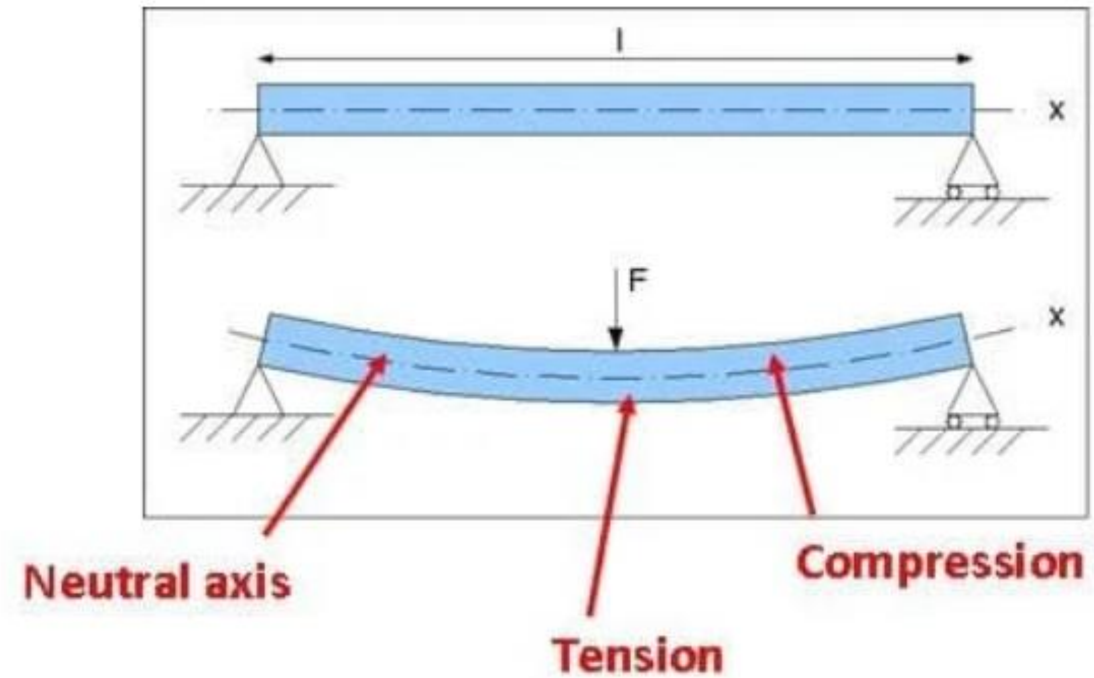
فصل پنجم

خمش

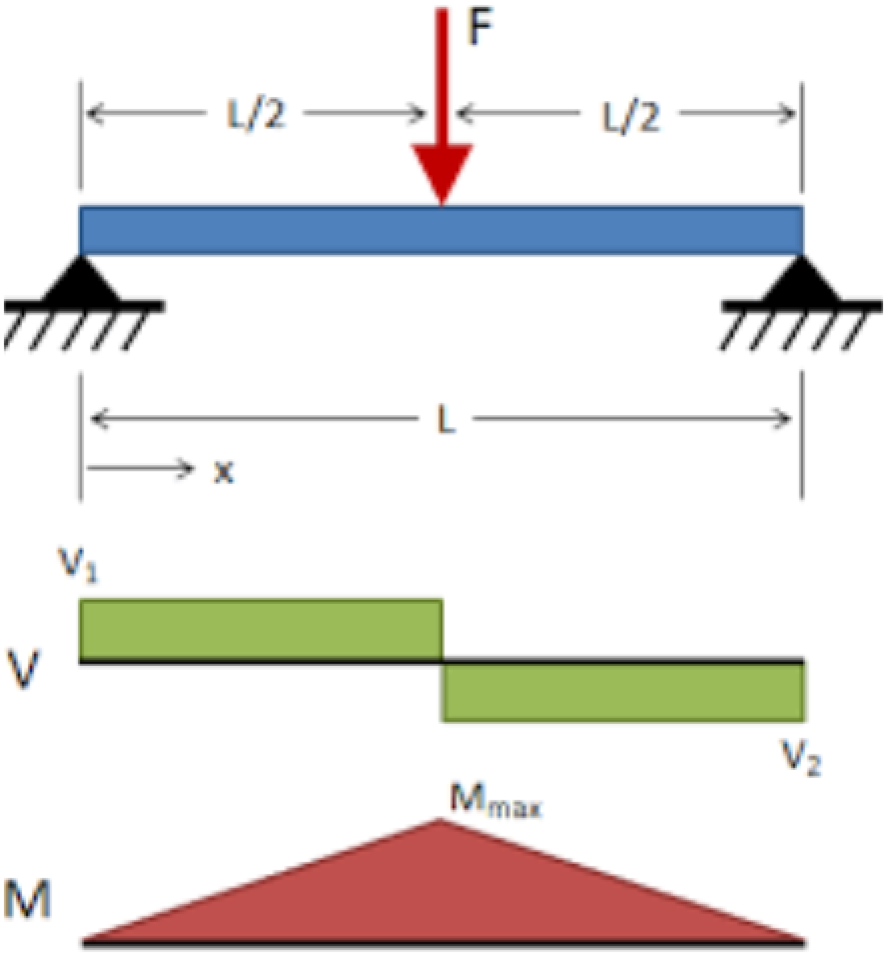
BENDING

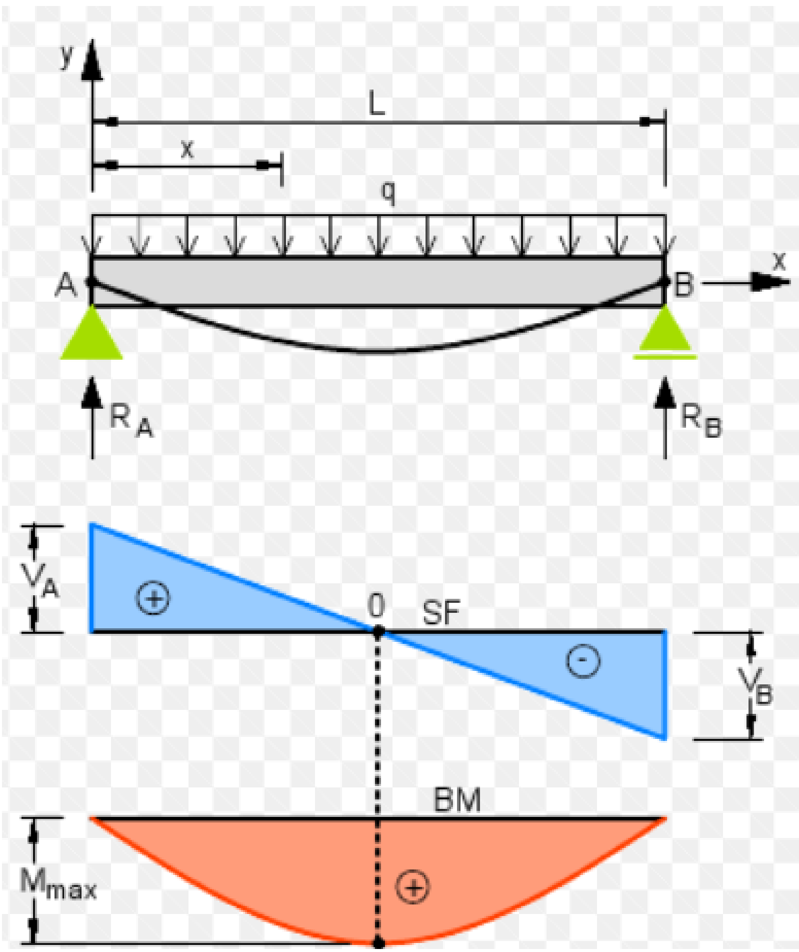
BENDING

فصل پنجم: خمش

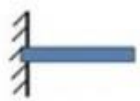


SHEAR AND MOMENT DIAGRAMS





SHEAR AND MOMENT DIAGRAMS

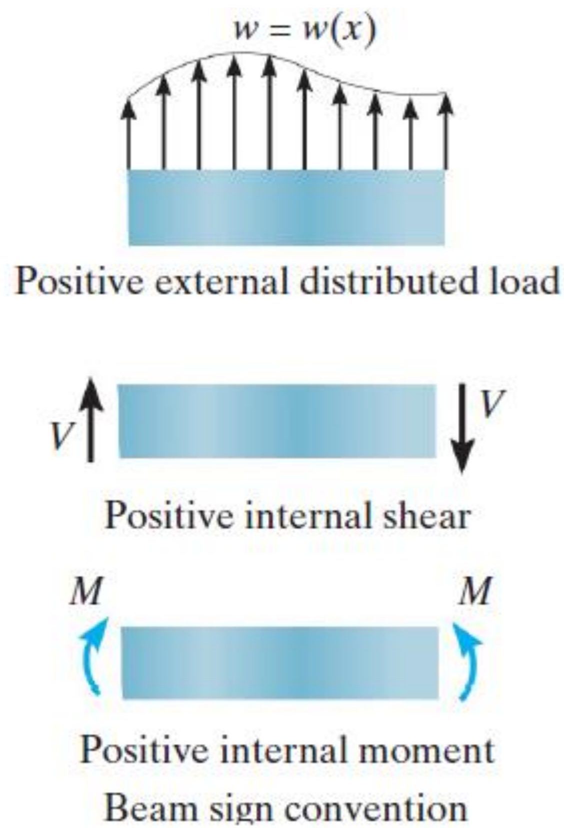
S.no	Types of Support	Representation by	Reaction Force	Resisting Load
1.	**Roller Support**		Vertical	Vertical loads
2.	**Pinned Support**		Horizontal and vertical	Vertical and horizontal loads
3.	**Fixed Support**		Horizontal, vertical and moments	All types of loads Horizontal, vertical and Moments
4.	**Simple Support**		Vertical	Vertical loads

Simply supported beam

Cantilevered beam

SHEAR AND MOMENT DIAGRAMS

Beam Sign Convention.



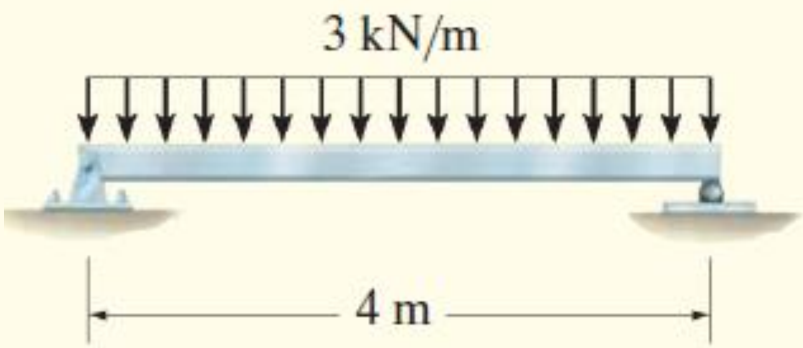
Positive external distributed load

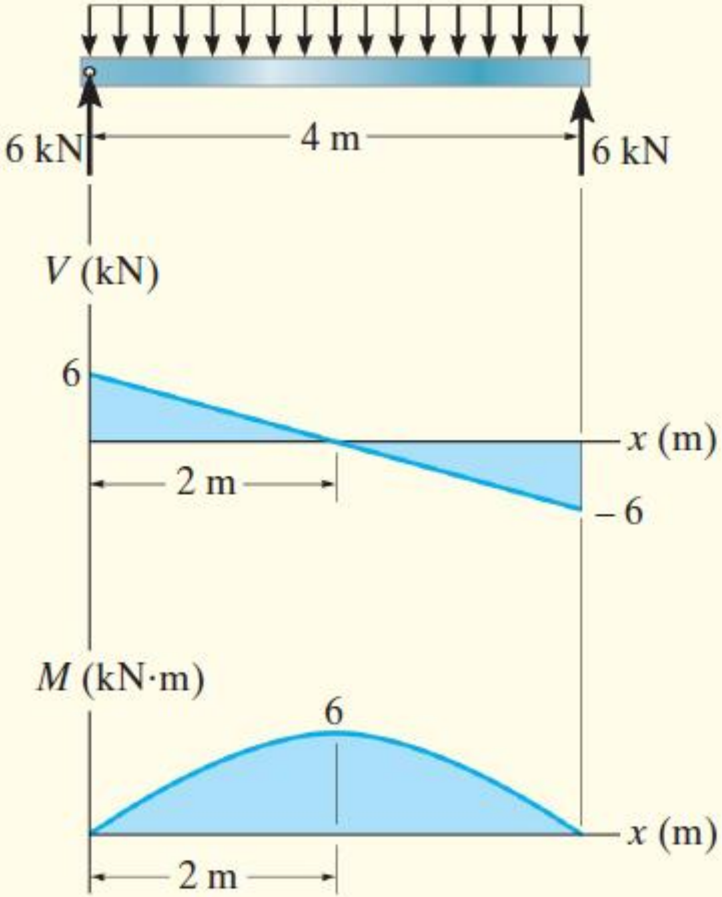
Positive internal shear

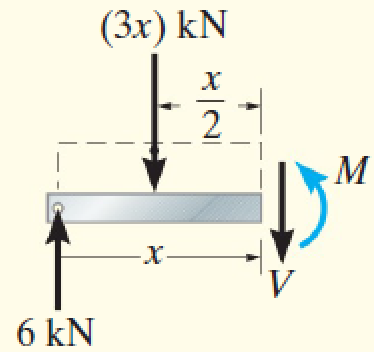
Positive internal moment

Beam sign convention

Draw the shear and moment diagrams for the beam shown in Fig.







