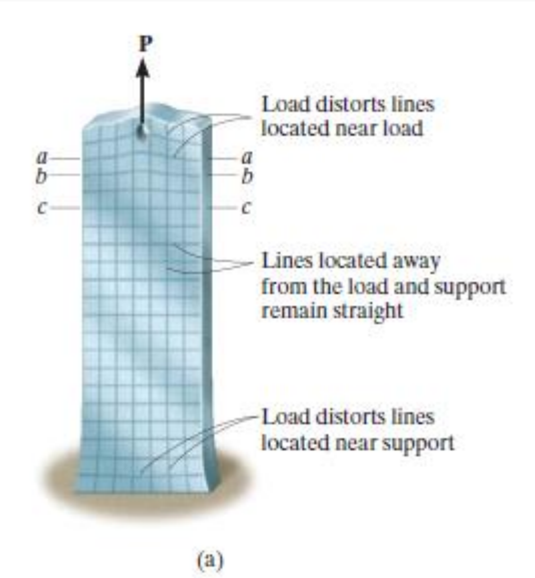
فصل سوم :

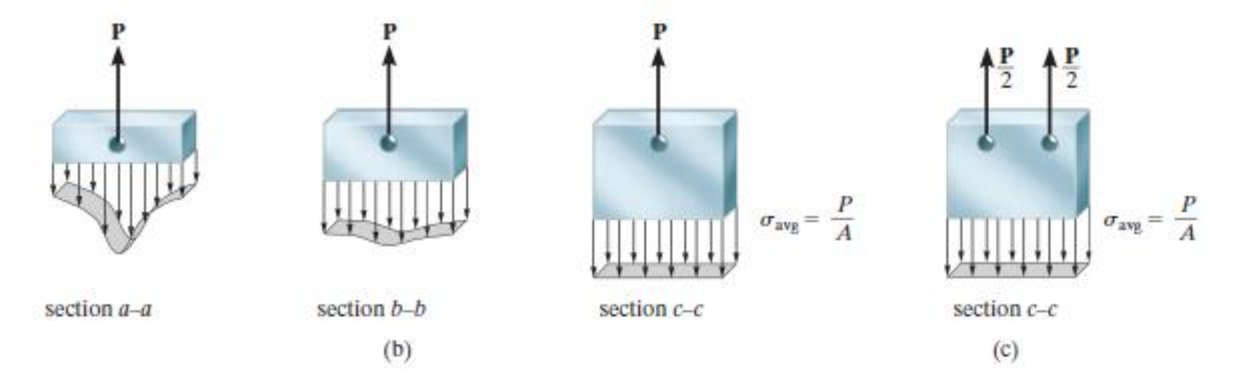
بارگذاری محوری

AXIAL LOAD

فصل سوم : بارگذاری محوری

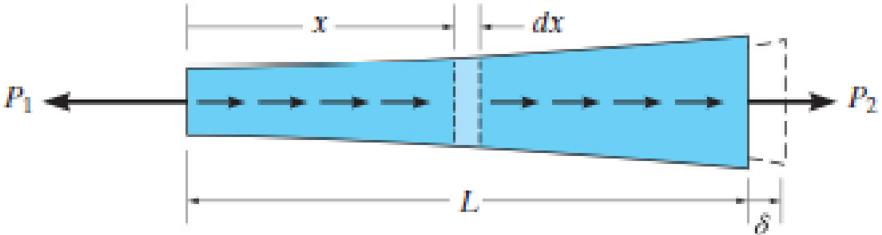
SAINT-VENANT'S PRINCIPLE





طبق اصل سن و نان، هرگاه مقطعی تحت بارگذاری قرار گیرد، توزیع تنش‌ها در مقطعی در فاصله‌ی برابر یا بزرگتر از عرض عضو، همواره یکسان خواهد بود. به عبارت دیگر، به جز در مجاورت نقاط اعمال بارها، توزیع تنش را می‌توان مستقل از مد واقعی اعمال بارها فرض کرد. این بیان نه تنها در بارگذاری محوری، بلکه عملا برای هر بارگذاری صادق است. به افتخار مهندس و ریاضیدان فرانسوی آدمار باره سن و نان، اصل سن و نان نامگذاری کرده‌اند.

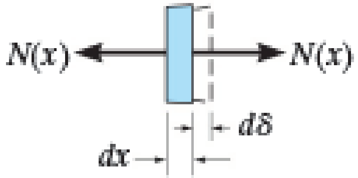
ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER



(a)

$$\sigma = \frac{N(x)}{A(x)} \text{ and } \epsilon = \frac{d\delta}{dx}$$

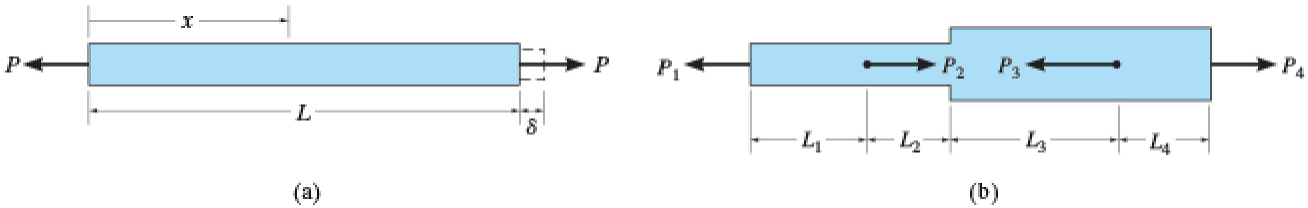
$$\frac{N(x)}{A(x)} = E(x)\left(\frac{d\delta}{dx}\right)$$



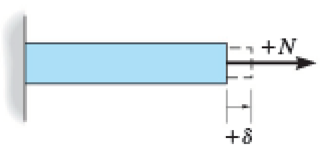
$$\delta = \int_0^L \frac{N(x)dx}{A(x)E(x)}$$

$$\delta = \frac{NL}{AE}$$

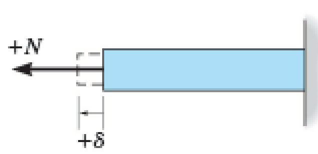
ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER



$$\delta = \sum \frac{NL}{AE}$$

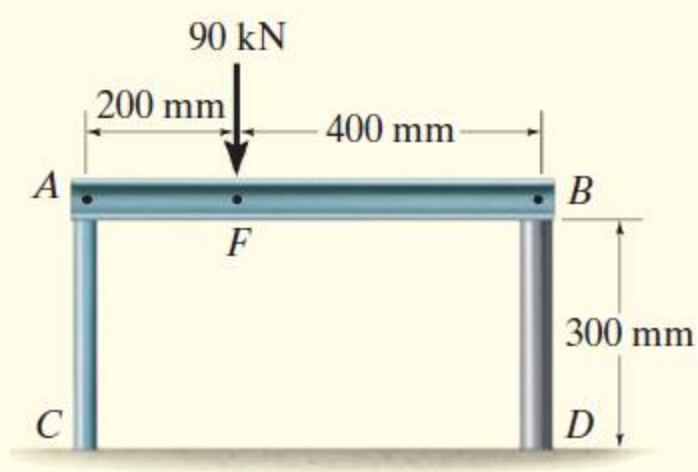


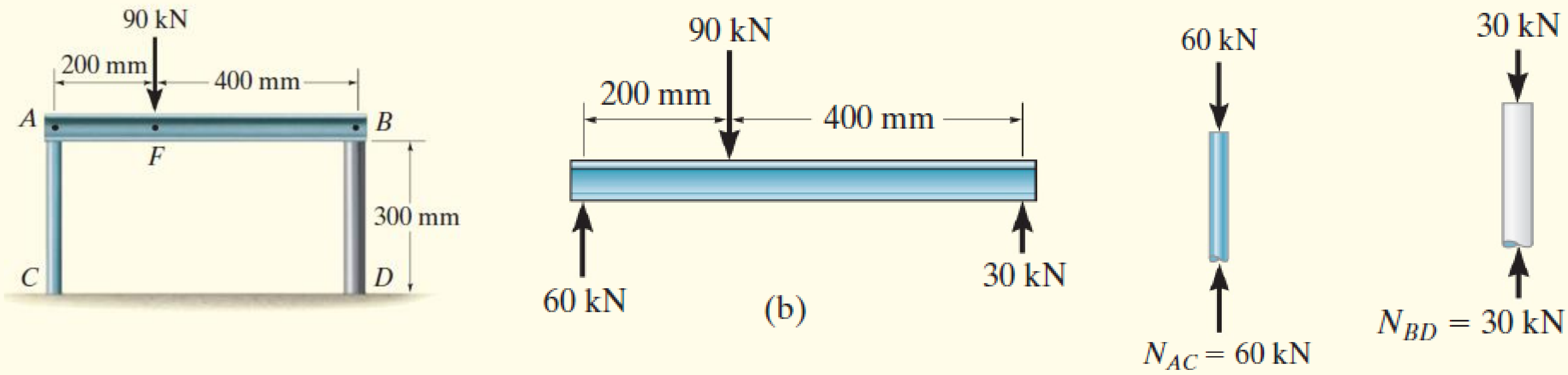
Sign Convention.

