (\mathbf{H}_O)_2$$

Kinetics of Particles: Impulse and Momentum

❑ Principle of Angular Impulse and Momentum

$$(\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O \, dt = (\mathbf{H}_O)_2$$

This equation is referred to as the *principle of angular impulse and momentum*. The initial and final angular momenta $(\mathbf{H}_O)_1$ and $(\mathbf{H}_O)_2$ are defined as the moment of the linear momentum of the particle ($\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$) at the instants t_1 and t_2, respectively. The second term on the left side, $\Sigma \int \mathbf{M}_O \, dt$, is called the *angular impulse*. It is determined by integrating, with respect to time, the moments of all the forces acting on the particle over the time period t_1 to t_2. Since the moment of a force about point O is $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$, the angular impulse may be expressed in vector form as

$$\text{angular impulse} = \int_{t_1}^{t_2} \mathbf{M}_O \, dt = \int_{t_1}^{t_2} (\mathbf{r} \times \mathbf{F}) \, dt$$

Kinetics of Particles: Impulse and Momentum

❑ Principle of Angular Impulse and Momentum

$$m\mathbf{v}_1 + \sum \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$$

$$(\mathbf{H}_O)_1 + \sum \int_{t_1}^{t_2} \mathbf{M}_O\, dt = (\mathbf{H}_O)_2$$

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$(H_O)_1 + \sum \int_{t_1}^{t_2} M_O\, dt = (H_O)_2$$

Kinetics of Particles: Impulse and Momentum

❑ Principle of Angular Impulse and Momentum

Conservation of Angular Momentum

When the angular impulses acting on a particle are all zero during the time t_1 to t_2

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

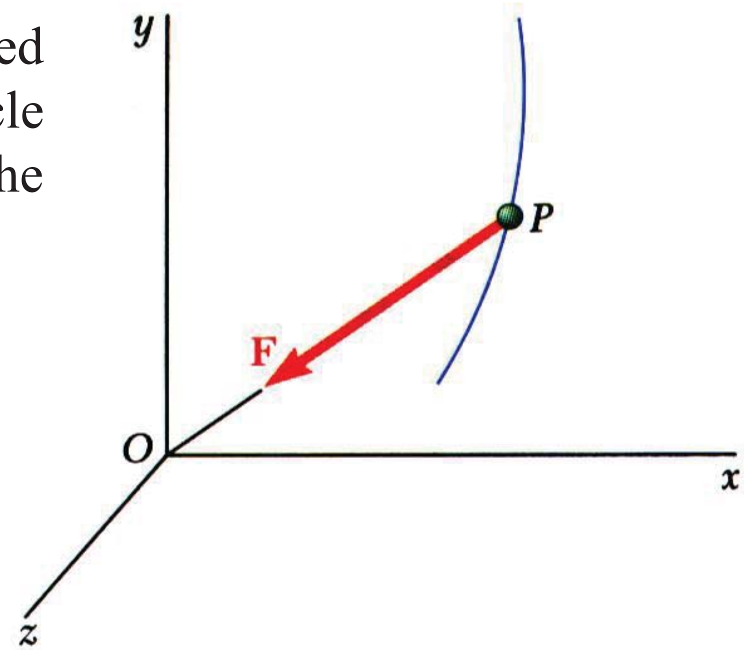
we can also write the conservation of angular momentum for a system of particles as

$$\Sigma(\mathbf{H}_O)_1 = \Sigma(\mathbf{H}_O)_2$$

Kinetics of Particles: Impulse and Momentum

Conservation of Angular Momentum

When only force acting on particle is directed toward or away from a fixed point O, the particle is said to be ***moving under a central force*** and the point O is referred to as the ***center of force.***



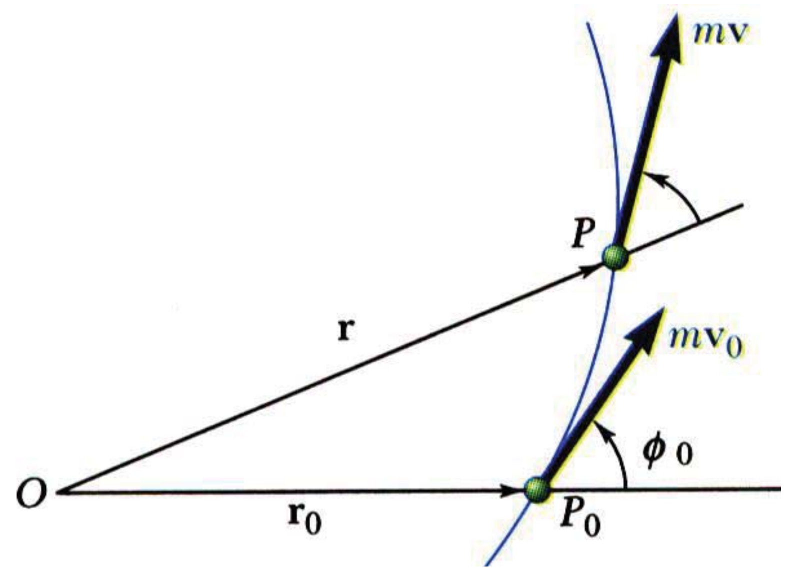
- Since the line of action of the central force passes through O,