$$+\uparrow \Sigma F_y = 0; \quad 6\text{ kN} - (3x)\text{ kN} - V = 0$$

$$V = (6 - 3x)\text{ kN}$$

$$\circlearrowleft + \Sigma M = 0; \quad -6\text{ kN}(x) + (3x)\text{ kN}\left(\tfrac{1}{2}x\right) + M = 0$$

$$M = (6x - 1.5x^2)\text{ kN}\cdot\text{m}$$

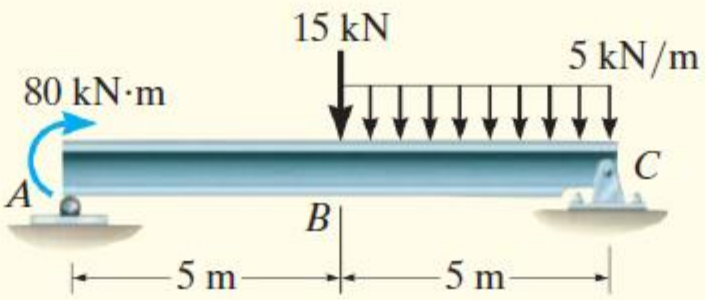
$$V = (6 - 3x)\text{ kN} = 0$$

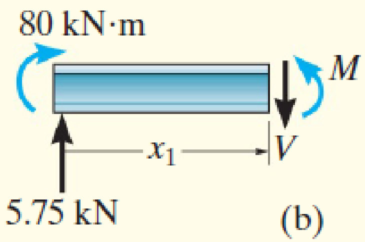
$$x = 2\text{ m}$$

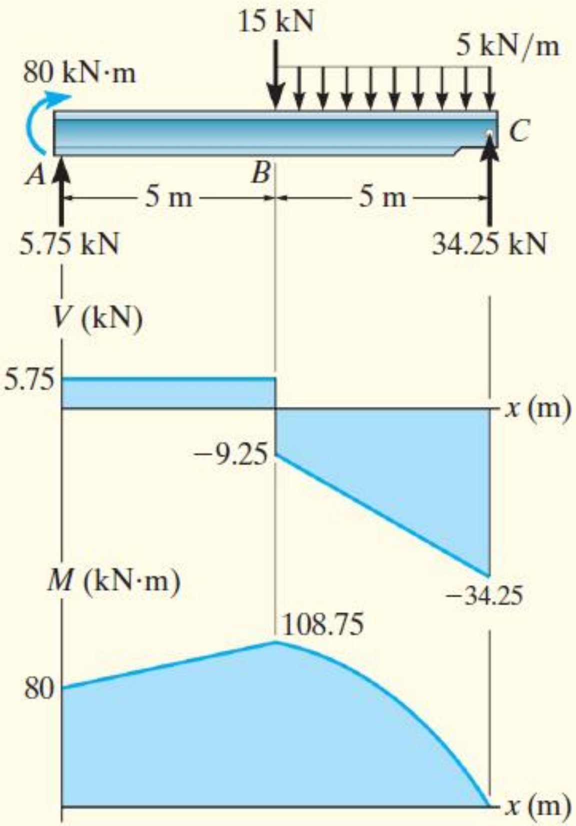
$$M_{max} = [6\,(2) - 1.5\,(2)^2]\text{ kN}\cdot\text{m}$$

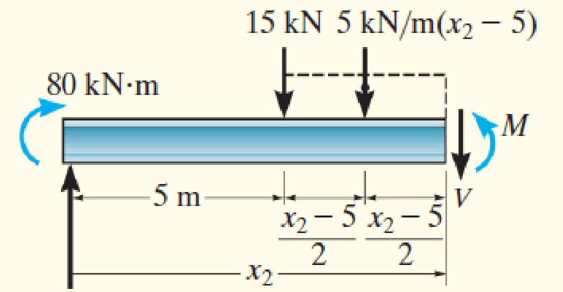
$$= 6\text{ kN}\cdot\text{m}$$

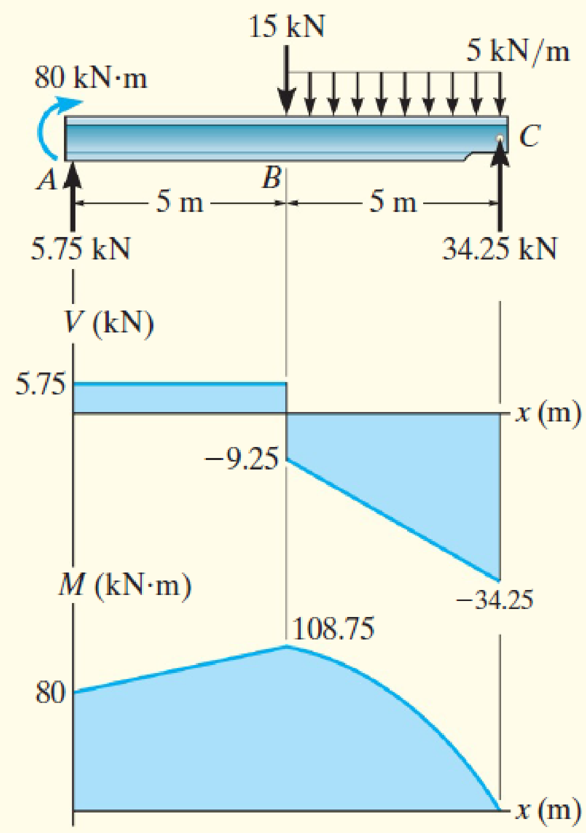
Draw the shear and moment diagrams for the beam shown in Fig.



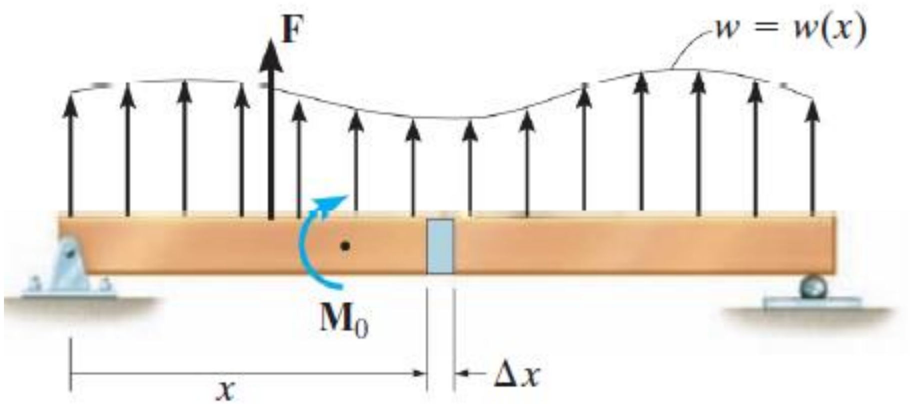

80 kN·m
M
x_1
V
5.75 kN
(b)

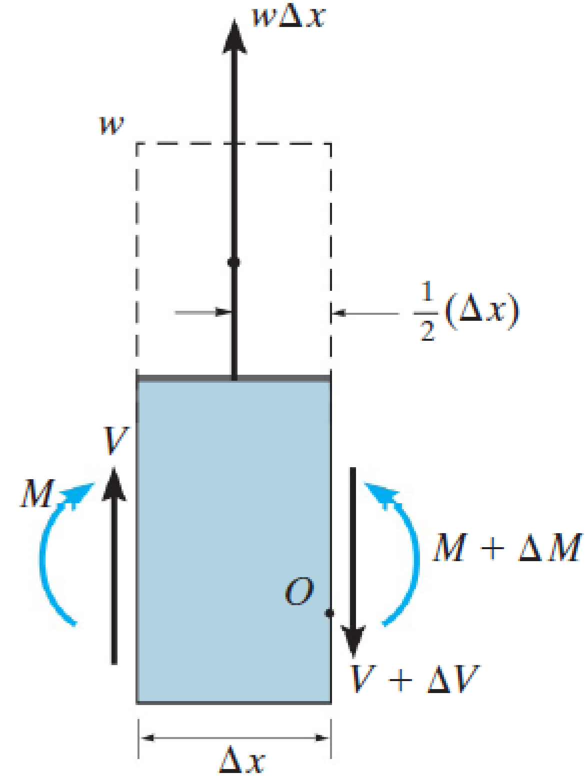

15 kN
5 kN/m
80 kN·m
C
A
B
5 m
5 m
5.75 kN
34.25 kN
V (kN)
5.75
x (m)
−9.25
−34.25
M (kN·m)
108.75
80
x (m)


15 kN
5 kN/m$(x_2 - 5)$
80 kN·m
M
5 m
$\frac{x_2-5}{2}$
$\frac{x_2-5}{2}$
V
x_2
5.75 kN
(c)


15 kN
5 kN/m
80 kN·m
C
A
B
5 m
5 m
5.75 kN
34.25 kN
V (kN)
5.75
x (m)
−9.25
−34.25
M (kN·m)
108.75
80
x (m)

GRAPHICAL METHOD FOR CONSTRUCTING SHEAR AND MOMENT DIAGRAMS





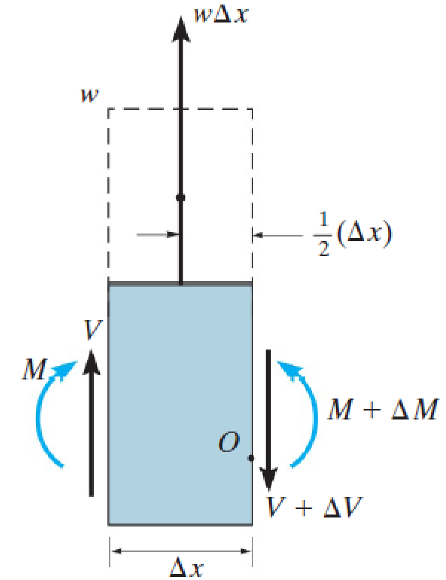
Free-body diagram of segment Δx

$$+\uparrow \Sigma F_y = 0; \qquad V + w\,\Delta x - (V + \Delta V) = 0$$

$$\Delta V = w\,\Delta x$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad -V\,\Delta x - M - w\,\Delta x\left[\tfrac{1}{2}(\Delta x)\right] + (M + \Delta M) = 0$$