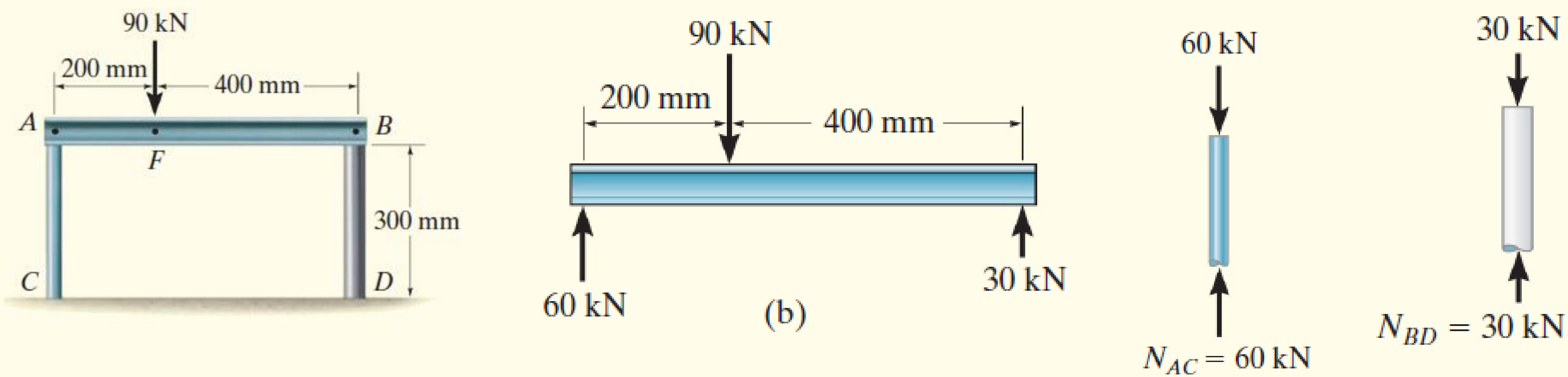


مثال

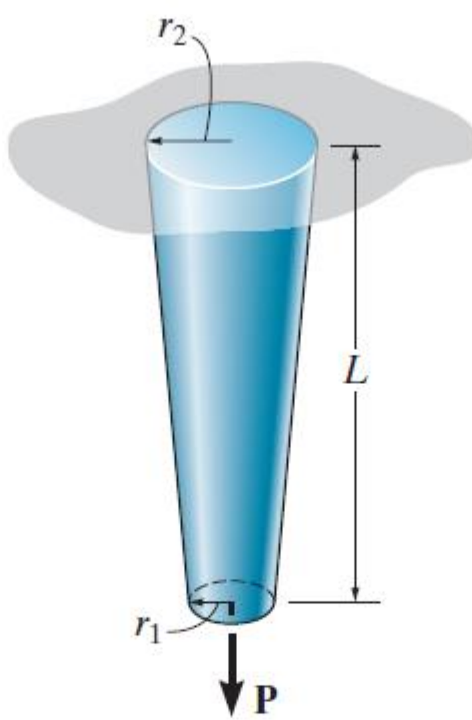
Rigid beam AB rests on the two short posts shown in Fig. 4–7a. AC is made of steel and has a diameter of 20 mm, and BD is made of aluminum and has a diameter of 40 mm. Determine the displacement of point F on AB if a vertical load of 90 kN is applied over this point. Take $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.

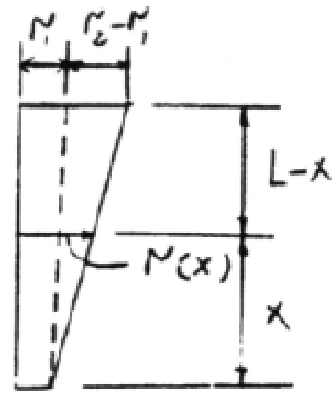


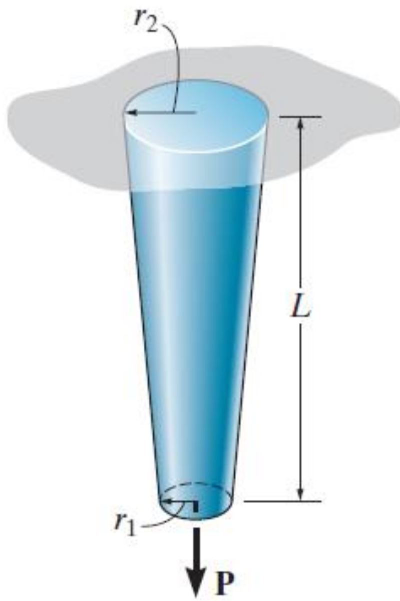

90 kN
200 mm
400 mm
A
B
F
300 mm
C
D
90 kN
200 mm
400 mm
60 kN
30 kN
(b)
60 kN
$N_{AC} = 60$ kN
30 kN
$N_{BD} = 30$ kN


90 kN
200 mm
400 mm
A
B
F
300 mm
C
D
90 kN
200 mm
400 mm
60 kN
30 kN
(b)
60 kN
N_AC = 60 kN
30 kN
N_BD = 30 kN

The rod has a slight taper and length L. It is suspended from the ceiling and supports a load **P** at its end. Show that the displacement of its end due to this load is $\delta = PL/(\pi E r_2 r_1)$. Neglect the weight of the material. The modulus of elasticity is E.




L-x
x

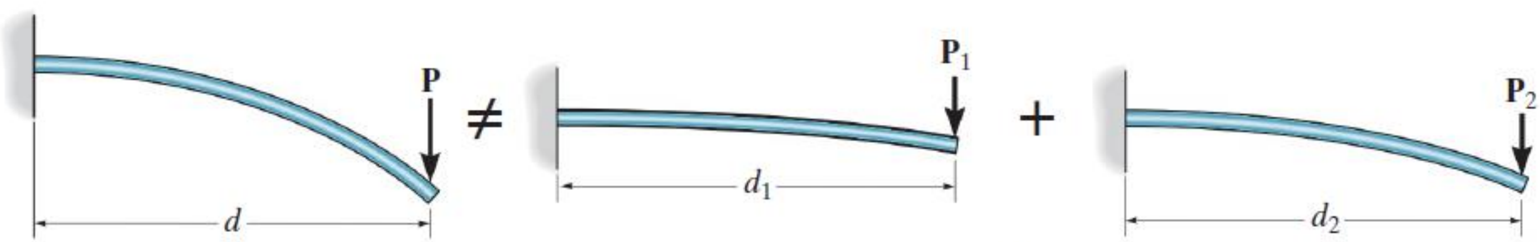

r_2
L
r_1
P

PRINCIPLE OF SUPERPOSITION

اصل جمع آثار قوا

The following two conditions must be satisfied if the principle of superposition is to be applied.

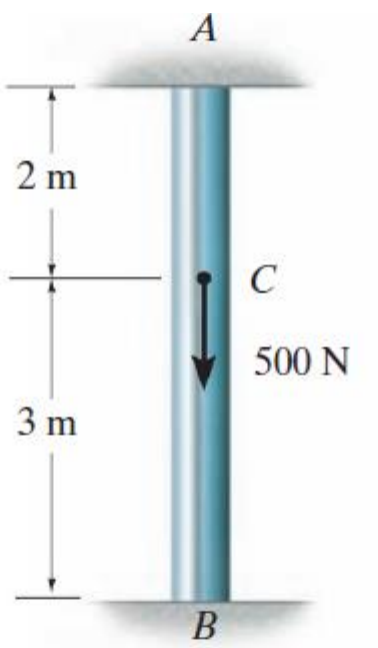
1. ***The loading N must be linearly related to the stress σ or displacement δ that is to be determined.*** For example, the equations $\sigma = N/A$ and $\delta = NL/AE$ involve a linear relationship between σ and N, and δ and N.
2. ***The loading must not significantly change the original geometry or configuration of the member.*** If significant changes do occur, the direction and location of the applied forces and their moment arms will change. For example, consider the slender rod shown in Fig. 4–9*a*, which is subjected to the load **P**. In Fig. 4–9*b*, **P** is replaced by two of its components, $\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$. If **P** causes the rod to deflect a large amount, as shown, the moment of this load about its support, Pd, will not equal the sum of the moments of its component loads, $Pd \neq P_1d_1 + P_2d_2$, because $d_1 \neq d_2 \neq d$.