$$\sum \vec{M}_O = \dot{\vec{H}}_O = 0 \quad \Rightarrow \quad \boxed{\vec{H}_O = \vec{r} \times m\vec{V} = cte}$$

We thus conclude that the angular momentum of a particle moving under a central force is constant, in both magnitude and direction.

❑ Conservation of Angular Momentum

moving under a central force



- Position vector and motion of particle are in a plane perpendicular to $\vec{H}_O$.

- Magnitude of angular momentum,

$$H_O = r_0 m V_0 \sin\phi_0 = rmV \sin\phi = cte$$

or

$$H_O = mr^2\dot{\theta} = \text{cte} \quad \Rightarrow \quad \frac{H_O}{m} = r^2\dot{\theta} = h = \frac{\text{angular momentum}}{\text{unit mass}}$$

Kinetics of Particles: Impulse and Momentum

❑ Conservation of Angular Momentum

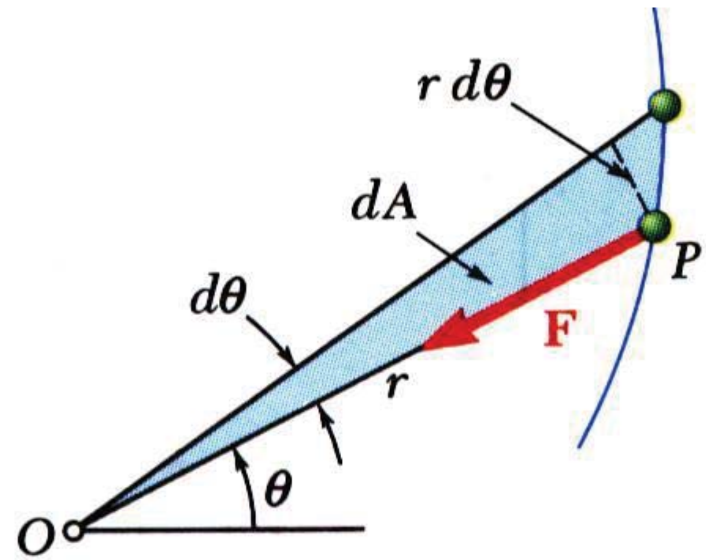
- Radius vector *OP* sweeps infinitesimal area

$$dA = \tfrac{1}{2} r^2 d\theta$$

- Define $\dfrac{dA}{dt} = \tfrac{1}{2} r^2 \dfrac{d\theta}{dt} = \tfrac{1}{2} r^2 \dot{\theta} =$ ***areal velocity***