$$\Delta M = V\,\Delta x + w\,\tfrac{1}{2}(\Delta x)^2$$



Free-body diagram of segment Δx

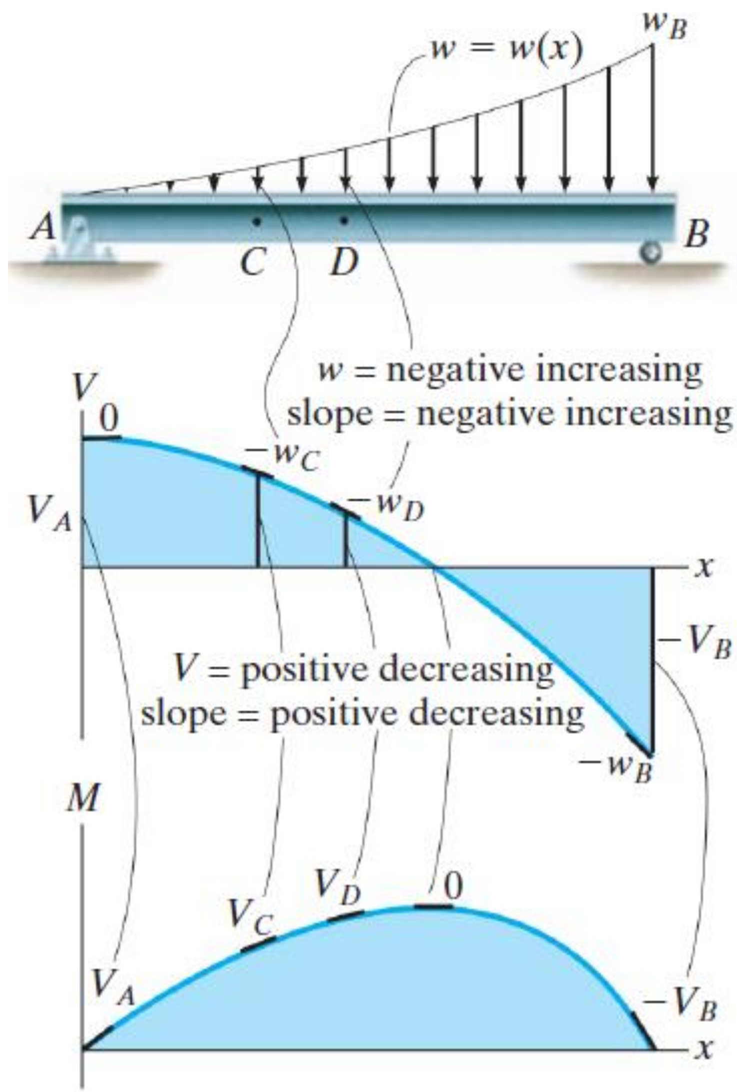
Dividing by Δx and taking the limit as $\Delta x \to 0$, the above two equations become

$$\frac{dV}{dx} = w$$

slope of shear diagram at each point = distributed load intensity at each point

$$\frac{dM}{dx} = V$$

slope of moment diagram at each point = shear at each point



$$\frac{dV}{dx} = w$$

slope of shear diagram at each point = distributed load intensity at each point

$$\frac{dM}{dx} = V$$

slope of moment diagram at each point = shear at each point

$$\frac{dV}{dx} = w$$

slope of shear diagram at each point = distributed load intensity at each point

$$\Delta V = \int w\, dx$$

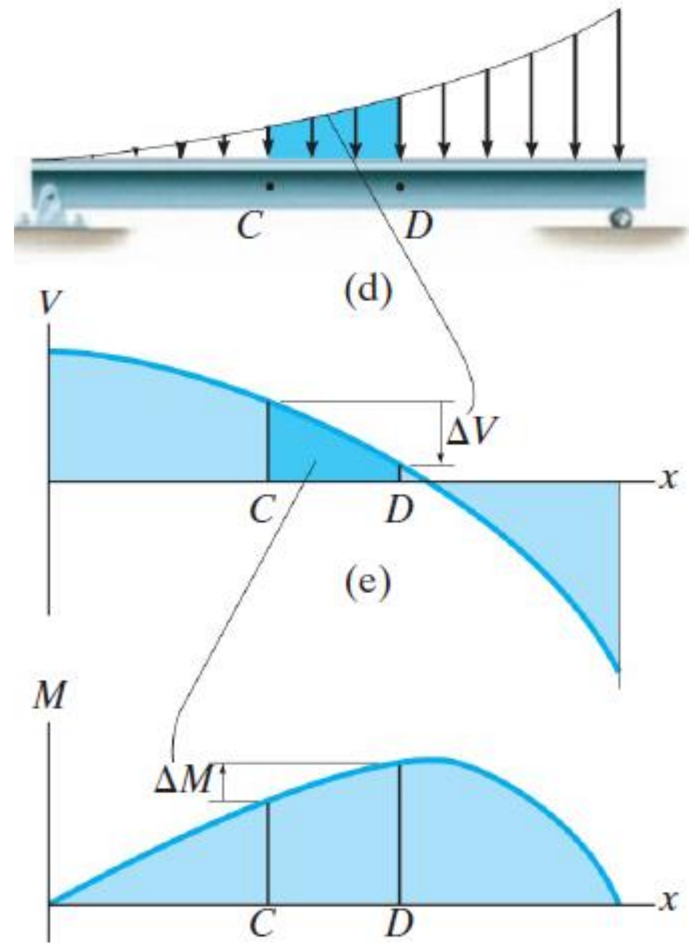
change in shear = area under distributed loading

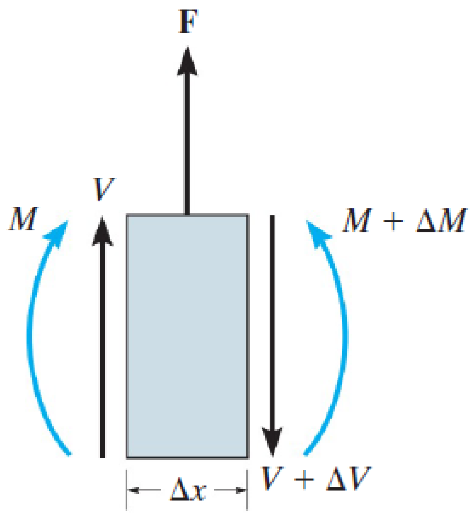
$$\frac{dM}{dx} = V$$

slope of moment diagram at each point = shear at each point

$$\Delta M = \int V\, dx$$

change in moment = area under shear diagram



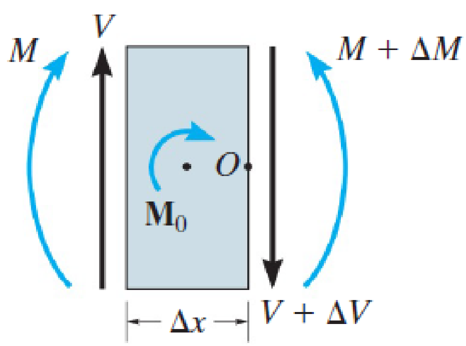


(a)

$$+\uparrow \Sigma F_y = 0; \qquad V + F - (V + \Delta V) = 0$$

$$\Delta V = F$$

Thus, when **F** acts *upward* on the beam, then the change in shear, *V*, is *positive* so the values of the shear on the shear diagram will "jump" *upward.*



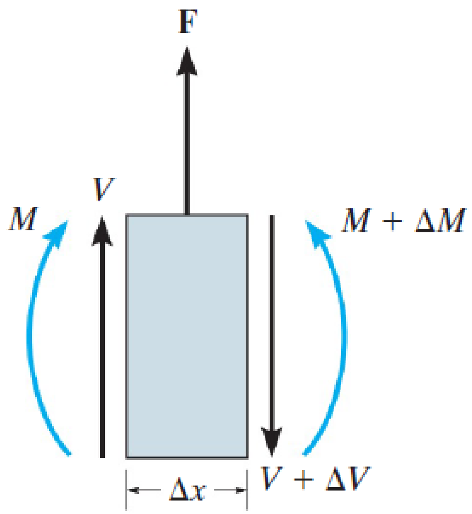
(b)

$$\circlearrowleft + \Sigma M_O = 0; \quad M + \Delta M - M_0 - V\,\Delta x - M = 0$$

Letting $\Delta x \approx 0$, we get

$$\Delta M = M_0$$

In this case, if $\mathbf{M}_0$ is applied *clockwise,* the change in moment, *M*, is *positive* so the moment diagram will "jump" *upward.* Likewise, when $\mathbf{M}_0$ acts *counterclockwise,* the jump (*M*) will be *downward*

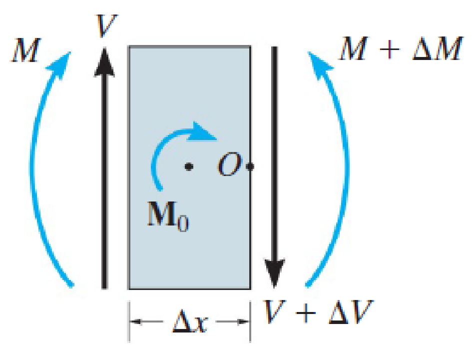


(a)

$$+\uparrow \Sigma F_y = 0; \qquad V + F - (V + \Delta V) = 0$$

$$\Delta V = F$$

Thus, when **F** acts *upward* on the beam, then the change in shear, *V*, is *positive* so the values of the shear on the shear diagram will "jump" *upward.*



(b)

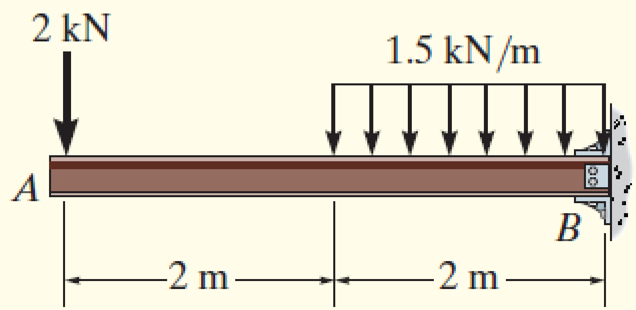
$$\circlearrowleft +\Sigma M_O = 0; \quad M + \Delta M - M_0 - V\,\Delta x - M = 0$$

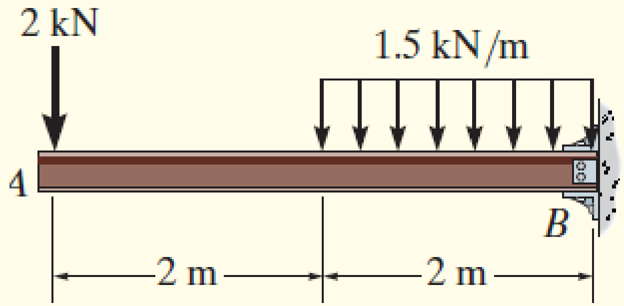
Letting $\Delta x \approx 0$, we get

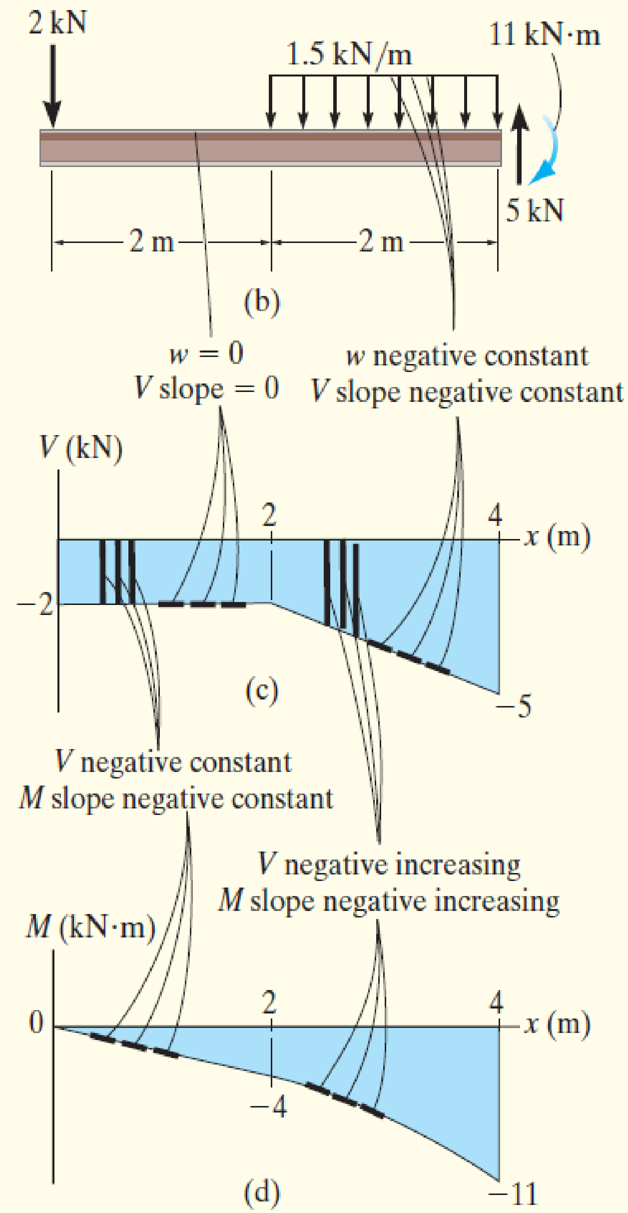
$$\Delta M = M_0$$

In this case, if $\mathbf{M}_0$ is applied *clockwise,* the change in moment, *M*, is *positive* so the moment diagram will "jump" *upward.* Likewise, when $\mathbf{M}_0$ acts *counterclockwise,* the jump (*M*) will be *downward*

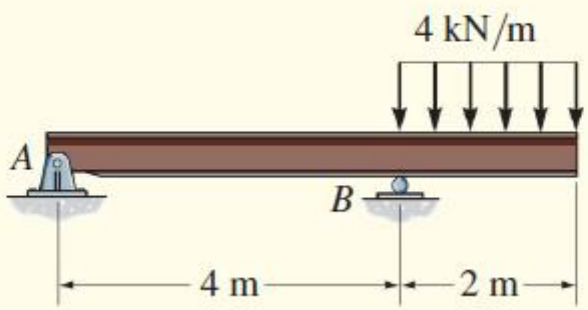
Draw the shear and moment diagrams for the cantilever beam in Fig.