(a) (b)

STATICALLY INDETERMINATE AXIALLY LOADED MEMBERS

مسایل استاتیکی نامعین برای اعضای تحت نیروی محوری



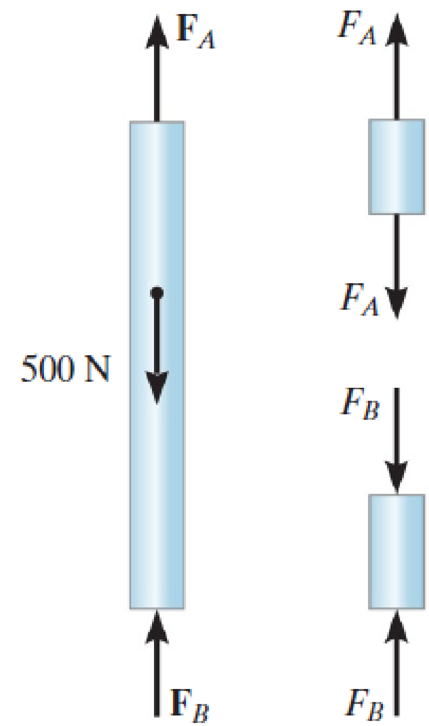
$$+\uparrow \Sigma F = 0; \qquad F_B + F_A - 500\text{ N} = 0$$

compatibility condition $\qquad \delta_{A/B} = 0$

شرط سازگاری

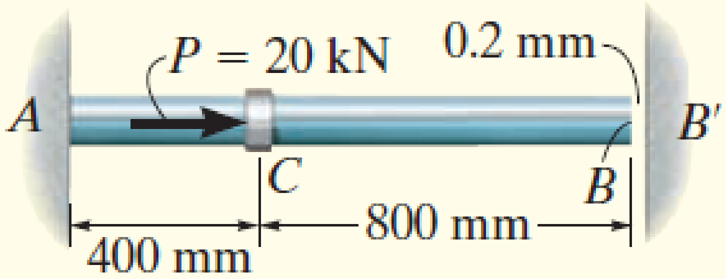
$$\frac{F_A(2\text{ m})}{AE} - \frac{F_B(3\text{ m})}{AE} = 0$$

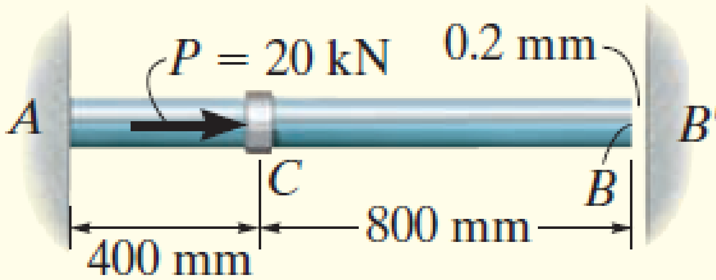
$$F_A = 300\text{ N} \quad \text{and} \quad F_B = 200\text{ N}$$

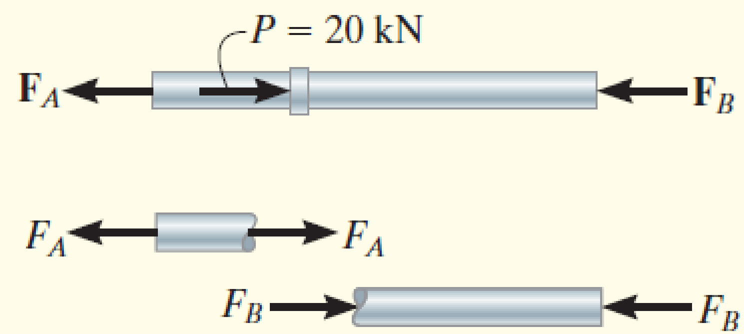


Example

The steel rod shown in Fig. 4–11*a* has a diameter of 10 mm. It is fixed to the wall at A, and before it is loaded, there is a gap of 0.2 mm between the wall at B' and the rod. Determine the reactions on the rod if it is subjected to an axial force of $P = 20$ kN. Neglect the size of the collar at C. Take $E_{st} = 200$ GPa.



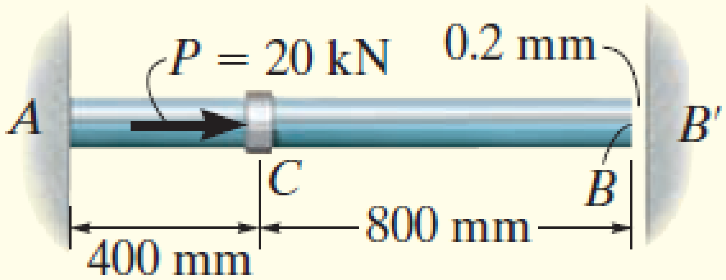


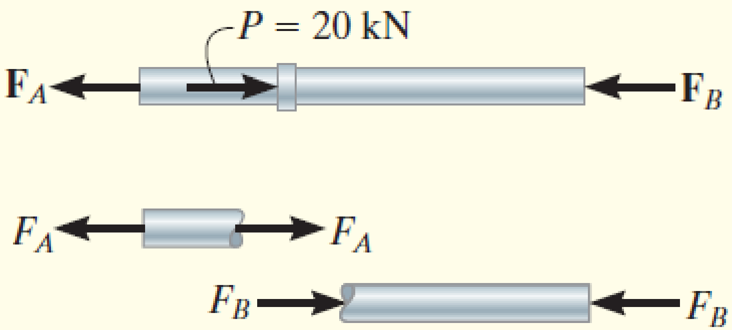


Equilibrium.

Compatibility.

Load–Displacement.

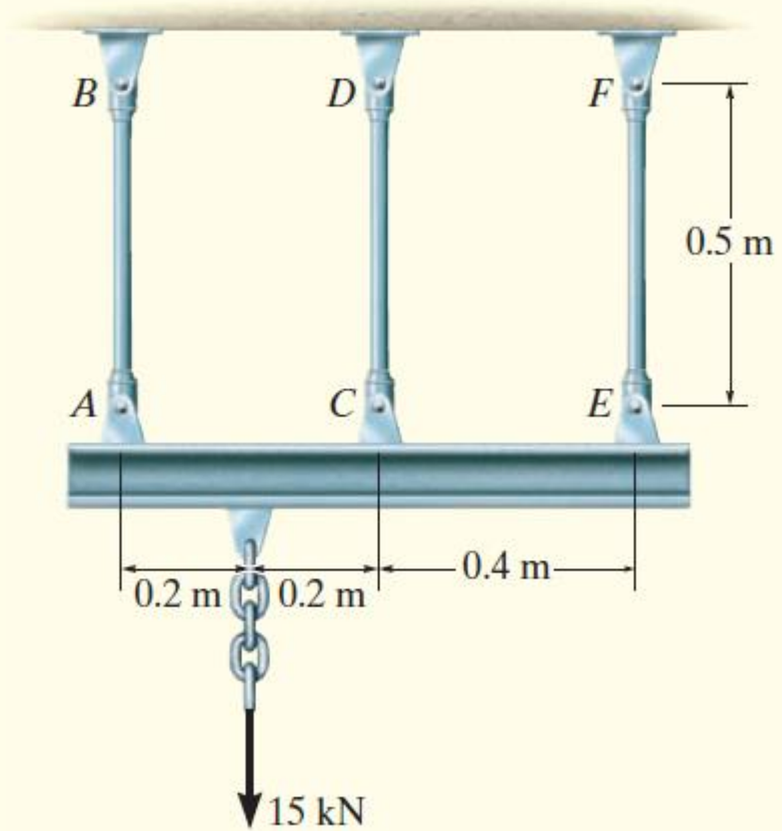


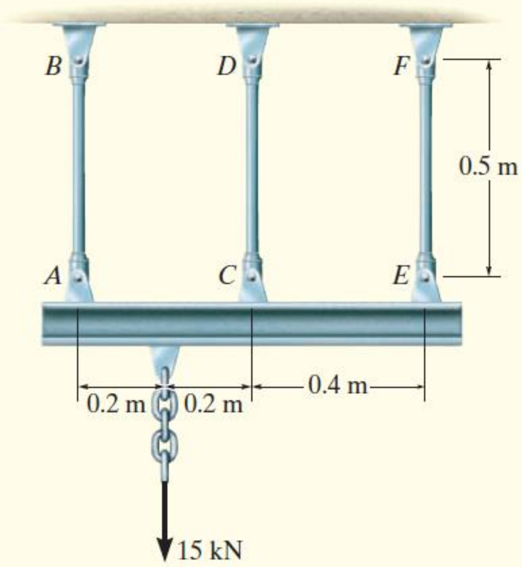


Load–Displacement.

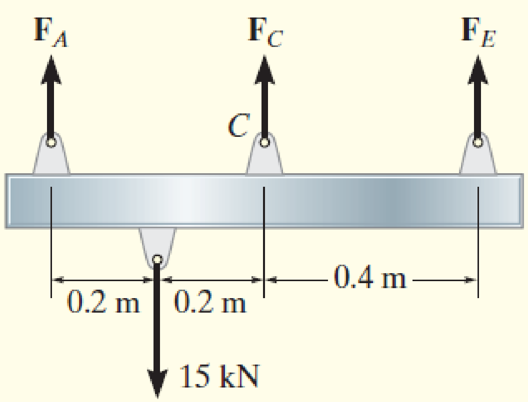
Example

The three A992 steel bars shown in Fig. 4–13*a* are pin connected to a *rigid* member. If the applied load on the member is 15 kN, determine the force developed in each bar. Bars *AB* and *EF* each have a cross-sectional area of 50 mm^2, and bar *CD* has a cross-sectional area of 30 mm^2.

