# Linear Control Systems

**Homework 3:** Transient Response, Steady State Error & Sensitivity

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## 1. Transient Response

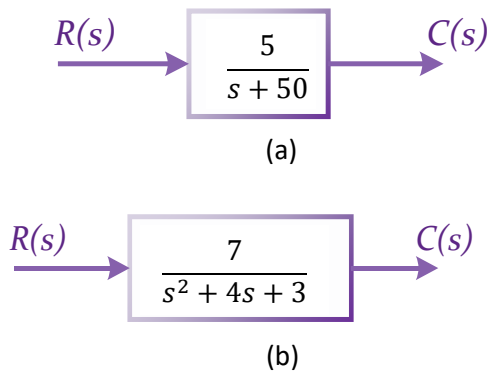1.1 Find the step response of the systems shown in Figure 1.1, also find the time constant for the first-order system.

$$R(s) \rightarrow \boxed{\dfrac{5}{s+50}} \rightarrow C(s)$$

(a)

$$R(s) \rightarrow \boxed{\dfrac{7}{s^2+4s+3}} \rightarrow C(s)$$

(b)

Figure 1. 1

1.2 Find $\zeta, \omega_n, T_s, T_p, T_r$, and $\%OS$, for each of the following second-order systems.

a) $T(s) = \dfrac{20.25}{s^2+9s+20.25}$                           b) $T(s) = \dfrac{14.24}{s^2+4s+14.24}$

1.3 Use following pair of second-order specifications to find the transfer function.

a) $\%OS = 16.5\%$; $T_p = 1.21\ s$

b) $T_s = 2.667\ s$; $T_p = 0.847\ s$

1.4 Which of the following third-order systems can be approximated as a second-order system? Assume a unit step input. (Write down the reason)

a) $G(s) = \dfrac{195}{s^3+19s^2+73s+195}$

b) $G(s) = \dfrac{6}{s^3+5s^2+8s+6}$

c) $G(s) = \dfrac{200.28(s+7)}{s^3+37s^2+410.3s+1402}$

1.5 An oscillatory system is known to have a transfer function of the following form:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Assume that a record of a damped oscillation is available as shown in Figure 1.2. Determine the damping ratio z of the system from the graph.
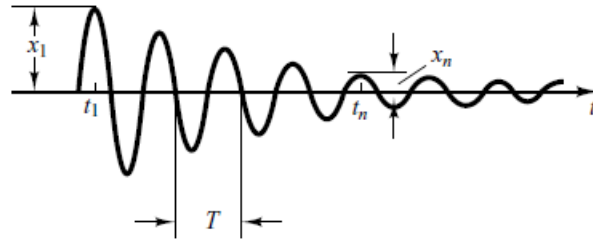


Figure 1. 2

1.6 The unity negative feedback system in Figure 1.3 has the process and controller given by

$$G(s) = \frac{1}{s(s+15)(s+25)} \qquad\qquad G_c(s) = K_p + \frac{K_I}{s}$$

The controller is a proportional plus integral controller with gains $K_p$ and $K_I$. The objective is to design the controller gains such that the dominant roots have a damping ratio $\zeta$ equal to $0.707$. Determine the resulting peak time and settling time (with a 2% criterion) of the system to unit step input.
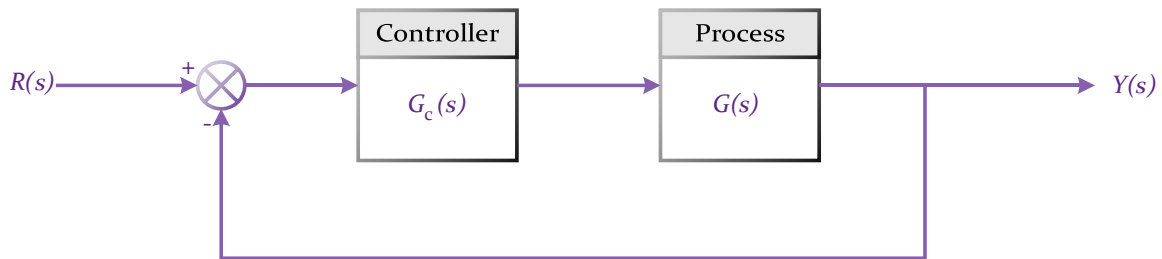


Figure 1. 3

## 2. Steady-State Error

2.1 For the unity feedback system shown in Figure 2.1 where

$$G(s) = \frac{20(s + 6)(s + 9)}{s(s + 1)(s + 2)(s + 3)(s + 5)}$$

Find the steady-state error for inputs of $20u(t), 15tu(t), and \ \frac{1}{2}t^2u(t)$.

2.2 Consider the feedback control system shown in Figure 2.2, determine the steady-state error for a step input for $k = 30$ and $k = 90$.
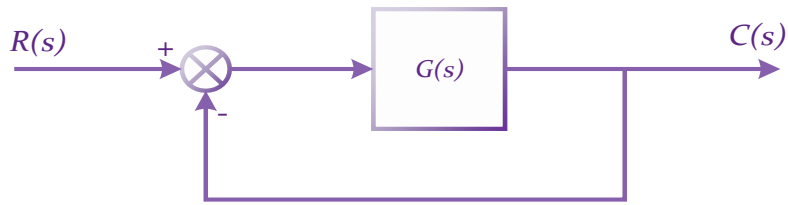
[2]

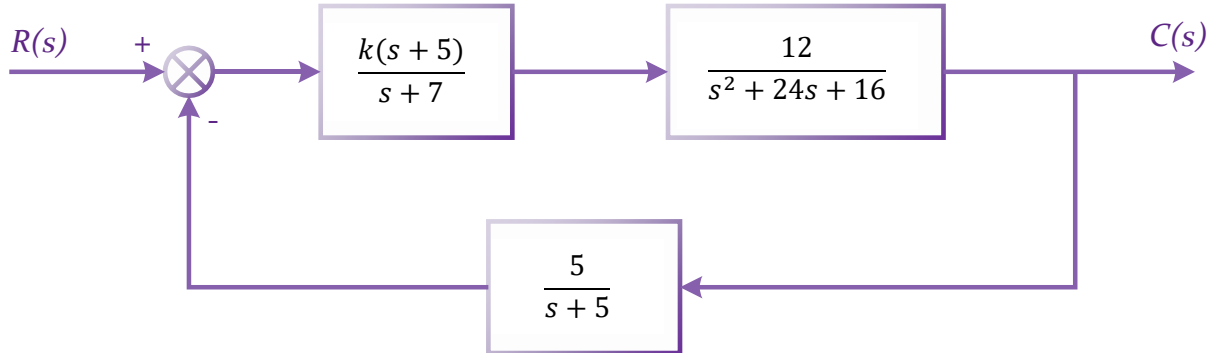$R(s)$     $+$     $G(s)$     $C(s)$     $-$

Figure 2. 1

$R(s)$     $+$     $\dfrac{k(s+5)}{s+7}$     $\dfrac{12}{s^2+24s+16}$     $C(s)$     $-$

$\dfrac{5}{s+5}$

Figure 2. 2

$T_d(s)$

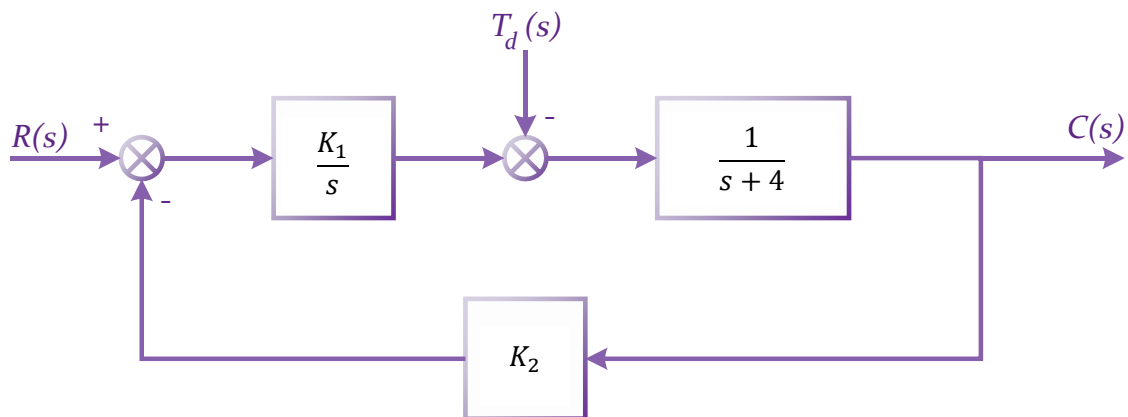$R(s)$     $+$     $\dfrac{K_1}{s}$     $-$     $\dfrac{1}{s+4}$     $C(s)$     $-$

$K_2$

Figure 2. 3

2.3 For the speed control system of Figure 2.3

a) Determine the range of $K_2$ allowable so that the steady state is $e_{ss} \leq 1\%$.

b) Determine a suitable value for $K_1$ and $K_2$ so that the magnitude of steady-state error to a wind disturbance $T_d(t) = 2t \frac{mrad}{s}, 0 \leq t < 5\ s$, is less than $0.1\ mrad$.

2.4 Consider the unity feedback system shown in Figure 2.4, this system is to be designed to meet the following specifications:

a) steady-state error for a unit ramp input=0.15; $\zeta = 0.4; \omega_n = \sqrt{5}$, find $n, K, \alpha, and\ \beta$.

b) steady-state error for a unit ramp input=0.15; the closed loop poles will be located at $-1 \pm j2$, find $n, K, \alpha, and\ \beta$.
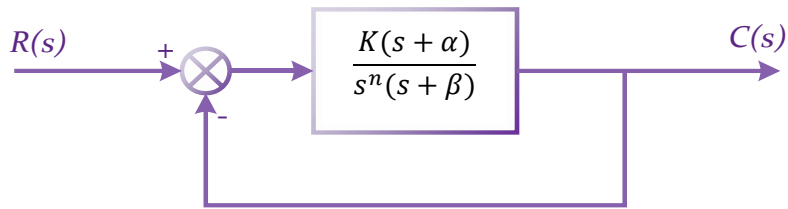
[3]

Figure 2. 4

2.5 The block diagram model of an armature-current-controlled DC motor is shown in Figure 2.5.

a) Determine the steady-state tracking error to a ramp input $r(t) = t, t \geq 0$, in terms of $K, K_b$, and $K_m$.

b) Let $K_m = 0.05$, and select $K$ so that steady-state tracking error is equal to 1.

c) Plot the response to a unit step input and a unit ramp input for 20 seconds. Are the responses acceptable?
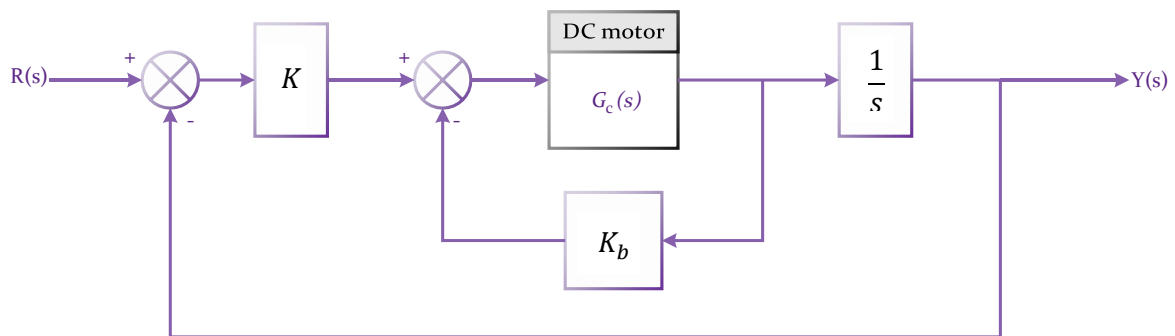


Figure 2. 5

## 3. Sensitivity

3.1 The block diagram of a control system is as shown in Figure 3.1,

a) Determine the overall transfer function $T(s) = Y(s)/R(s)$
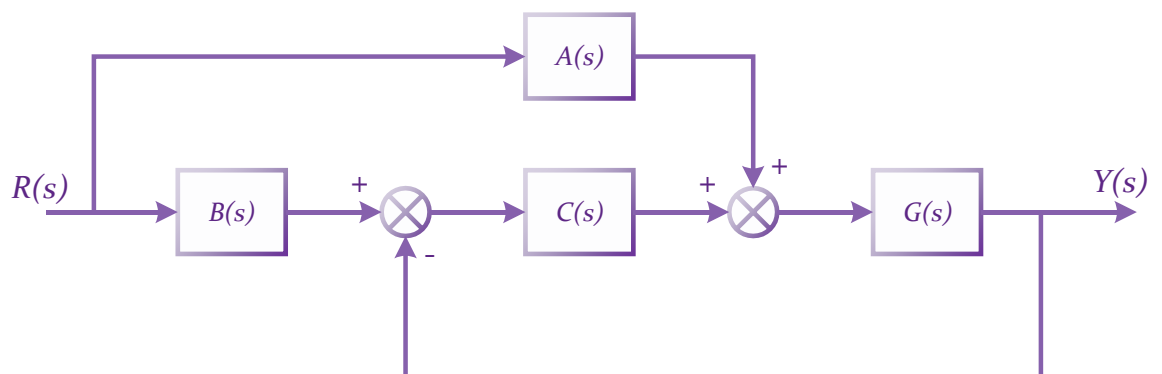
b) Calculate the sensitivity $S_{T:G}$.



Figure 3. 1

[4]

3.2 For the system of Figure 3.2, find the sensitivity of steady-state error to parameter $a$, assume a step input. Also find the sensitivity $S_{T:K}$.
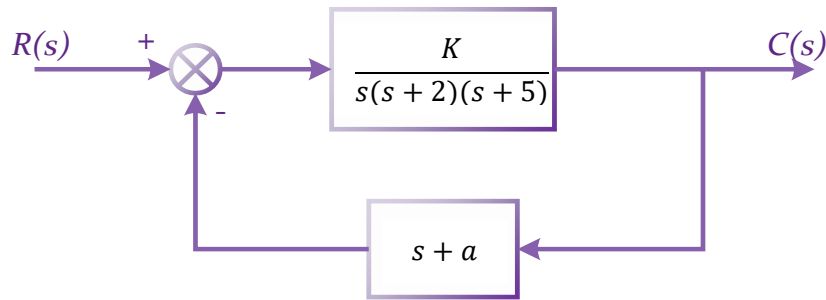


Figure 3. 2

## 4. MATLAB Simulation

4.1 Use MATLAB to obtain settling time, peak time, rise time, and percent overshoot for the system in problem 1.2 (a).

4.2 Use MATLAB to plot the step response of the system represented in state space.

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -6 & 1 \\ 0 & 0 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = [\,0\ 1\ 1\,]\mathbf{x}$$

4.3 Consider a control system defined by the following transfer function

$$G(s) = \frac{4}{s^2 + 4\zeta s + 4}$$

Plot unit-step response when $\zeta$ assumes the following values.

$$\zeta = 0, \quad 0.2, \quad 0.4, \quad 0.6, \quad 0.8, \quad 1$$

Also plot a three-dimensional plot.

4.4 For the system of Figure 4.1 use MATLAB to evaluate the static error constants and find the expected error for the standard step, ramp, and parabolic inputs.
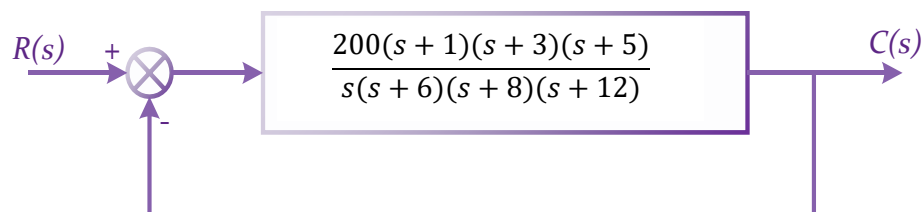


Figure 4. 1

[5]

4.5 Consider the closed-loop system in Figure 4.2. Develop
   an m-file to accomplish the following tasks:

a) Determine the closed-loop transfer function $T(s) = Y(s)/R(s)$

b) Plot the closed-loop system response to an impulse input
   $R(s) = 1$, a unit step input $R(s) = \frac{1}{s}$, and a unit ramp input
   $R(s) = \frac{1}{s^2}$. Use the subplot function to display the three
   system responses.

"Try to strengthen your words not your volume of voice, if
you shout your voice will be heard but if you speak slowly
others will listen to your words."