- Recall, for a body *moving under a central force*,

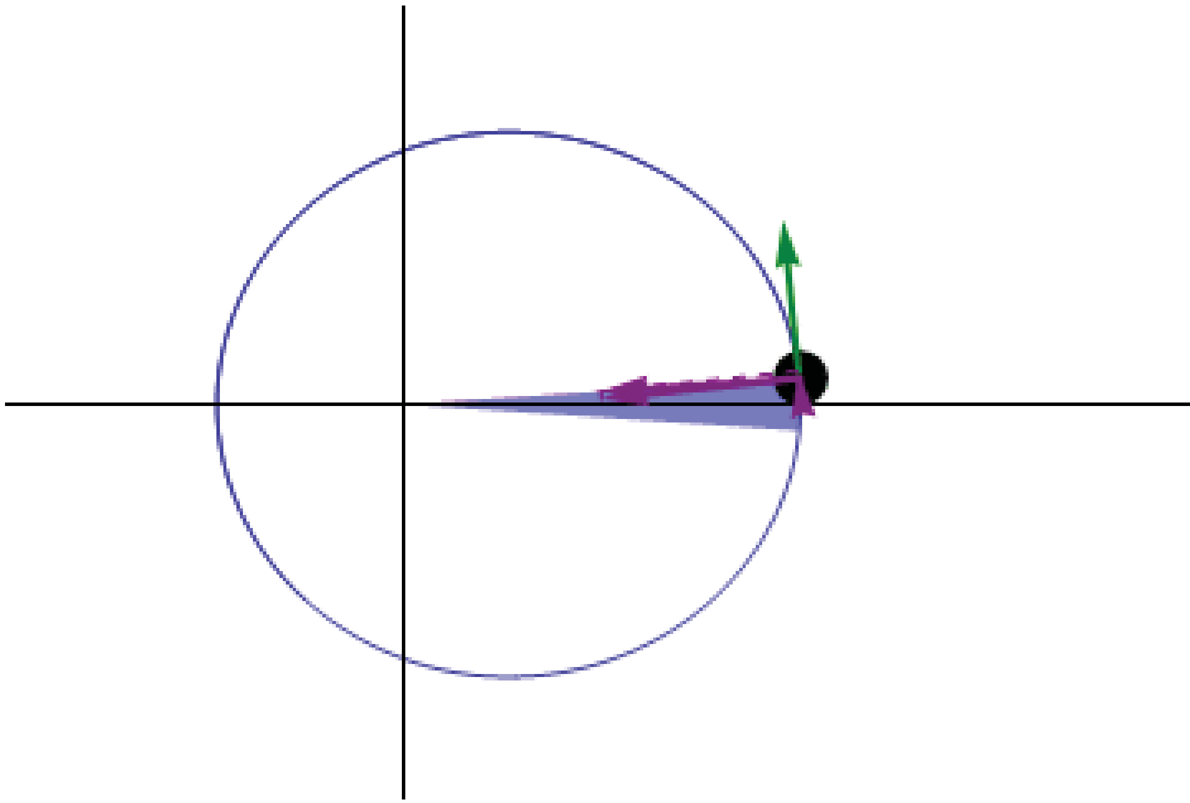
$$h = r^2 \dot{\theta} = \text{constant}$$

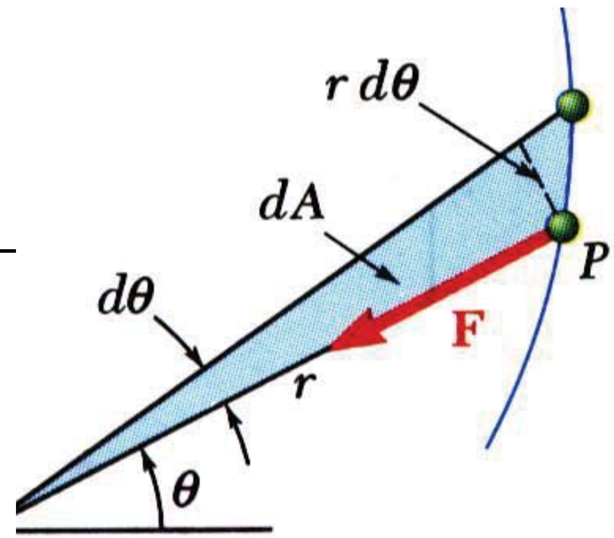


In classical mechanics, **areal velocity** is the rate at which area is swept out by a particle as it moves along a curve.