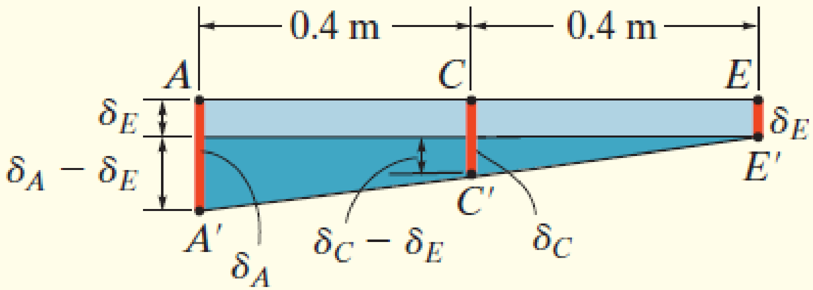




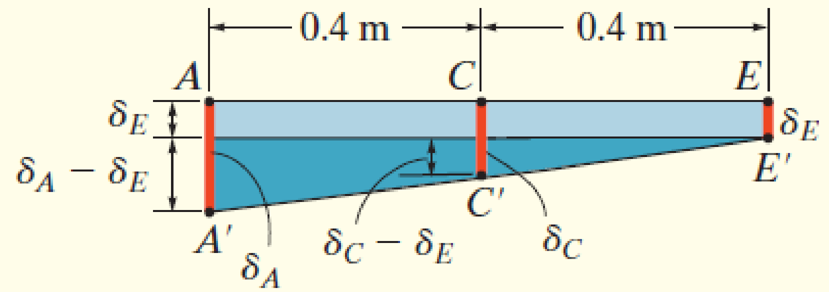
Equilibrium.



Compatibility. شرط سازگاری



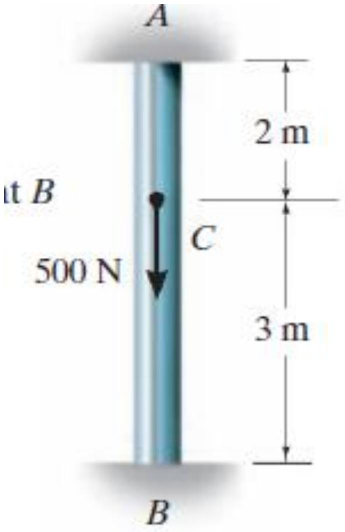
Load–Displacement.



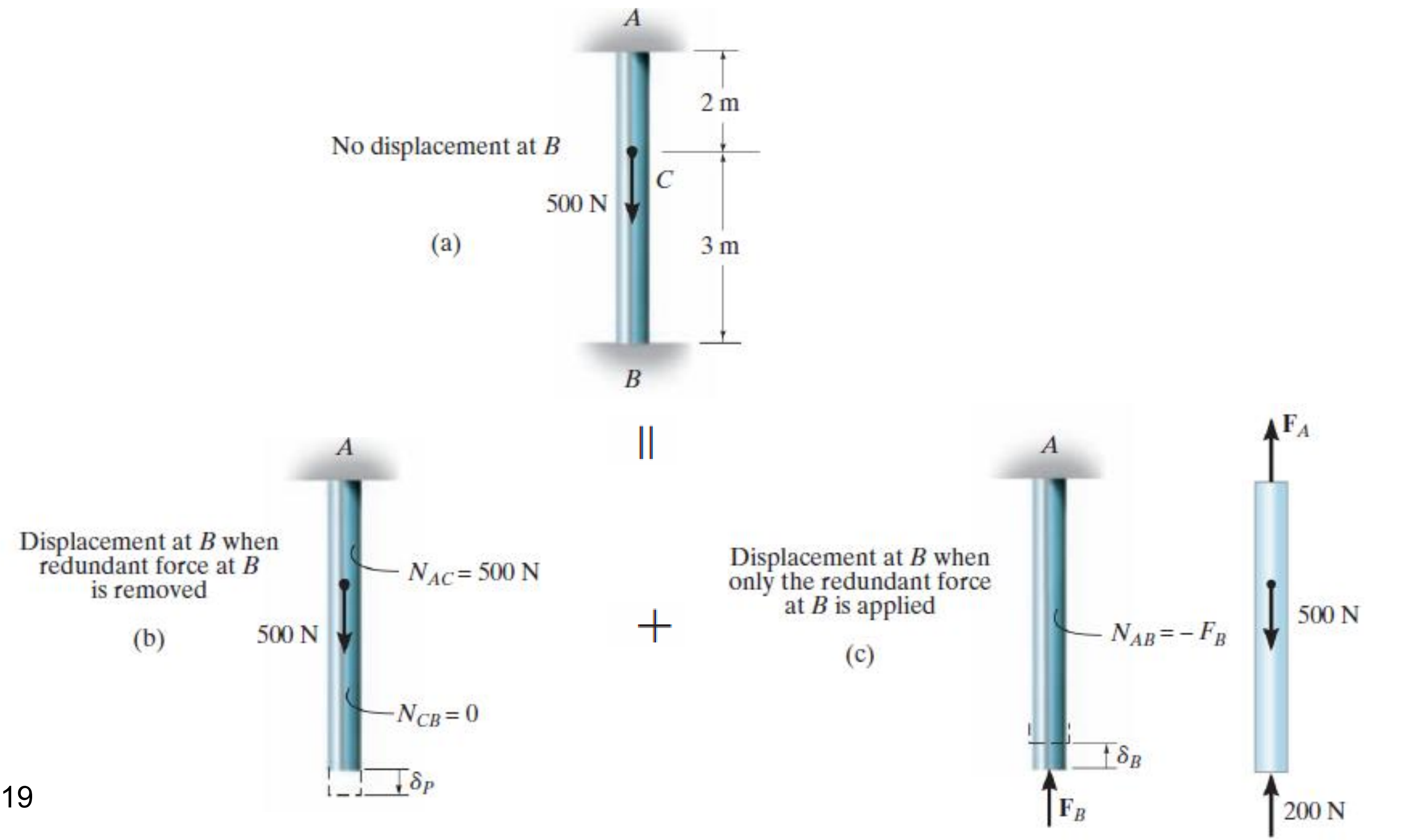
THE FORCE METHOD OF ANALYSIS FOR AXIALLY LOADED MEMBERS

روش نیرو برای تحلیل اعضای تحت نیروی محوری

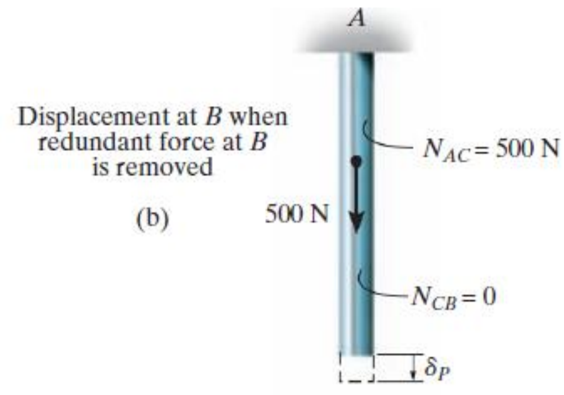
- Used to also solve statically indeterminate problems by using superposition of the forces acting on the free-body diagram
- First, choose any one of the two supports as "redundant" and remove its effect on the bar
- Thus, the bar becomes statically determinate
- Apply principle of superposition and solve the equations simultaneously



THE FORCE METHOD OF ANALYSIS FOR AXIALLY LOADED MEMBERS



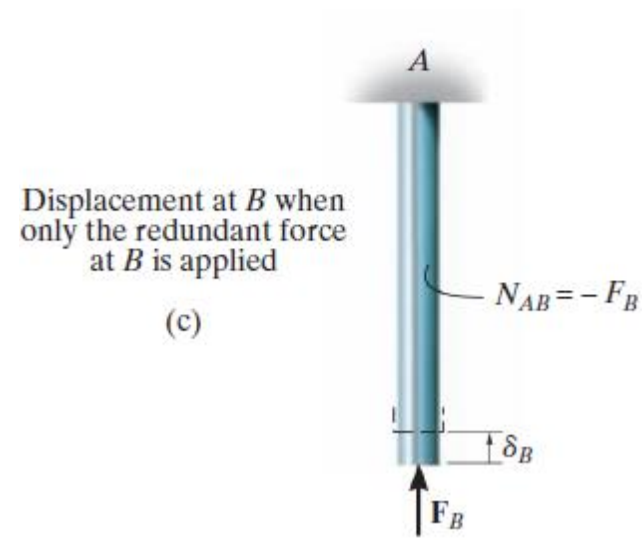
THE FORCE METHOD OF ANALYSIS FOR AXIALLY LOADED MEMBERS



$$0 = \delta_P - \delta_B$$

$$0 = \frac{500\text{ N}(2\text{ m})}{AE} - \frac{F_B(5\text{ m})}{AE}$$

$$F_B = 200\text{ N}$$

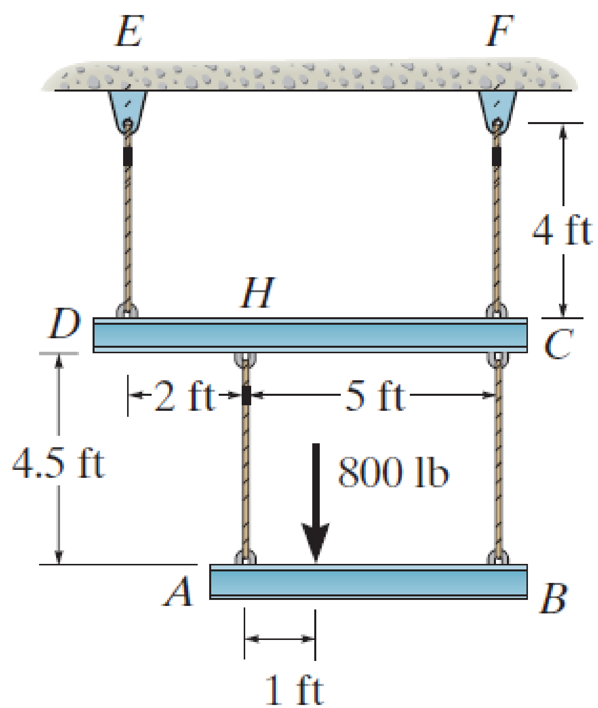


$$+\uparrow\Sigma F_y = 0; \qquad 200\text{ N} + F_A - 500\text{ N} = 0$$

Then

$$F_A = 300\text{ N}$$

The load of 800 lb is supported by the four 304 stainless steel wires that are connected to the rigid members *AB* and *DC*. Determine the vertical displacement of the load if the members were horizontal before the load was applied. Each wire has a cross-sectional area of 0.05 in^2.



THERMAL STRESS

تنش حرارتی

THERMAL STRESS

- From experiment, deformation of a member having length L is

$$\delta_T = \alpha \Delta T L$$

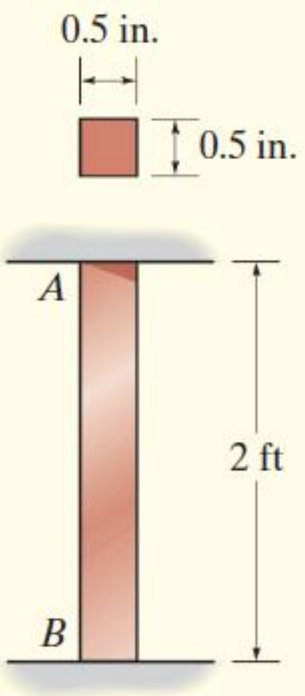
α = liner coefficient of thermal expansion. Unit measure strain per degree of temperature: $1/^{o}\mathrm{C}$ (Celsius) or $1/^{o}\mathrm{K}$ (Kelvin)

ΔT = algebraic change in temperature of member

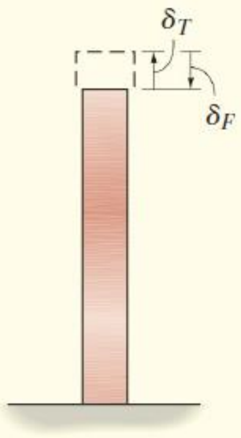
δT = algebraic change in length of member

Example

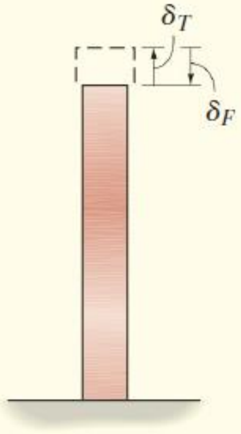
The A-36 steel bar shown in Fig. 4–17*a* is constrained to just fit between two fixed supports when $T_1 = 60°F$. If the temperature is raised to $T_2 = 120°F$, determine the average normal thermal stress developed in the bar.



Example

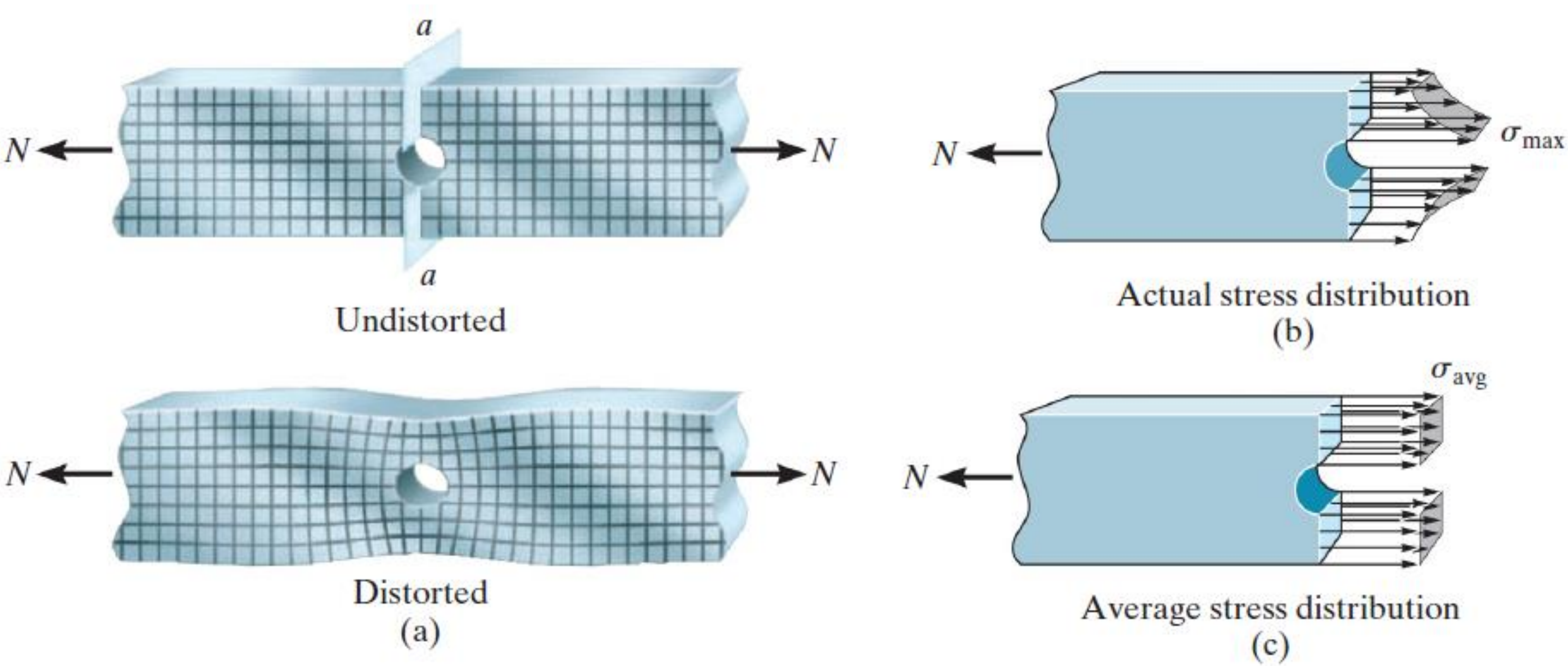


Example



STRESS CONCENTRATIONS

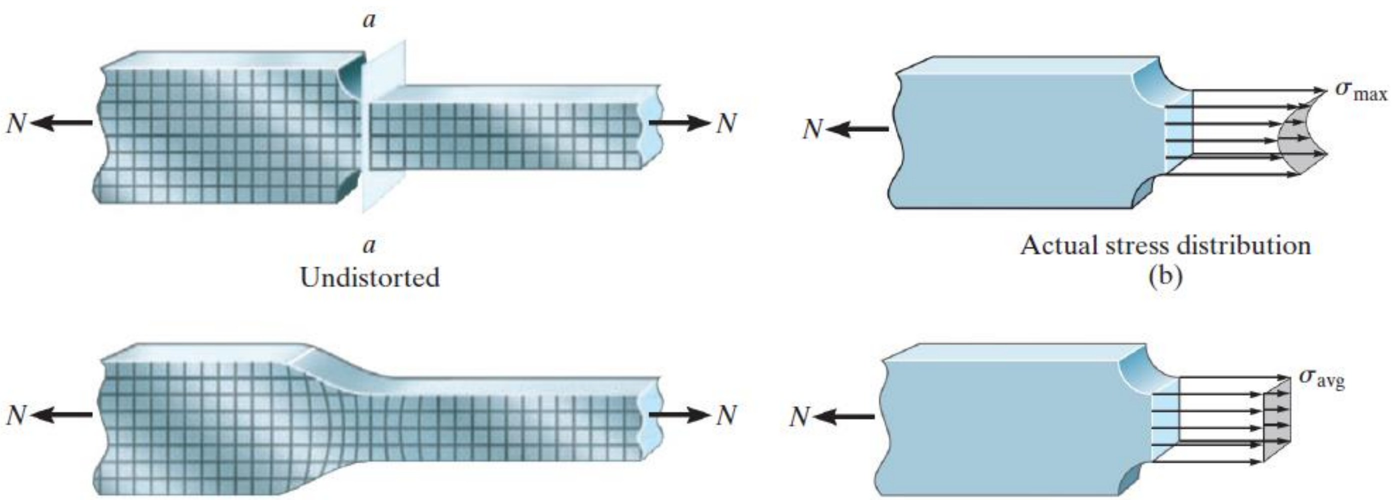
تمرکز تنش



STRESS CONCENTRATIONS

Stress concentrations often arise at sharp corners on heavy machinery. Engineers can mitigate this effect by using stiffeners welded to the corners.

STRESS CONCENTRATIONS



(a)

Actual stress distribution
(b)

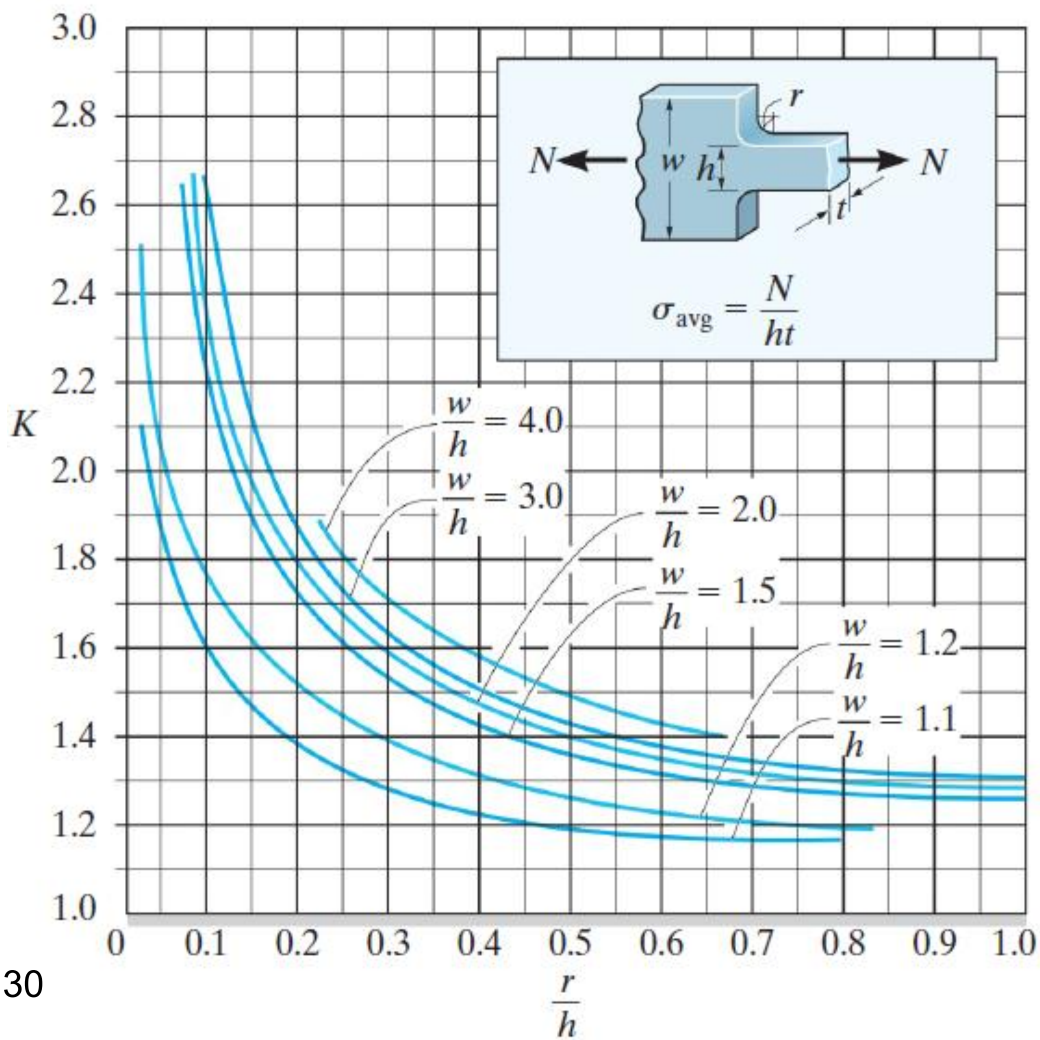
Average stress distribution
(c)

stress concentration factor K

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

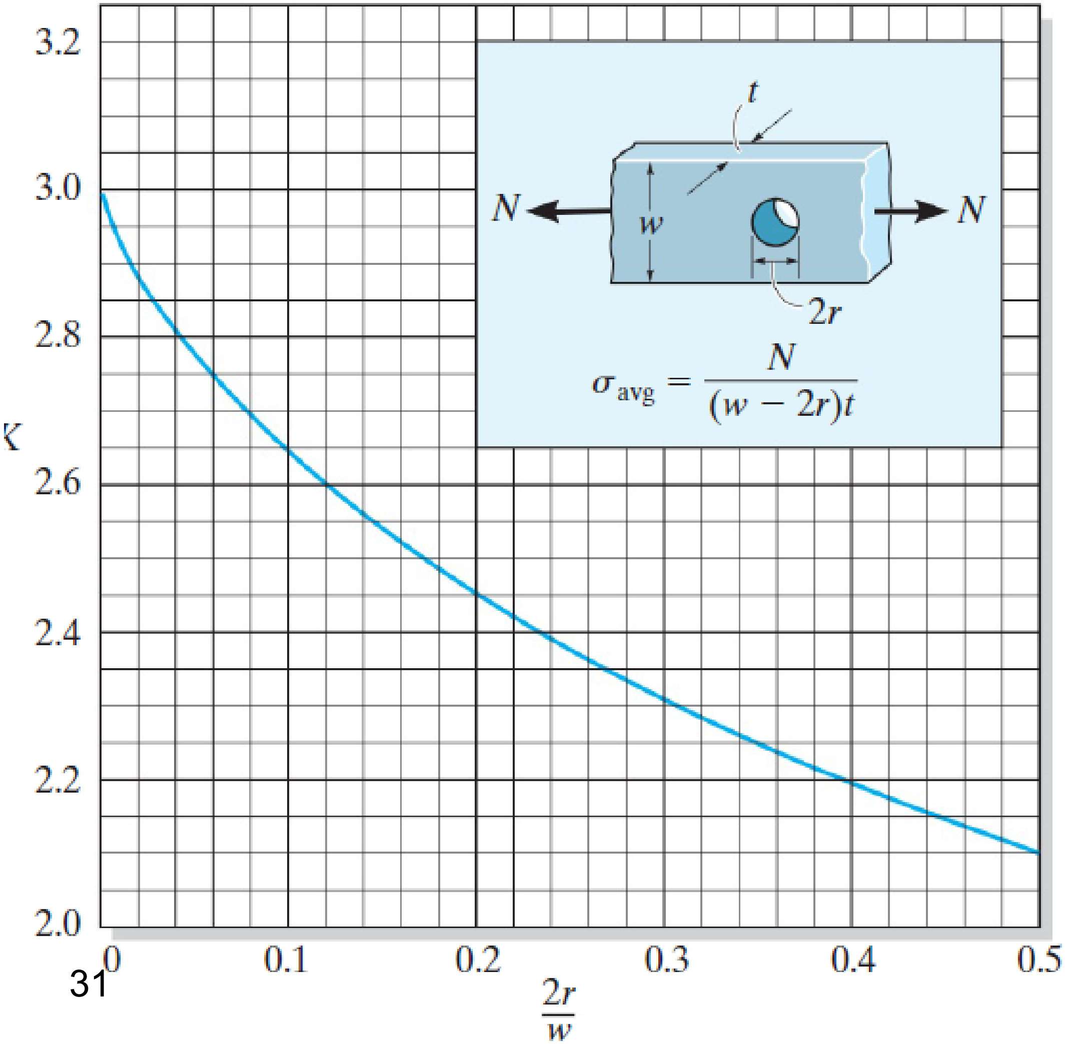
$$\sigma_{avg} = N/A$$

STRESS CONCENTRATIONS



$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

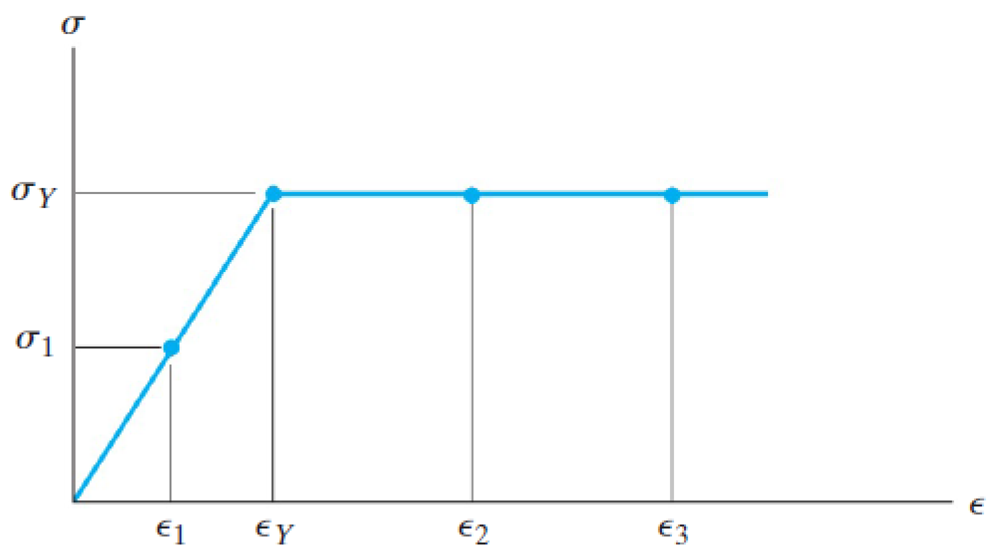
STRESS CONCENTRATIONS



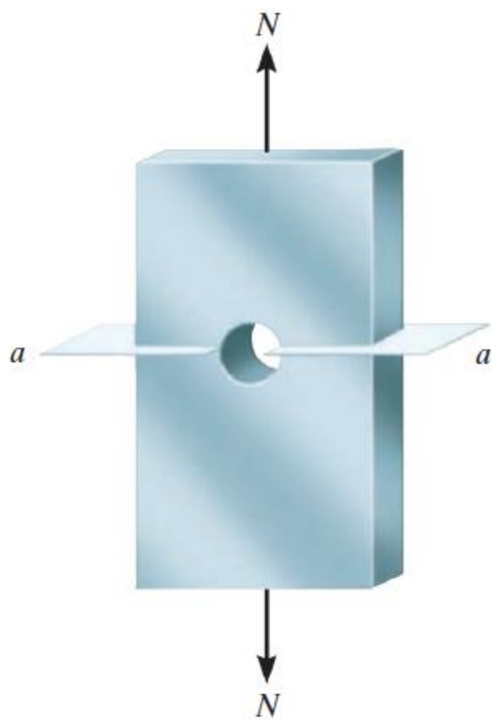
$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