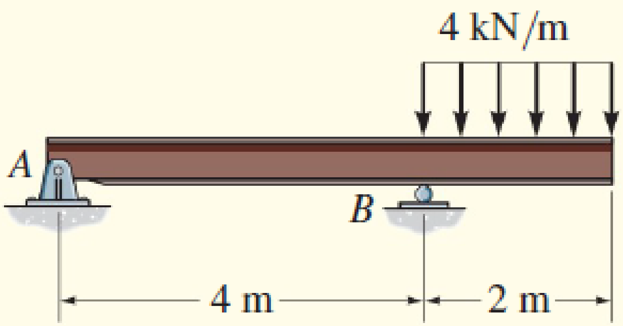


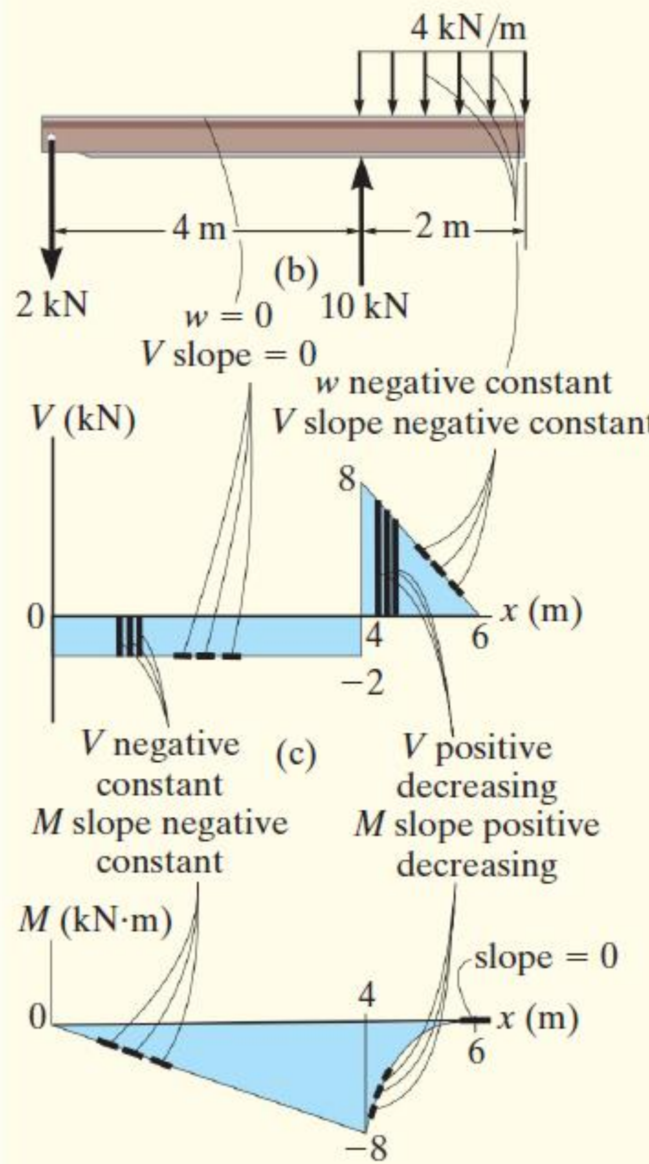

2 kN
1.5 kN/m
4
B
2 m
2 m


2 kN
1.5 kN/m
11 kN·m
5 kN
2 m
2 m
(b)
w = 0
V slope = 0
w negative constant
V slope negative constant
V (kN)
2
4
x (m)
−2
−5
(c)
V negative constant
M slope negative constant
V negative increasing
M slope negative increasing
M (kN·m)
0
2
4
x (m)
−4
−11
(d)

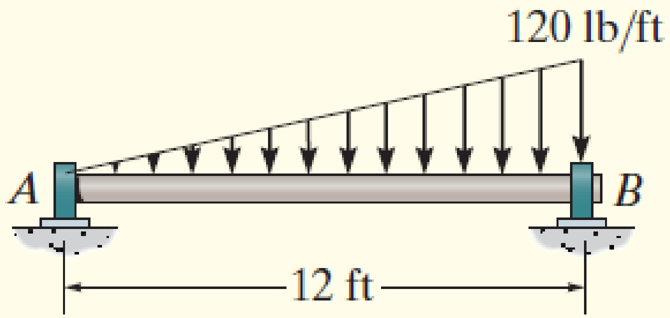
Draw the shear and moment diagrams for the overhang beam in Fig.

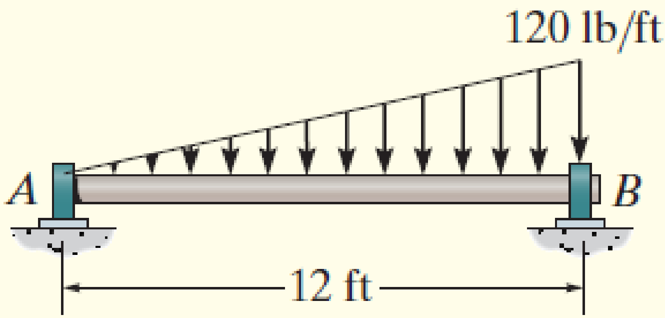


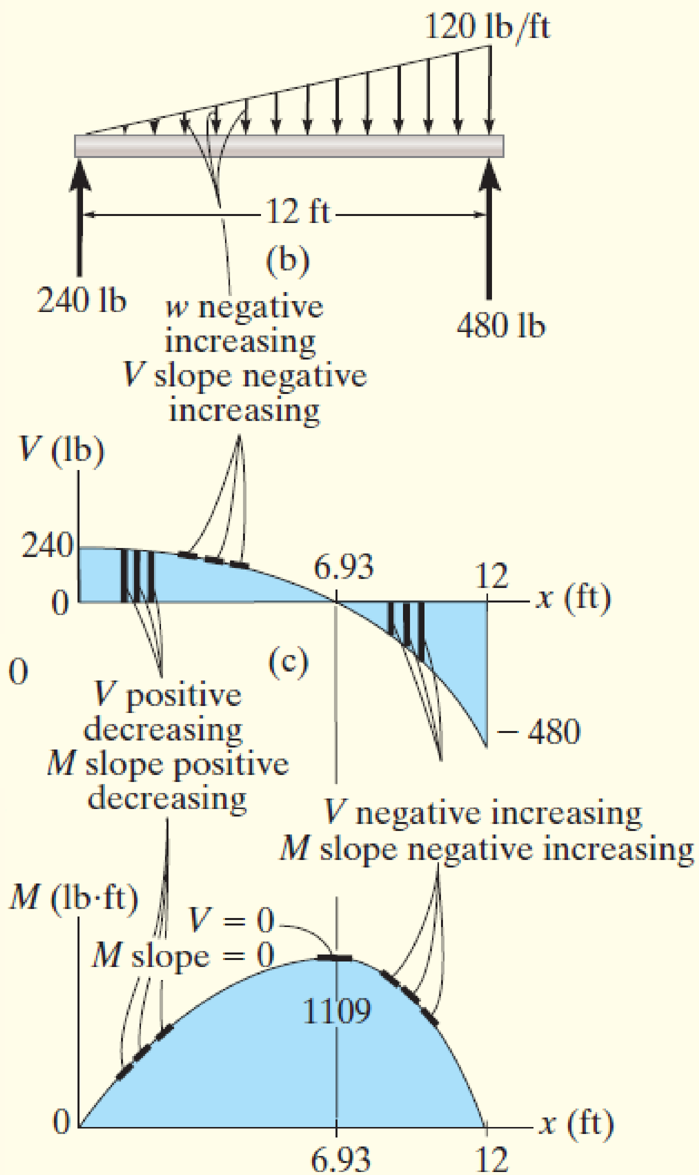

4 kN/m
A
B
4 m
2 m


4 kN/m
4 m
2 m
(b)
2 kN
10 kN
w = 0
V slope = 0
w negative constant
V slope negative constant
V (kN)
8
0
x (m)
4
6
−2
V negative
constant
M slope negative
constant
(c)
V positive
decreasing
M slope positive
decreasing
M (kN·m)
slope = 0
0
4
x (m)
6
−8

The shaft in Fig. 6–17*a* is supported by a thrust bearing at *A* and a journal bearing at *B*. Draw the shear and moment diagrams.





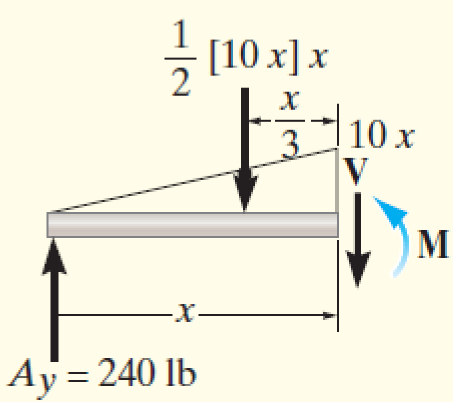


$$+\uparrow\Sigma F_y = 0; \qquad 240 \text{ lb} - \tfrac{1}{2}(10x)x = 0$$

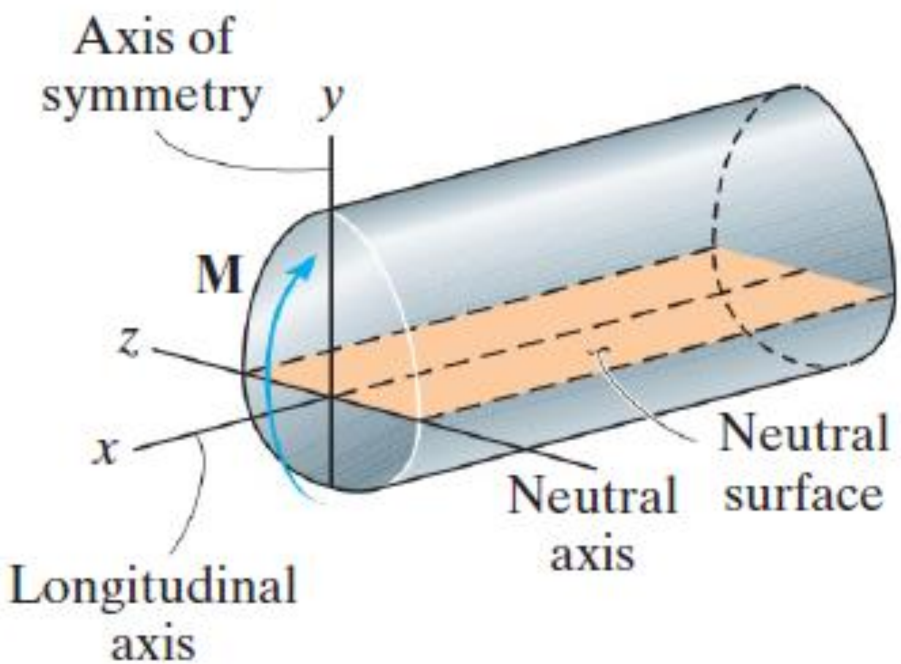
$$x = 6.93 \text{ ft}$$

$$\circlearrowleft+\Sigma M = 0; \qquad M_{max} + \tfrac{1}{2}[(10)(6.93)]\,6.93\left(\tfrac{1}{3}(6.93)\right) - 240(6.93) = 0$$