Kinetics of Particles: Impulse and Momentum

❑ Conservation of Angular Momentum



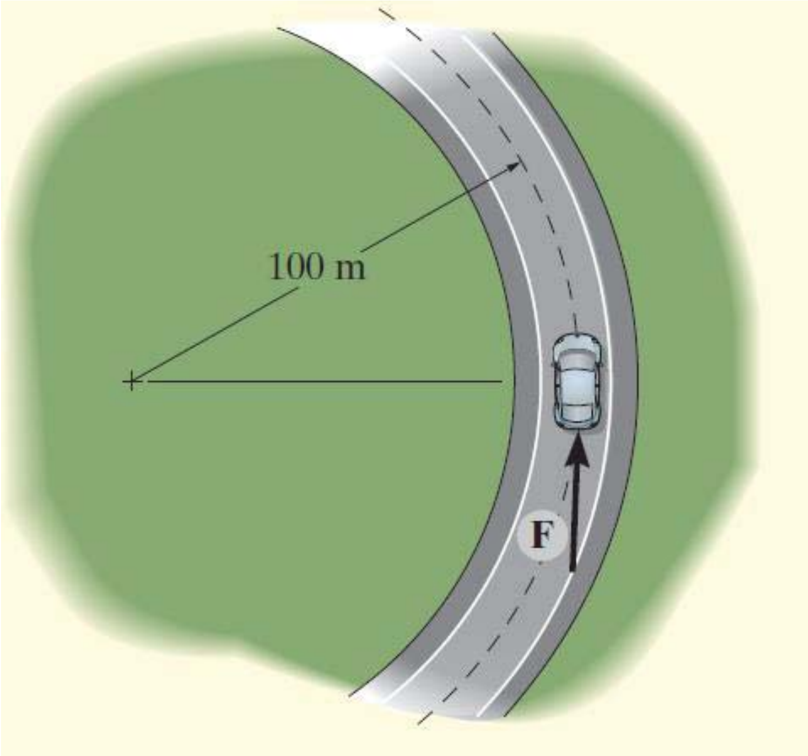


- *When a particle moves under a central force, its areal velocity is constant.*

Kinetics of Particles: Angular Momentum

❑ Sample Problem 15

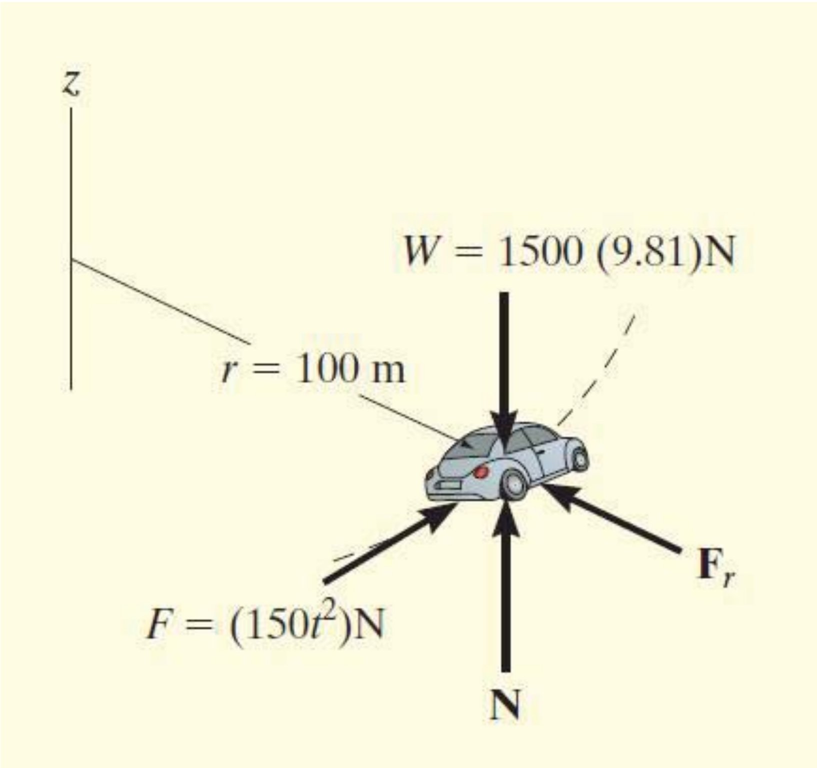
The 1.5-Mg car travels along the circular road as shown in Fig. 15–24*a*. If the traction force of the wheels on the road is $F = (150t^2)$ N, where t is in seconds, determine the speed of the car when $t = 5$ s. The car initially travels with a speed of 5 m/s. Neglect the size of the car.