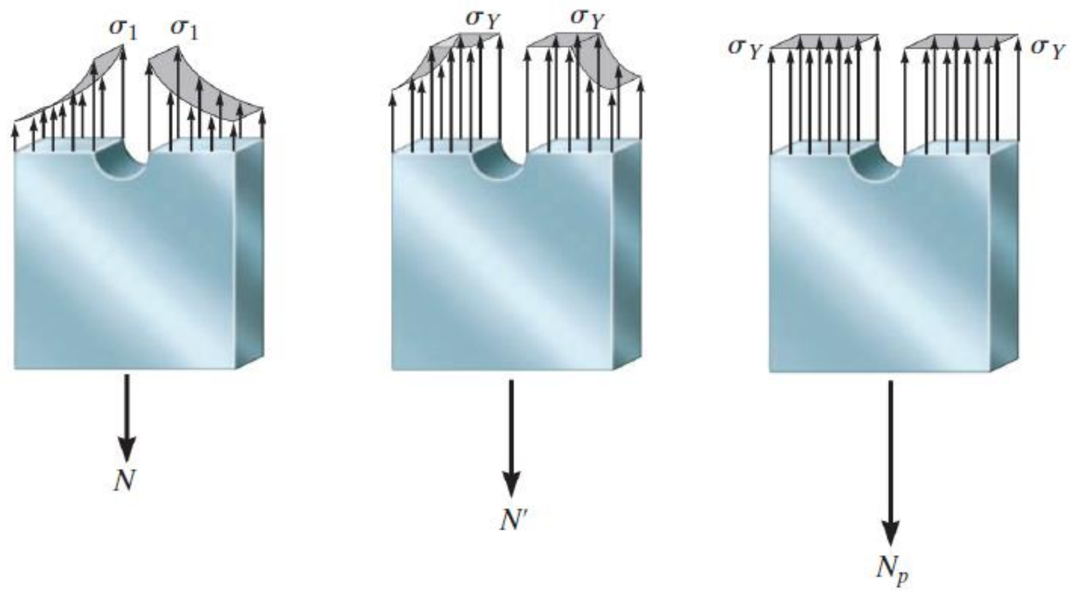
INELASTIC AXIAL DEFORMATION

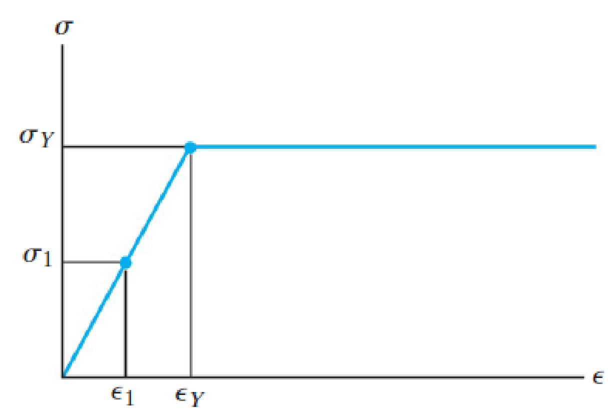
تغییرشکلهای محوری غیر الاستیک



INELASTIC AXIAL DEFORMATION

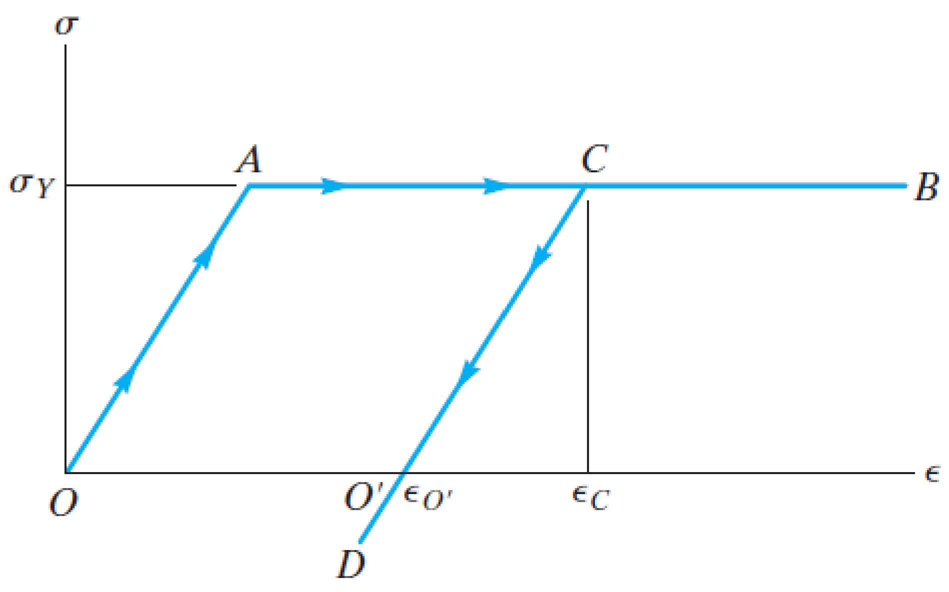






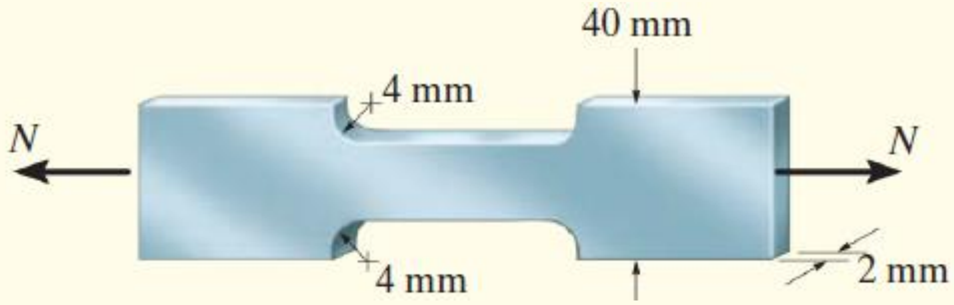
RESIDUAL STRESS

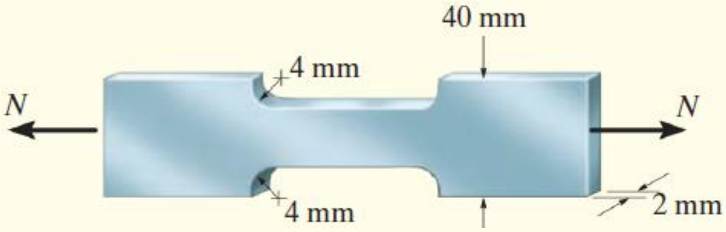
تنش محبوس

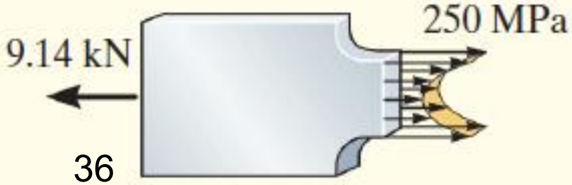


Example

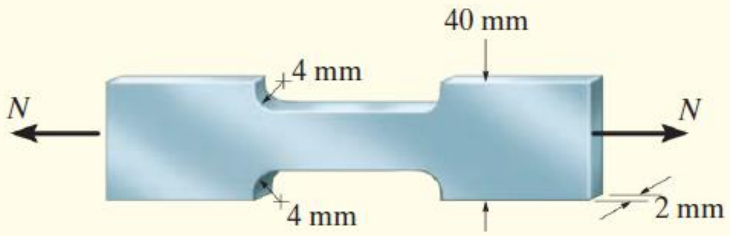
The bar in Fig. 4–28*a* is made of steel that is assumed to be elastic perfectly plastic, with $\sigma_Y = 250$ MPa. Determine (a) the maximum value of the applied load N that can be applied without causing the steel to yield and (b) the maximum value of N that the bar can support. Sketch the stress distribution at the critical section for each case.



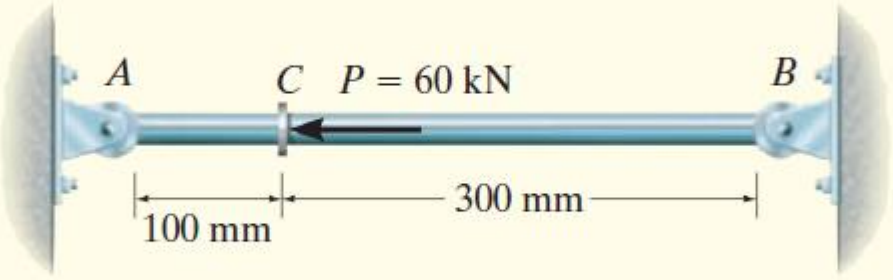

40 mm
4 mm
N
N
4 mm
2 mm

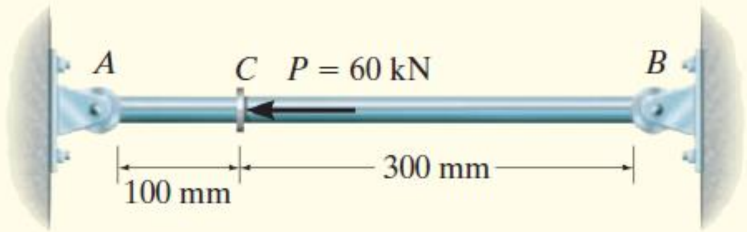

250 MPa
9.14 kN

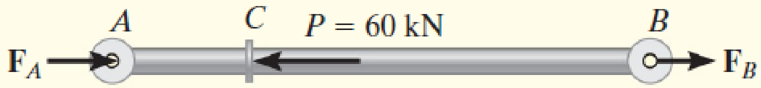

250 MPa
N_P

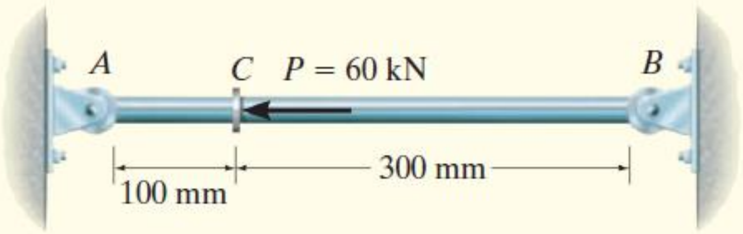

40 mm
4 mm
N
N
4 mm
2 mm

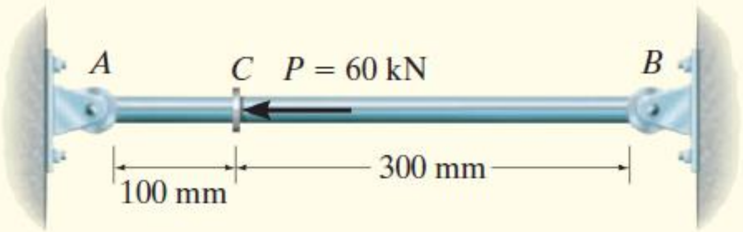
The rod shown in Fig. 4–30*a* has a radius of 5 mm and is made of an elastic perfectly plastic material for which $\sigma_Y = 420$ MPa, $E = 70$ GPa, Fig. 4–30*c*. If a force of $P = 60$ kN is applied to the rod and then removed, determine the residual stress in the rod.



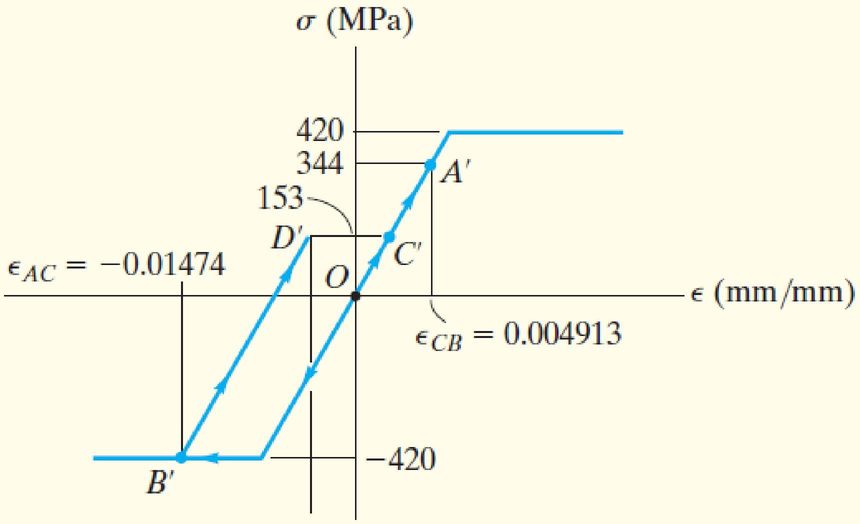

A
C
P = 60 kN
B
100 mm
300 mm

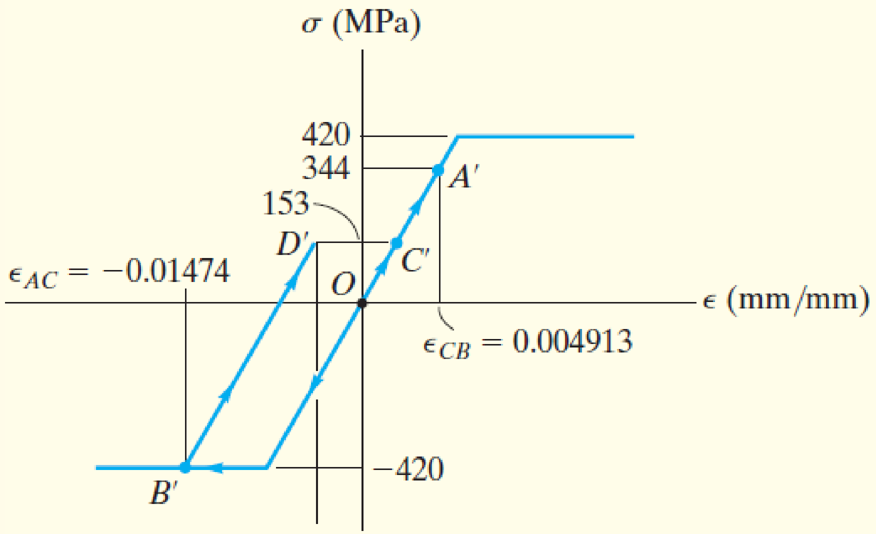

A
C
P = 60 kN
B
F_A
F_B


A
C
P = 60 kN
B
100 mm
300 mm



Residual Stress. In order to obtain the residual stress, it is also necessary to know the strain in each segment due to the loading. Since CB responds elastically,


σ (MPa)
420
344
153
A′
D′
C′
O
$\epsilon_{AC} = -0.01474$
ϵ (mm/mm)
$\epsilon_{CB} = 0.004913$
−420
B′


σ (MPa)
420
344
153
A′
D′
C′
O
ϵ_AC = −0.01474
ϵ (mm/mm)
ϵ_CB = 0.004913
−420
B′