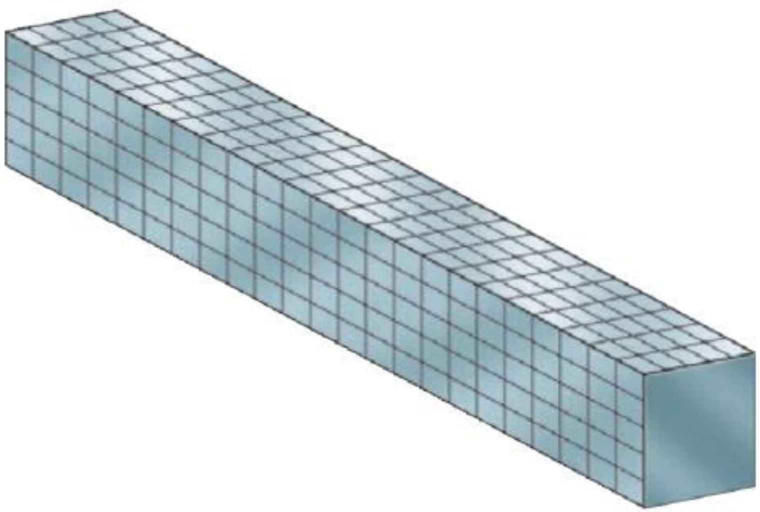
$$M_{max} = 1109 \text{ lb} \cdot \text{ft}$$

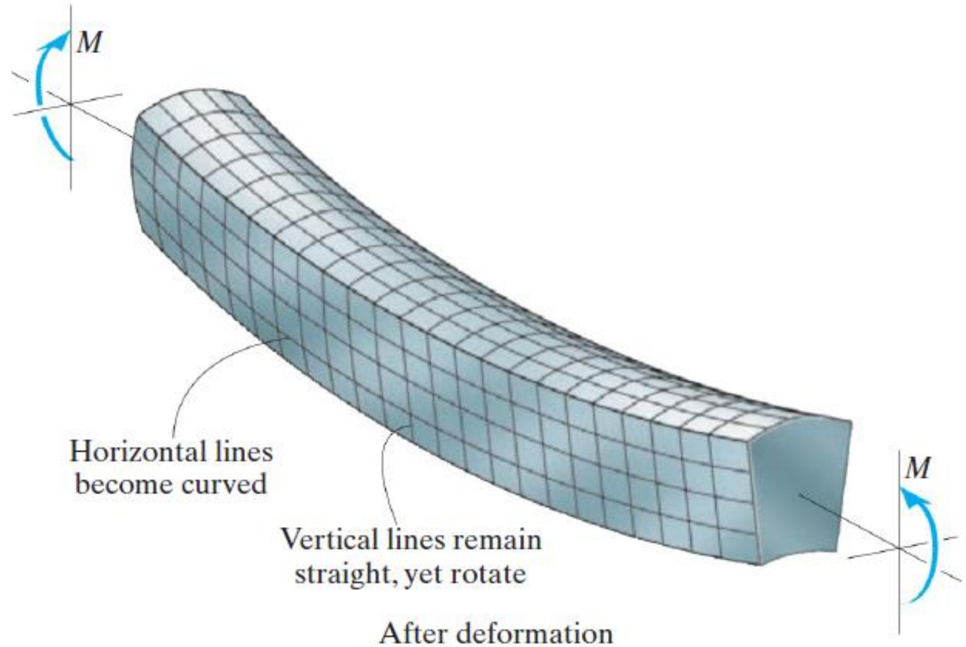


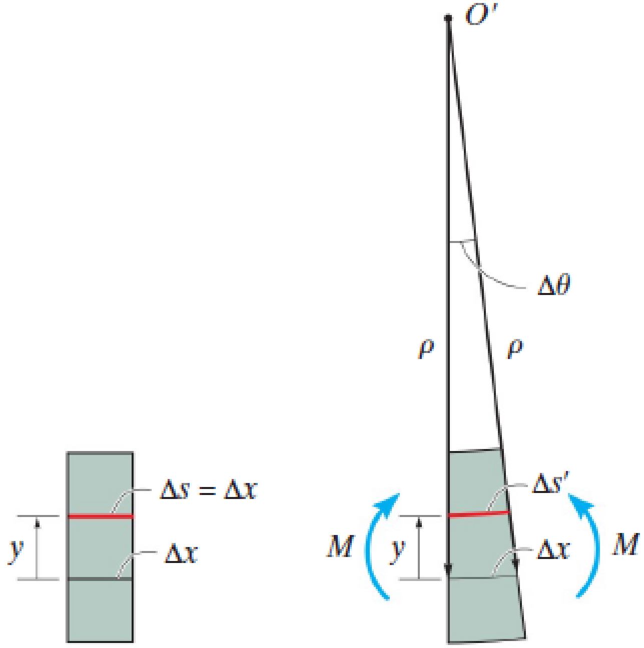
BENDING DEFORMATION OF A STRAIGHT MEMBER

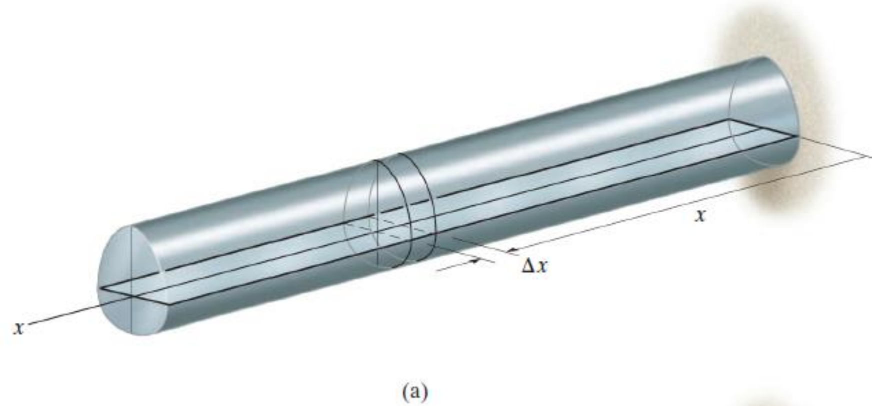


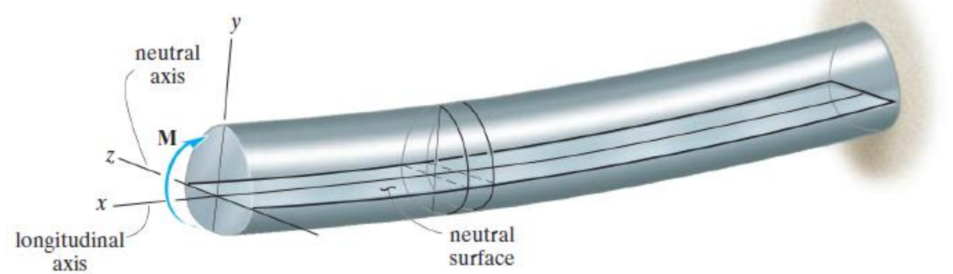
BENDING DEFORMATION OF A STRAIGHT MEMBER











$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

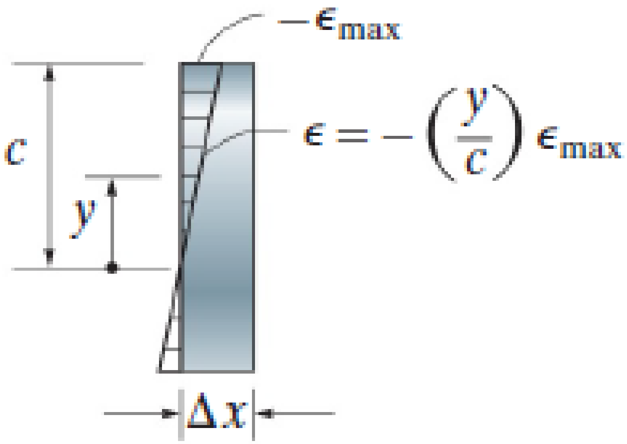
$$\Delta x = \Delta s = \rho \Delta\theta.$$

$$\Delta s' = (\rho - y)\Delta\theta.$$

$$\epsilon = \lim_{\Delta\theta \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta}$$

$$\epsilon = -\frac{y}{\rho}$$

$$\epsilon = -\frac{y}{\rho}$$



Normal strain distribution

$$\frac{\epsilon}{\epsilon_{max}} = -\left(\frac{y/\rho}{c/\rho}\right)$$

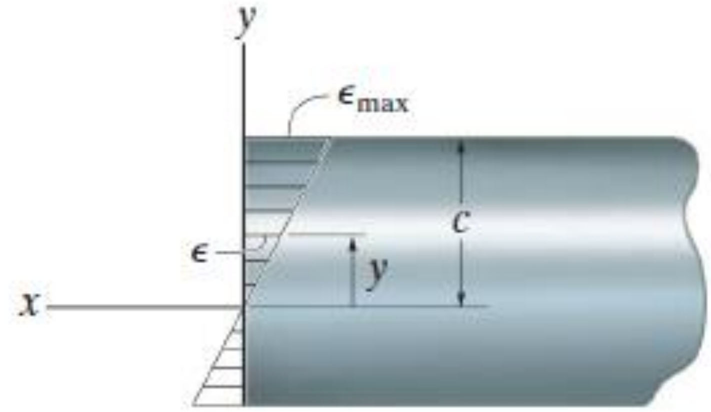
$$\epsilon = -\left(\frac{y}{c}\right)\epsilon_{max}$$

THE FLEXURE FORMULA

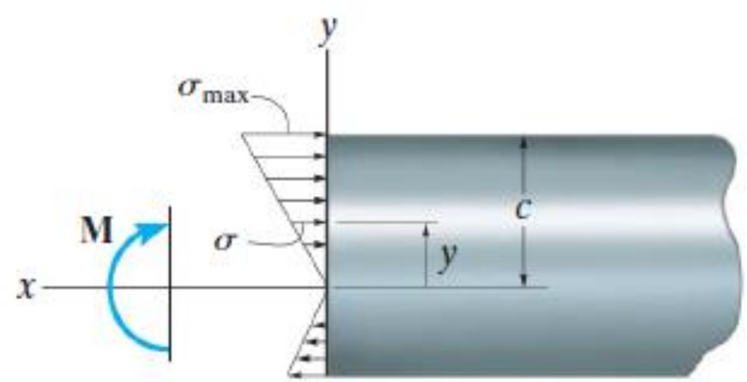
$$\sigma = E\epsilon$$

$$\epsilon = -\left(\frac{y}{c}\right)\epsilon_{max}$$

$$\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$



Normal strain variation
(profile view)



Bending stress variation
(profile view)

Location of Neutral Axis.

$$F_R = \Sigma F_x; \qquad 0 = \int_A dF = \int_A \sigma \, dA$$

$$= \int_A -\left(\frac{y}{c}\right)\sigma_{\max} \, dA$$

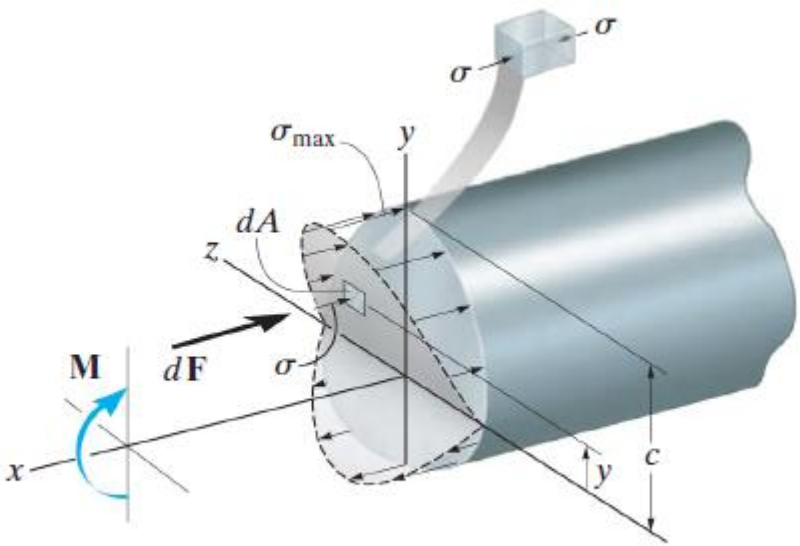
$$= \frac{-\sigma_{\max}}{c}\int_A y \, dA$$

Since $\sigma_{\max}/c$ is not equal to zero, then

$$\int_A y \, dA = 0$$

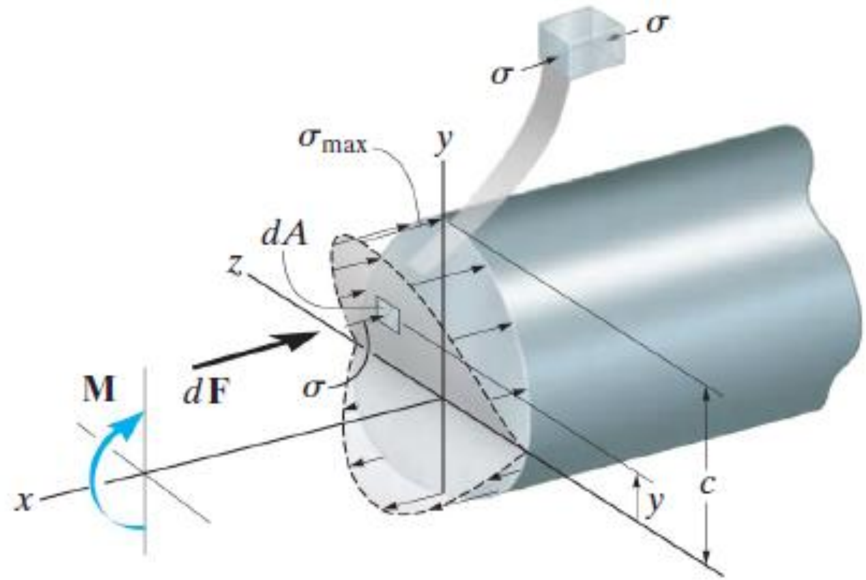
$$\bar{y} = \int y \, dA / \int dA.$$

$$\bar{y} = 0,$$



Bending stress variation

Bending Moment.



Bending stress variation

$$dM = y\,dF.$$

$$dF = \sigma\,dA$$

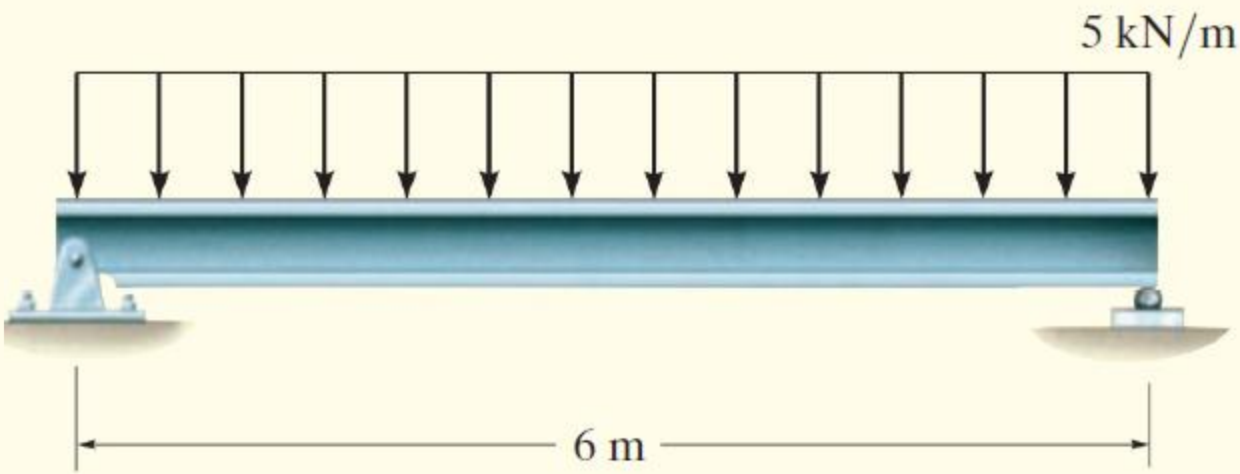
$$(M_R)_z = \Sigma M_z; \qquad M = \int_A y\,dF = \int_A y\,(\sigma\,dA) = \int_A y\left(\frac{y}{c}\sigma_{\max}\right)dA$$

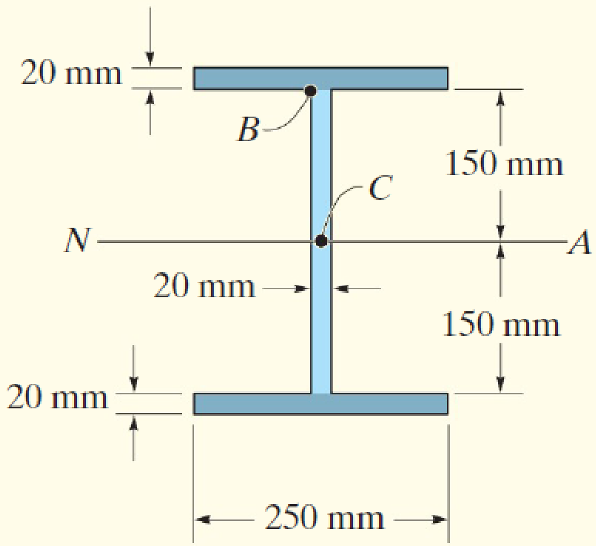
$$M = \frac{\sigma_{\max}}{c}\int_A y^2\,dA$$

$$\sigma_{\max} = \frac{Mc}{I}$$

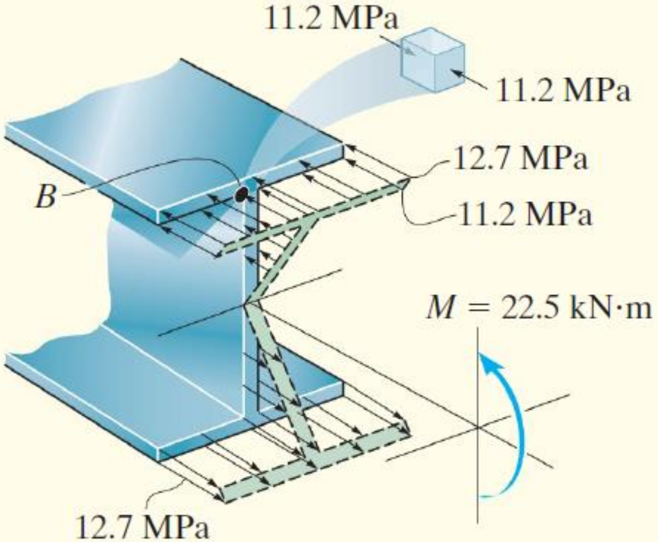
$$\sigma = -\frac{My}{I}$$

The simply supported beam in Fig. 6–26*a* has the cross-sectional area shown in Fig. 6–26*b*. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location. Also, what is the stress at point *B*?

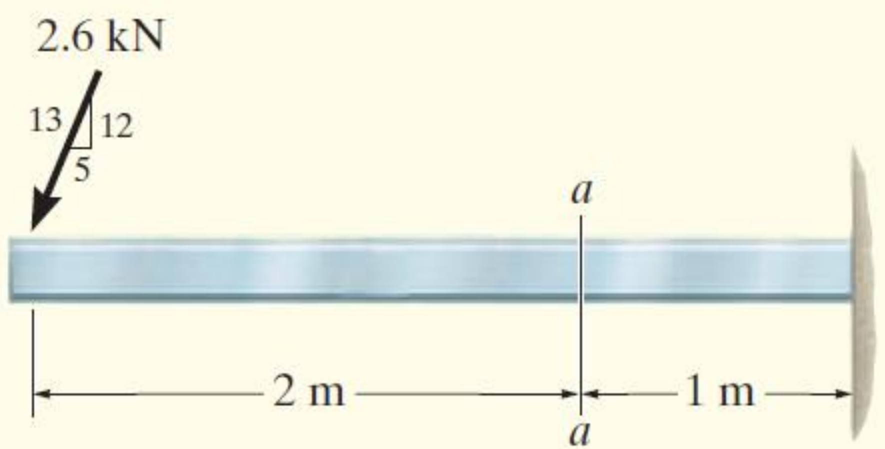


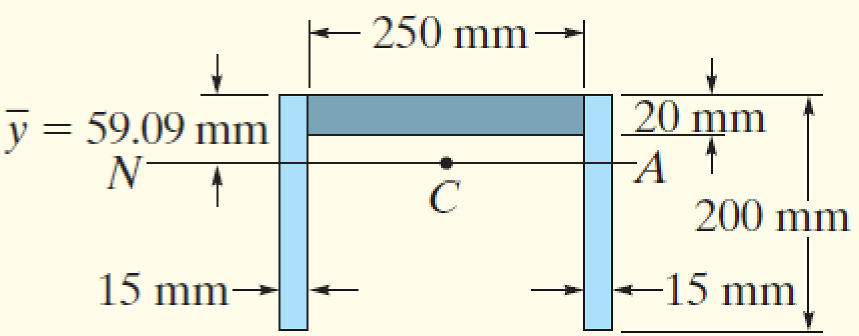


Section Property.

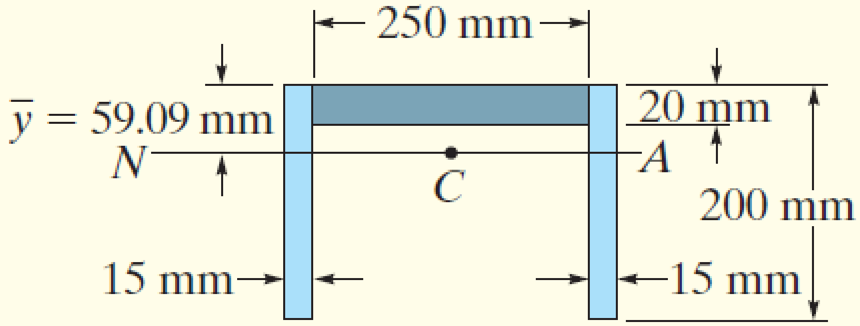


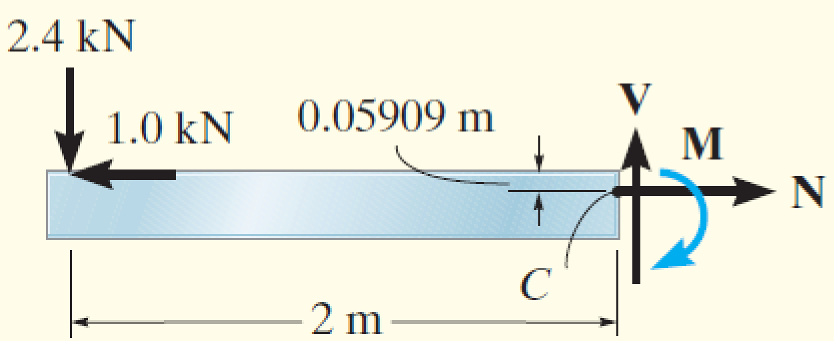
The beam shown in Fig. 6–27*a* has a cross-sectional area in the shape of a channel, Fig. 6–27*b*. Determine the maximum bending stress that occurs in the beam at section *a–a*.



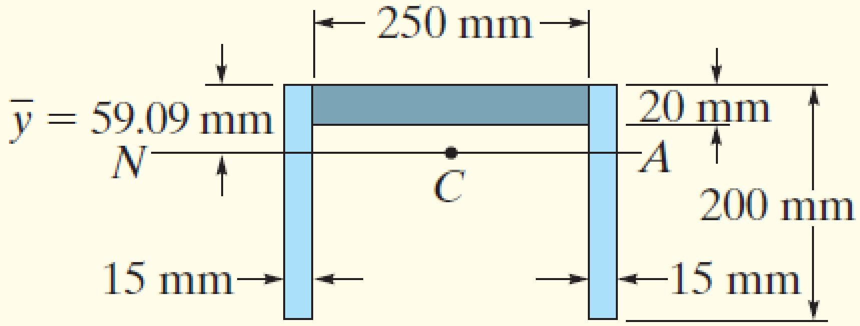


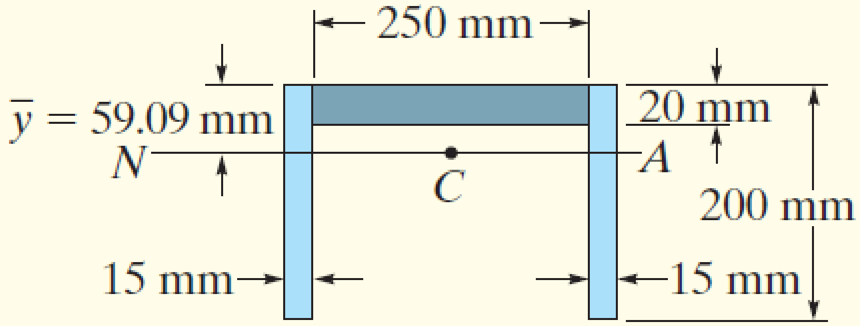
Internal Moment.





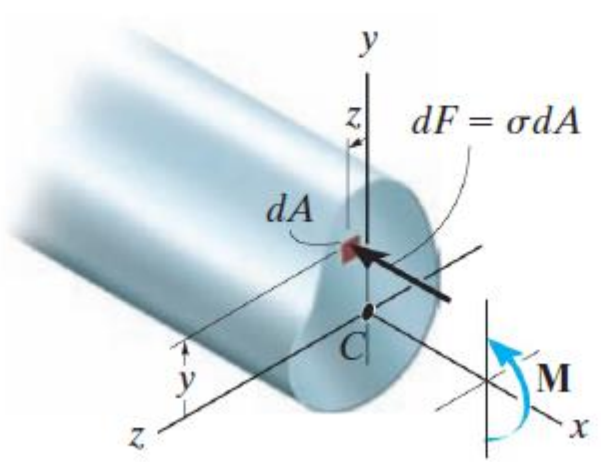
Section Property.

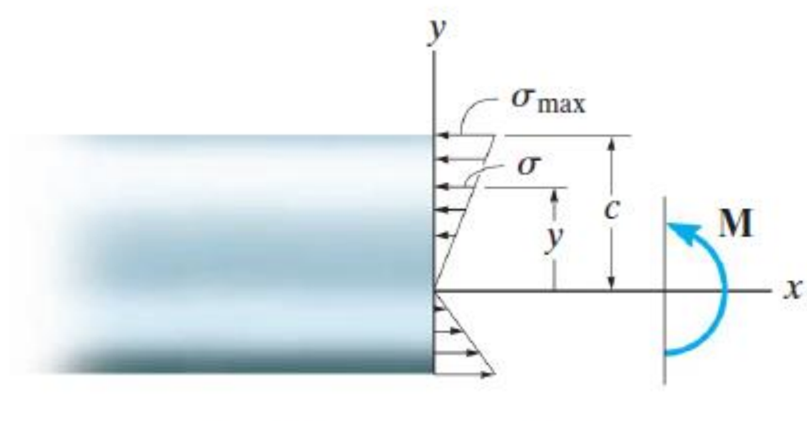



250 mm
$\bar{y}$ = 59.09 mm
20 mm
N
A
C
200 mm
15 mm
15 mm

UNSYMMETRIC BENDING

Moment Applied About Principal Axis.





Bending-stress distribution
(profile view)

$$F_R = \Sigma F_x; \qquad 0 = -\int_A \sigma \, dA$$

$$(M_R)_y = \Sigma M_y; \qquad 0 = -\int_A z\sigma \, dA$$

$$(M_R)_z = \Sigma M_z; \qquad M = \int_A y\sigma \, dA$$

UNSYMMETRIC BENDING

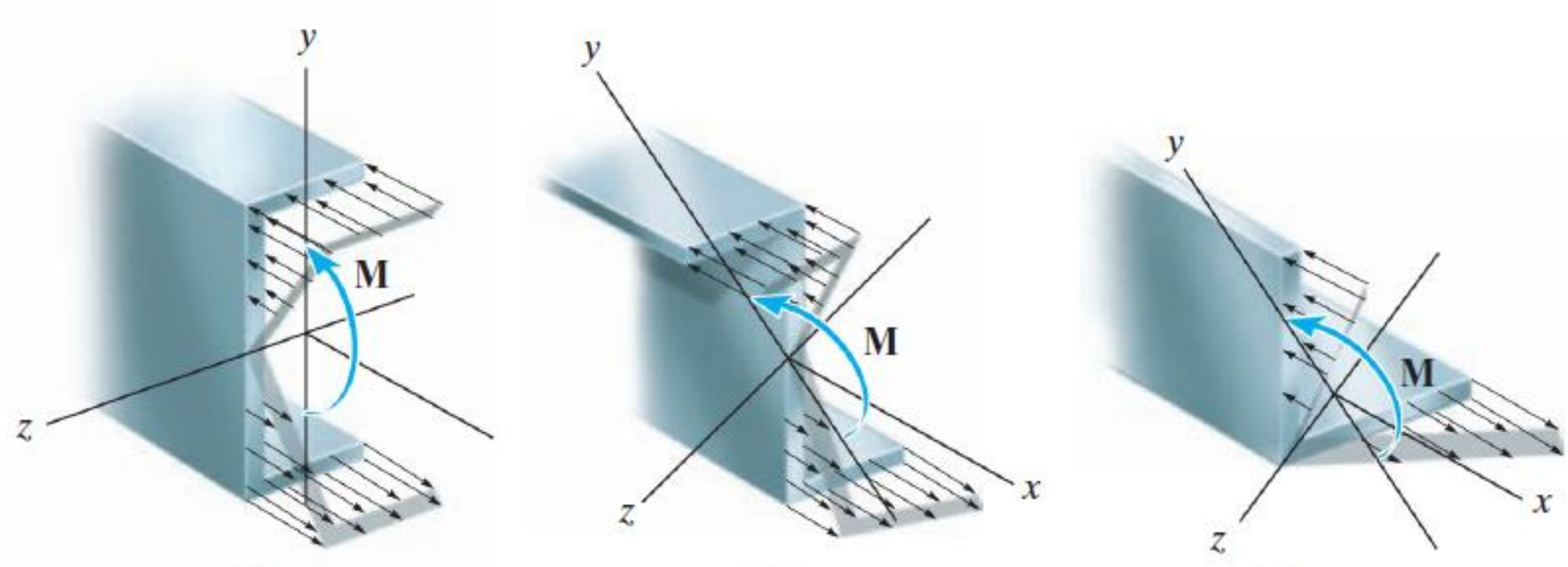
$$\sigma = -(y/c)\sigma_{max} \qquad M = \int_A y\sigma \, dA \qquad \Rightarrow \qquad \sigma_{max} = Mc/I.$$

$$\sigma = -(y/c)\sigma_{max} \qquad 0 = -\int_A z\sigma \, dA \qquad \Rightarrow \qquad 0 = \frac{-\sigma_{max}}{c}\int_A yz \, dA$$

$$\int_A yz \, dA = 0$$

product of inertia

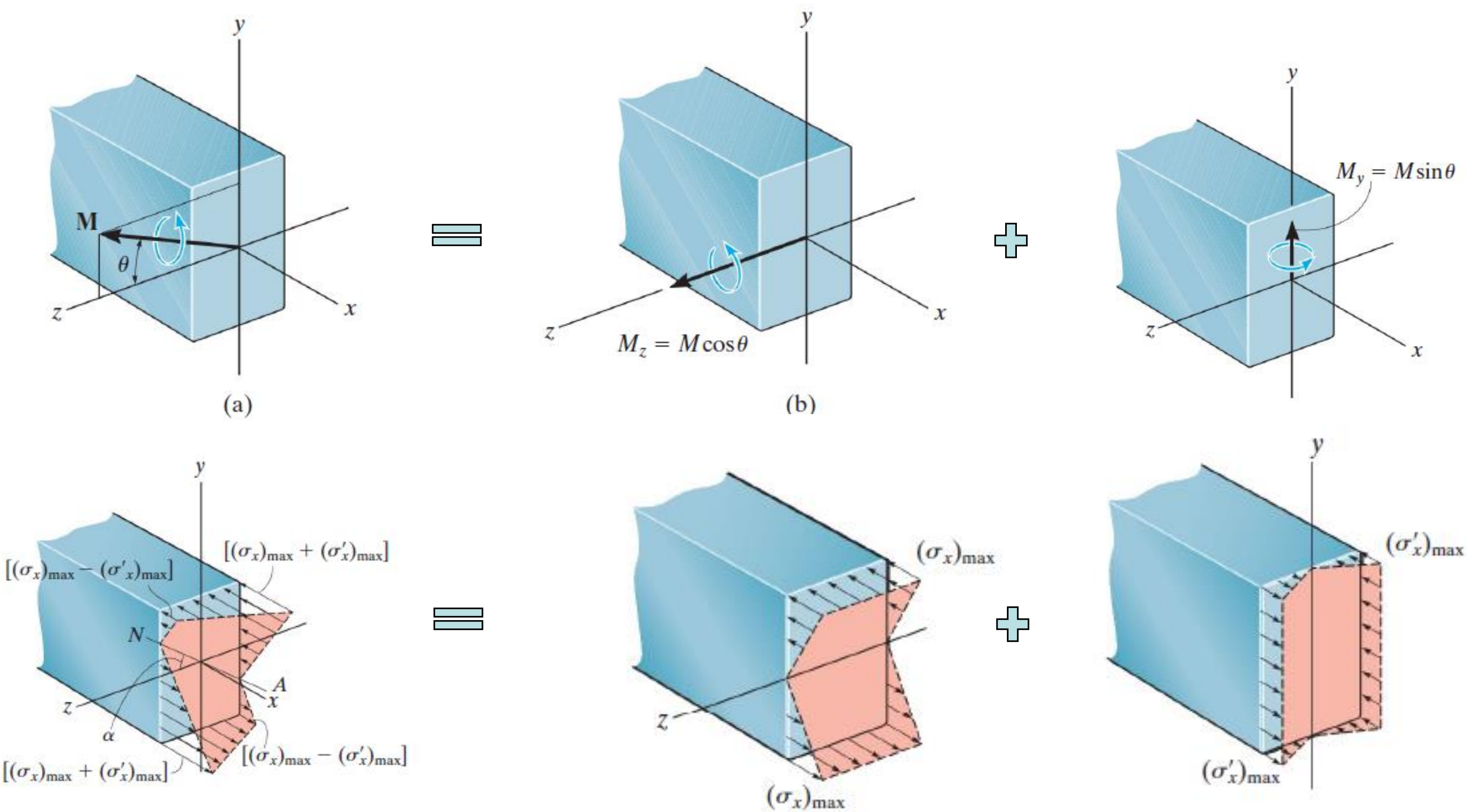
UNSYMMETRIC BENDING



$$\sigma = -My/I_z$$

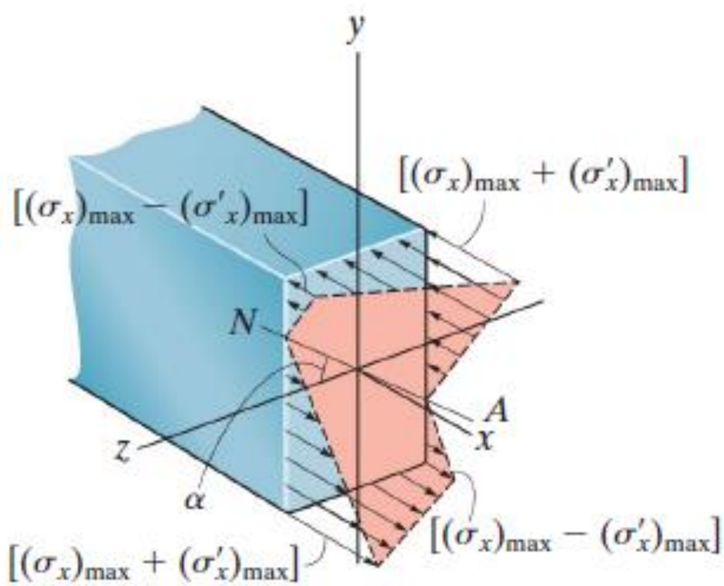
UNSYMMETRIC BENDING

Moment Arbitrarily Applied.



UNSYMMETRIC BENDING

Moment Arbitrarily Applied.



$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

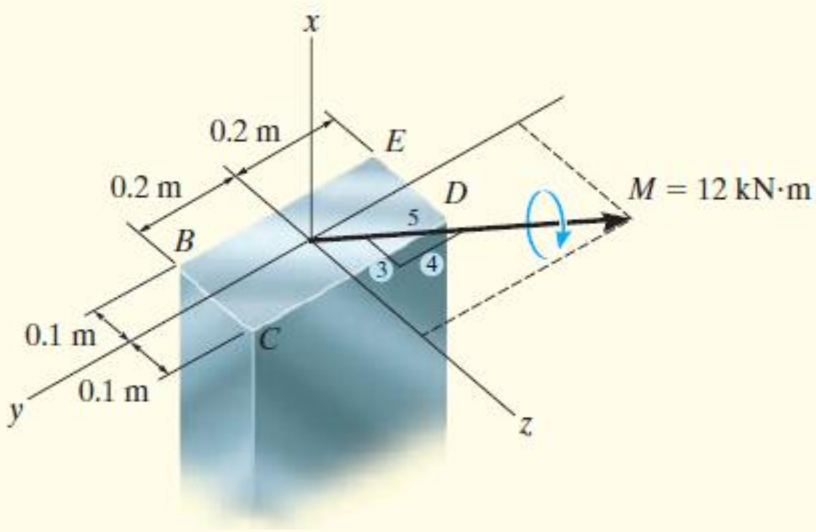
Orientation of the Neutral Axis.

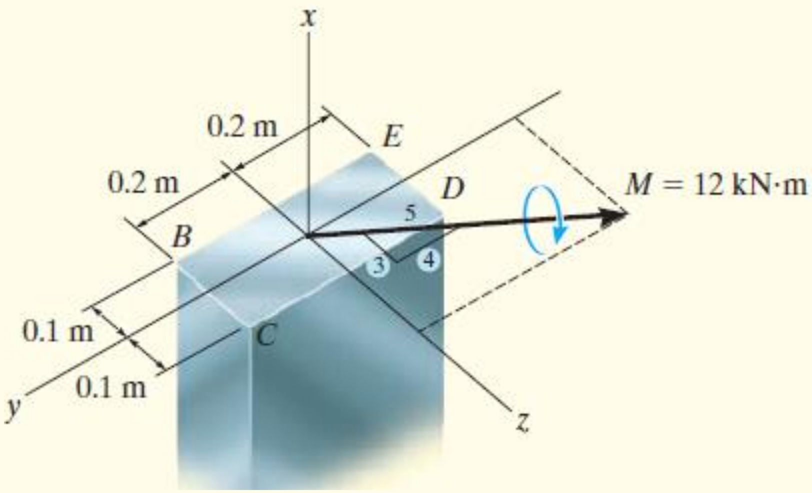
$$\sigma = 0, \qquad y = \frac{M_y I_z}{M_z I_y} z$$

Since $M_z = M\cos\theta$ and $M_y = M\sin\theta$, then

$$y = \left(\frac{I_z}{I_y}\tan\theta\right) z$$

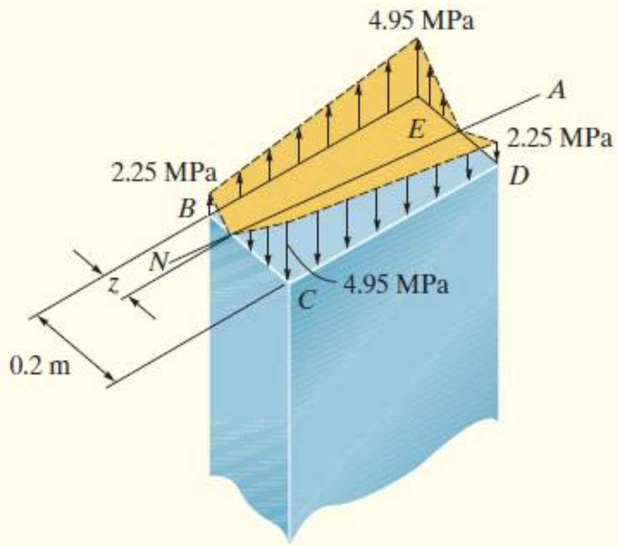
The rectangular cross section shown in Fig. 6–33*a* is subjected to a bending moment of $M = 12\ \text{kN}\cdot\text{m}$. Determine the normal stress developed at each corner of the section, and specify the orientation of the neutral axis.



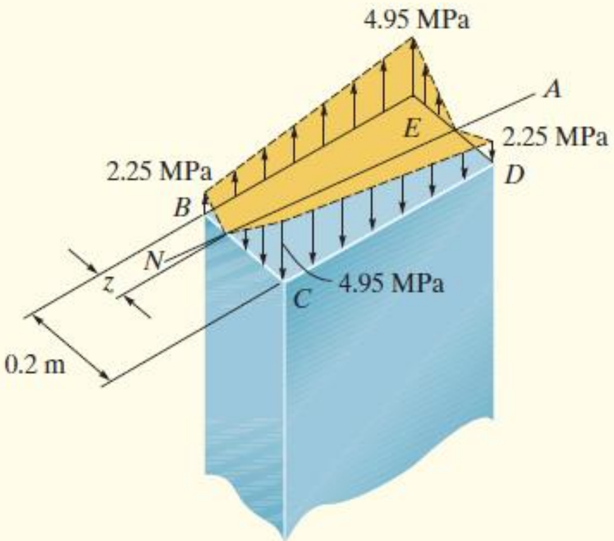


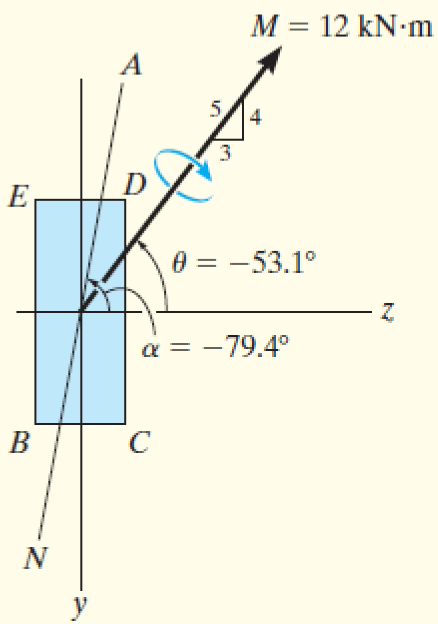
Internal Moment Components.

Bending Stress.

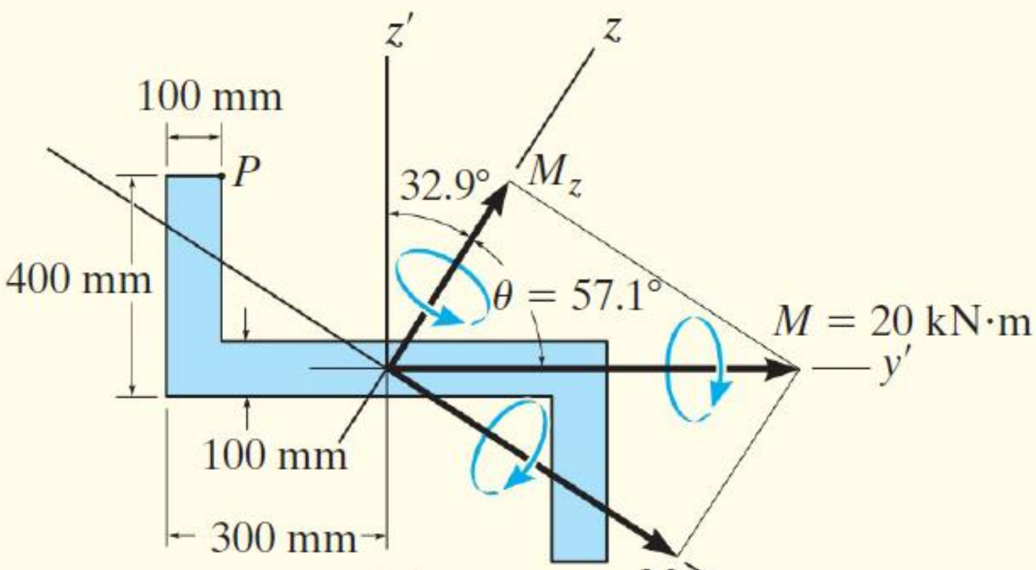


Orientation of Neutral Axis.

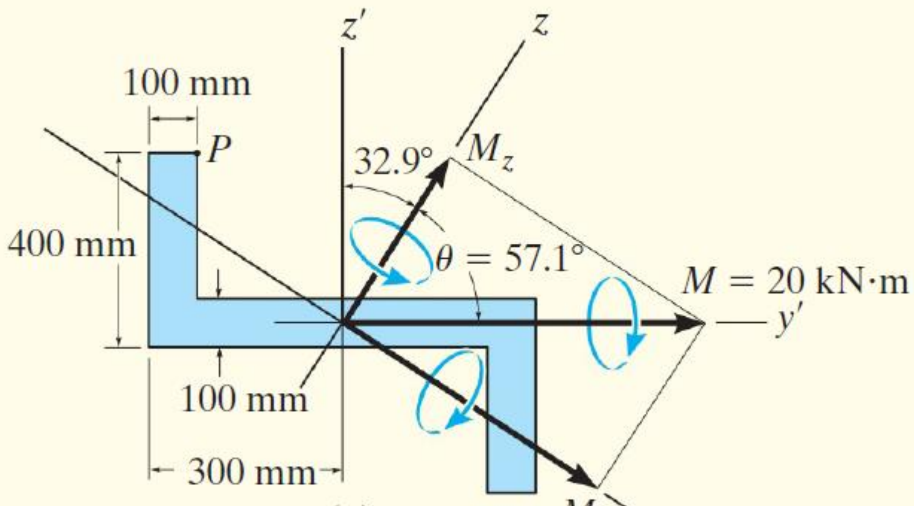




The Z-section shown in Fig. 6–34a is subjected to the bending moment of $M = 20\ \text{kN}\cdot\text{m}$. The principal axes y and z are oriented as shown, such that they represent the minimum and maximum principal moments of inertia, $I_y = 0.960(10^{-3})\ \text{m}^4$ and $I_z = 7.54(10^{-3})\ \text{m}^4$, respectively.* Determine the normal stress at point P and the orientation of the neutral axis.



(a)



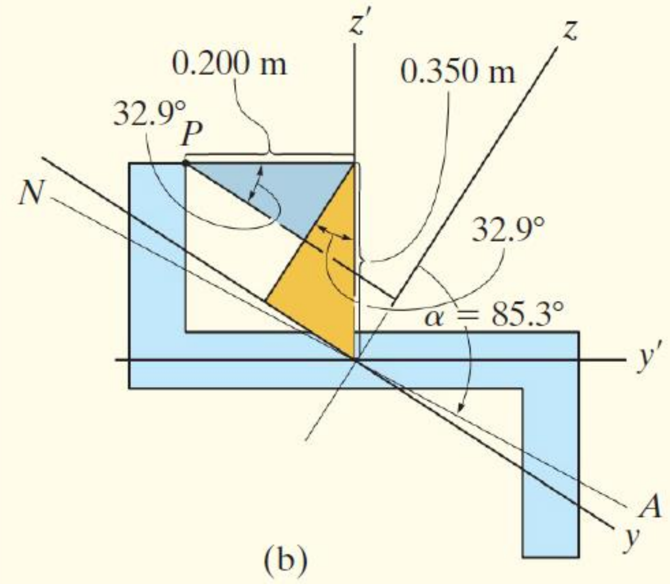
(a)

Internal Moment Components.

$$M_y = 20 \text{ kN} \cdot \text{m} \sin 57.1^\circ = 16.79 \text{ kN} \cdot \text{m}$$

$$M_z = 20 \text{ kN} \cdot \text{m} \cos 57.1^\circ = 10.86 \text{ kN} \cdot \text{m}$$

Bending Stress.



Orientation of Neutral Axis.