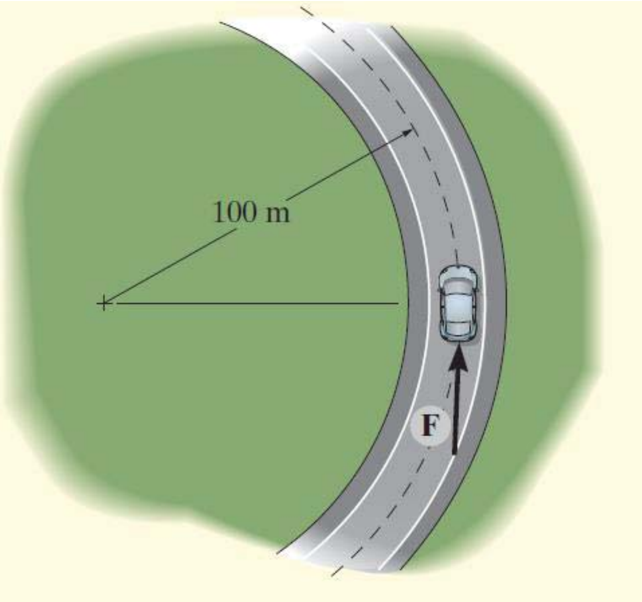


Kinetics of Particles: Angular Momentum

❑ Sample Problem 15

Free-Body Diagram

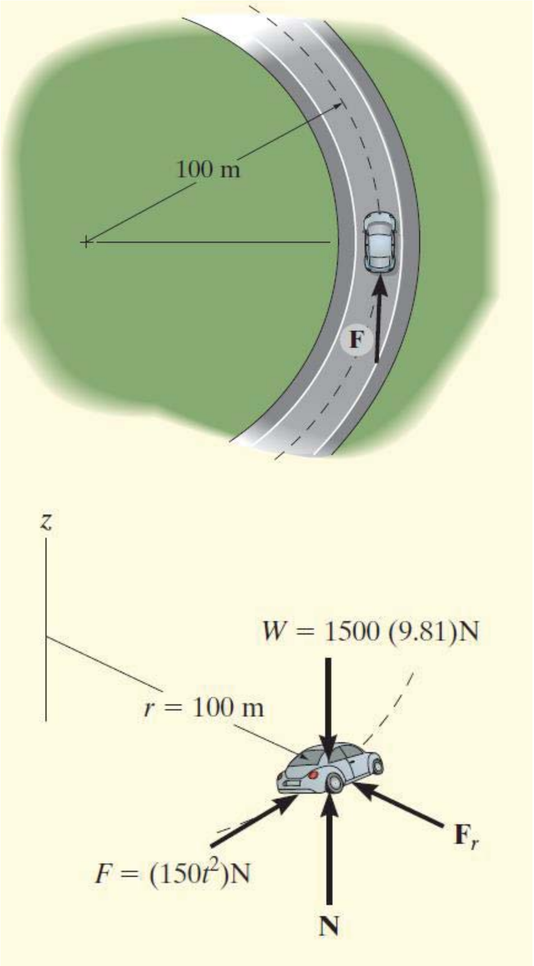




Kinetics of Particles: Angular Momentum

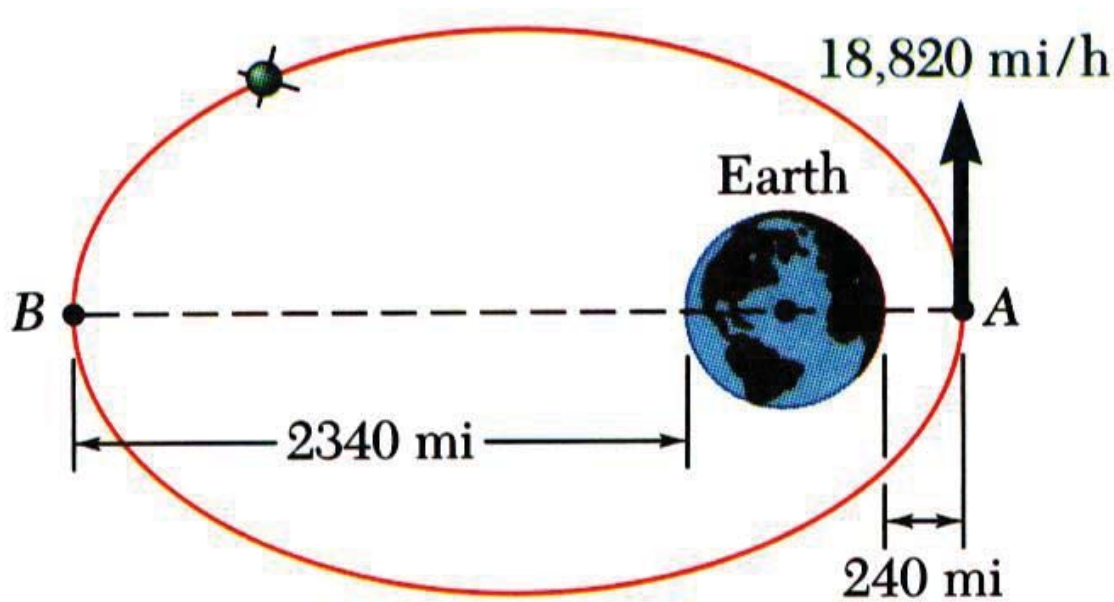
❑ Sample Problem 15

Principle of Angular Impulse and Momentum



❑ Sample Problem 16

A satellite is launched in a direction parallel to the surface of the earth with a velocity of 18820 mi/h from an altitude of 240 mi. Determine the velocity of the satellite as it reaches it maximum altitude of 2340 mi. The radius of the earth is 3960 mi.



❑ Sample Problem 16

SOLUTION: