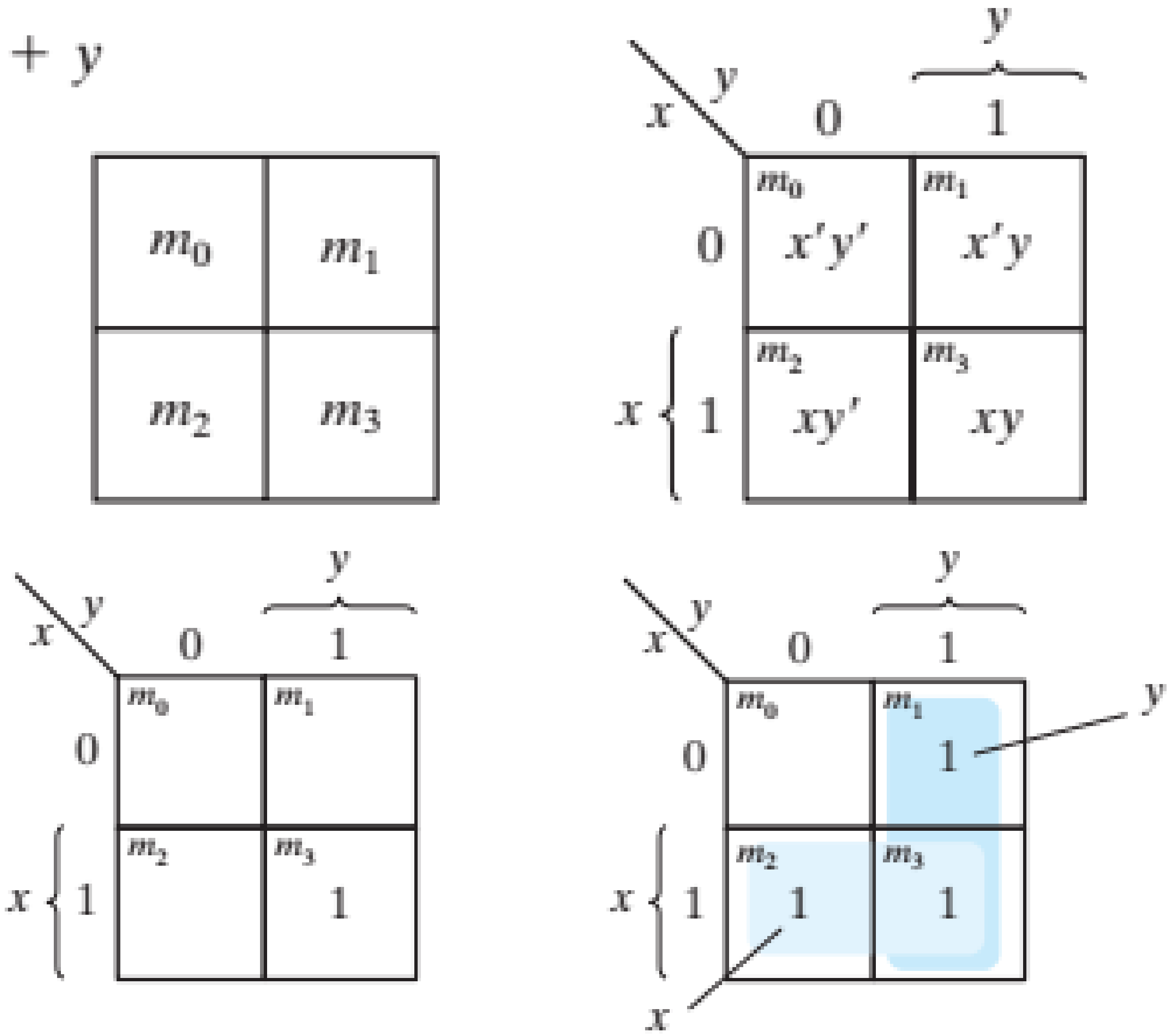
Digital

Gate-Level Minimization

Two-Variable K-Map

$$m_1 + m_2 + m_3 = x'y + xy' + xy = x + y$$

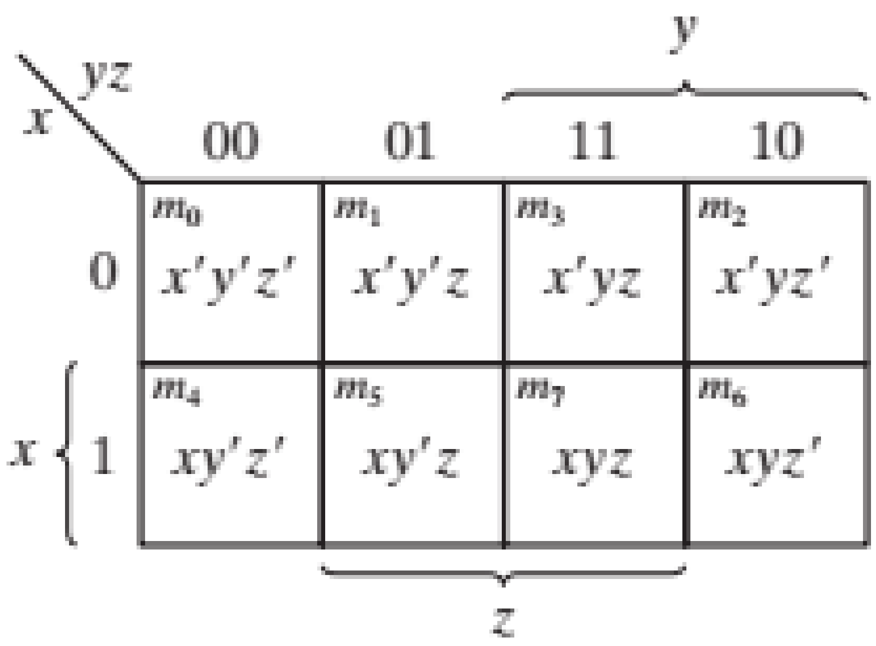
x	y
0	0
0	1
1	0
1	1



THE MAP METHOD

Three-Variable K-Map

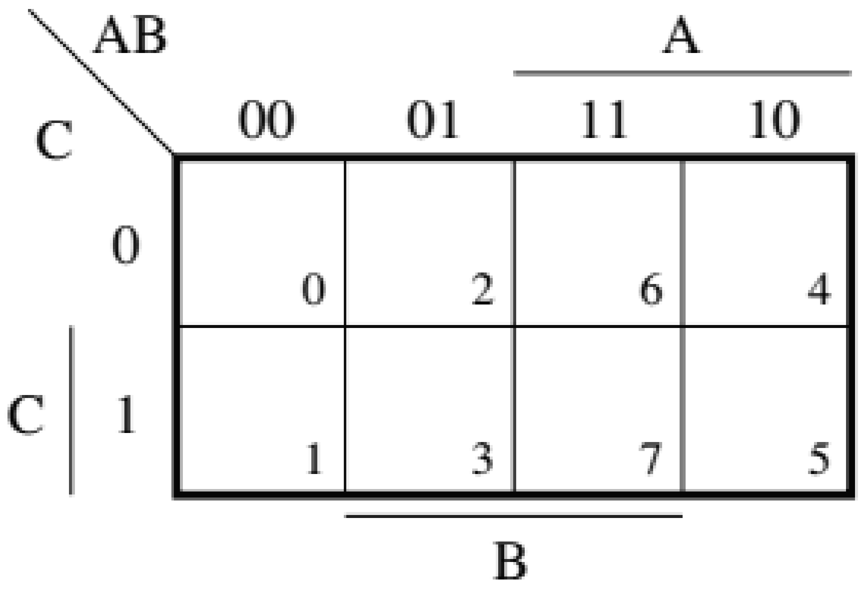
m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

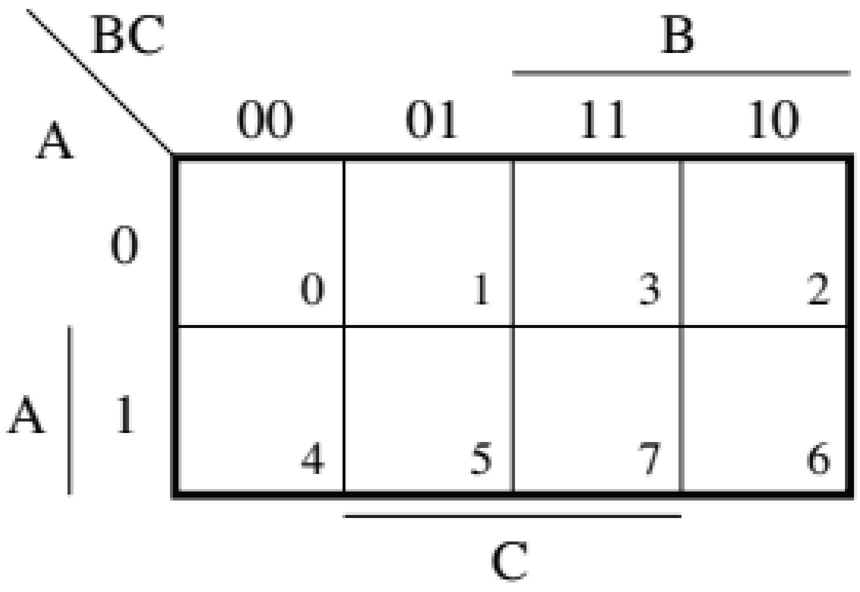


	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	3	2
A	4	5	7	6

THE MAP METHOD

Three-Variable K-Map



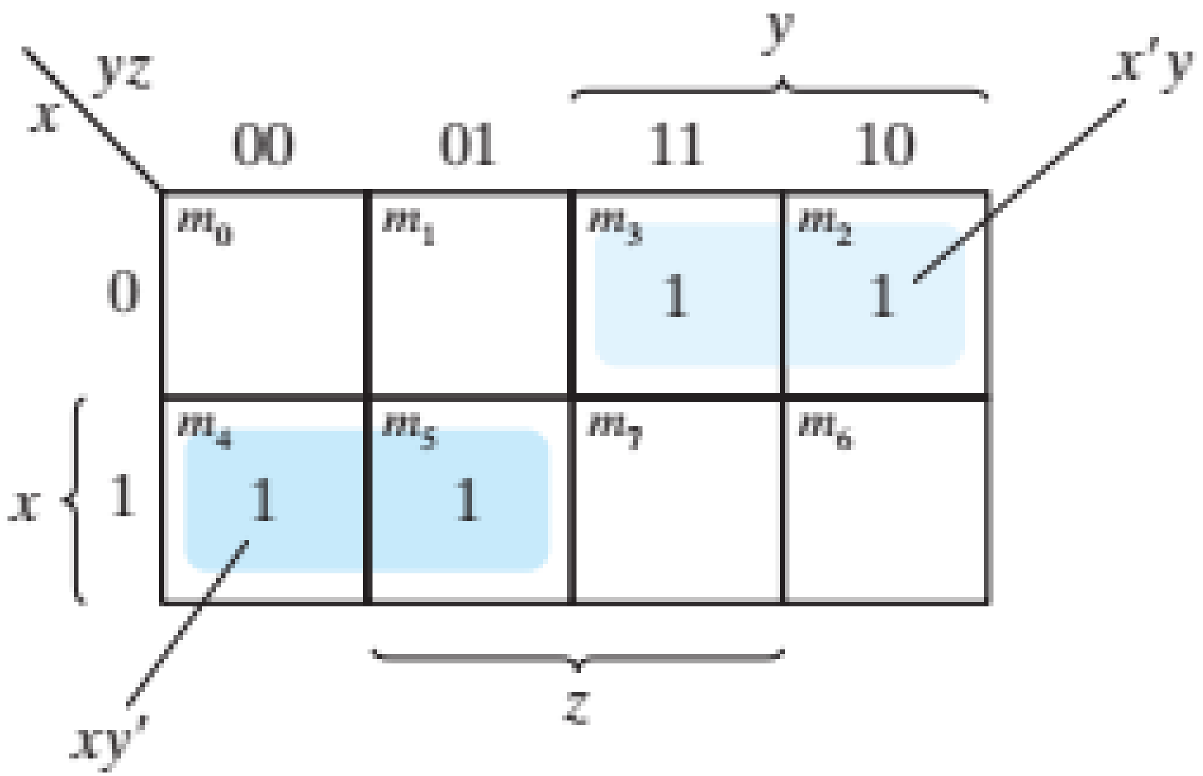


Three-Variable K-Map

Simplify the Boolean function

$$F(x, y, z) = \Sigma(2, 3, 4, 5)$$

x	y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



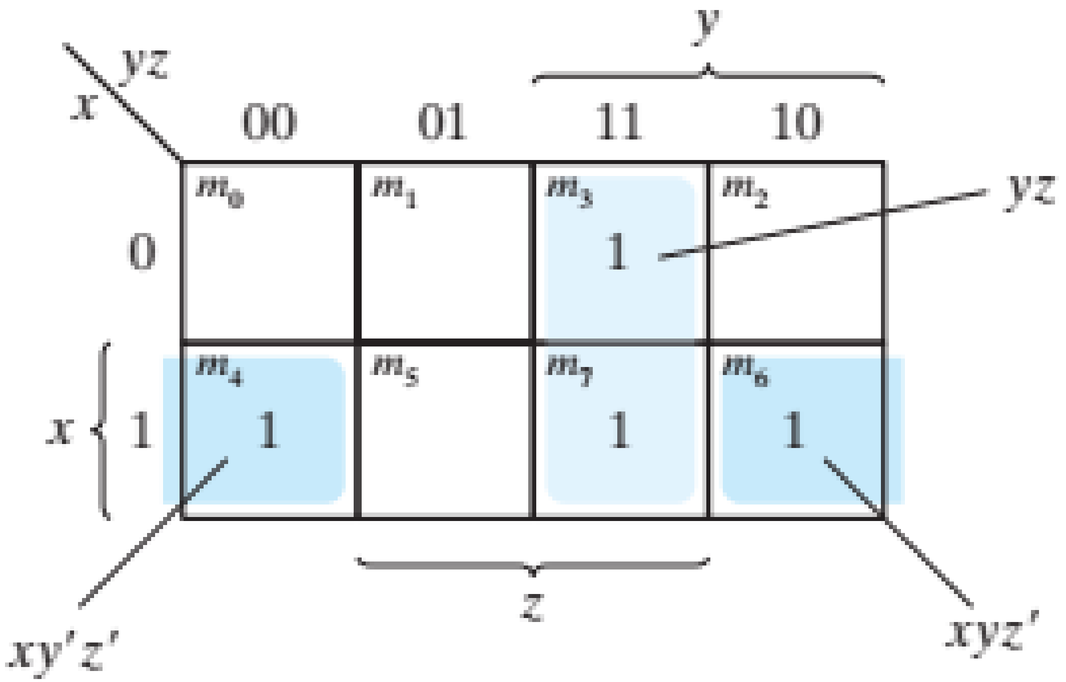
$$F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$$

Three-Variable K-Map

Simplify the Boolean function

$$F(x, y, z) = \Sigma(3, 4, 6, 7)$$

x	y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



Note: $xy'z' + xyz' = xz'$

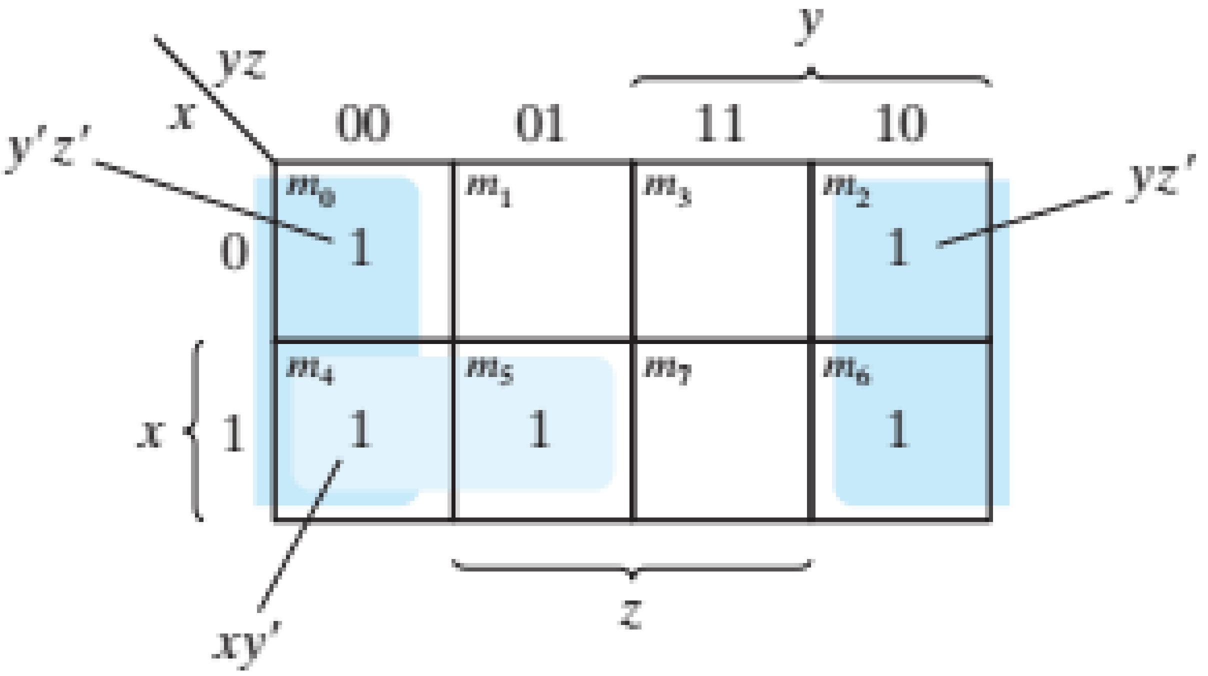
$$F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$$

Three-Variable K-Map

Simplify the Boolean function

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

x	y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



Note: $y'z' + yz' = z'$

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6) = z' + xy'$$

Three-Variable K-Map

For the Boolean function

$$F = A'C + A'B + AB'C + BC$$

(a) Express this function as a sum of minterms.

(b) Find the minimal sum-of-products expression.

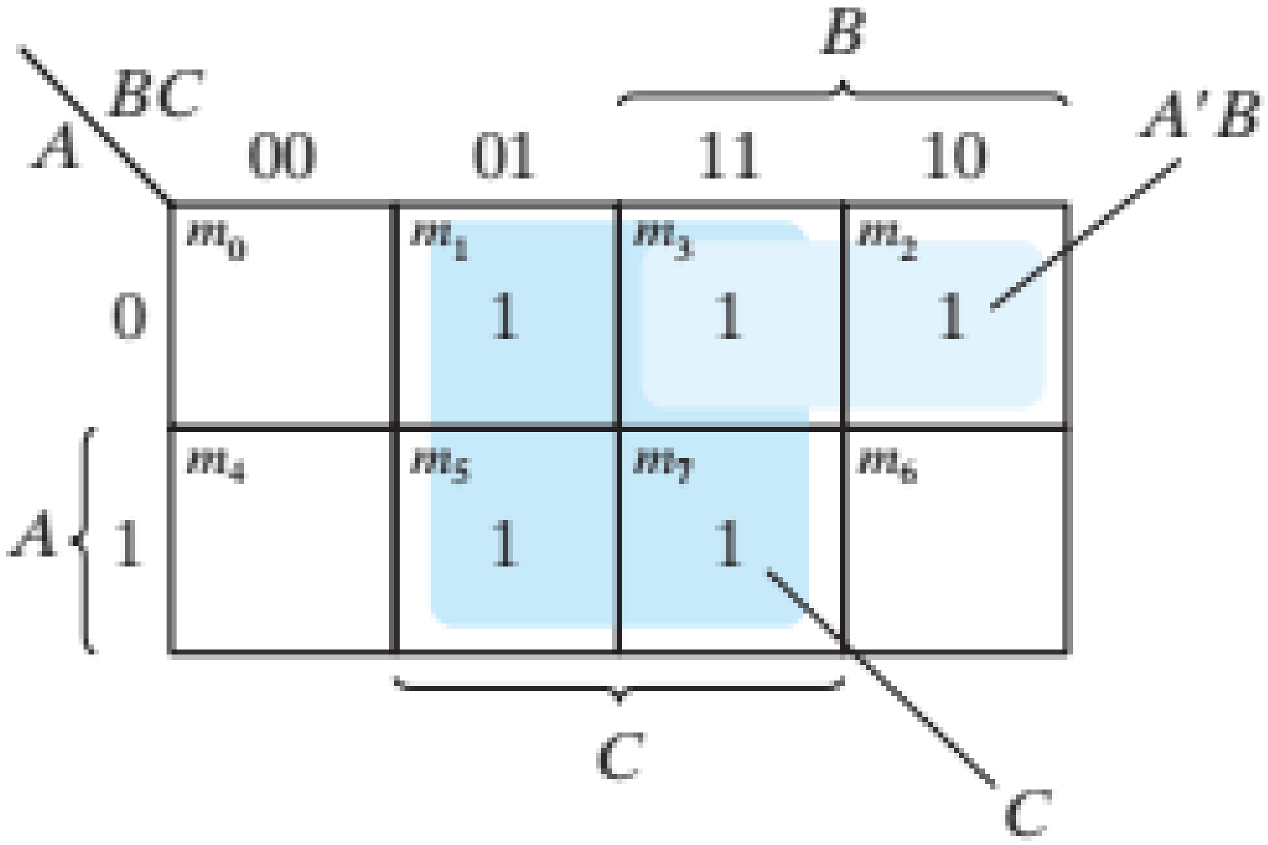
$$F = A'C(B + B') + A'B(C + C') + AB'C + BC(A + A')$$

$$= (A'BC + A'B'C) + (A'BC + A'BC') + AB'C + (ABC + A'BC)$$

$$= \quad m_3 + \quad m_1 + \quad m_2 + \quad m_5 + \quad m_7$$

Three-Variable K-Map

$$F(A, B, C) = \Sigma(1, 2, 3, 5, 7)$$



$$A'C + A'B + AB'C + BC = C + A'B$$

FOUR-VARIABLE K-MAP

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

wx \ yz	00	01	11	10
00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
10	m_8 $wx'y'z'$	m_9 $wx'y'z$	m_{11} $wx'yz$	m_{10} $wx'yz'$

y x w z

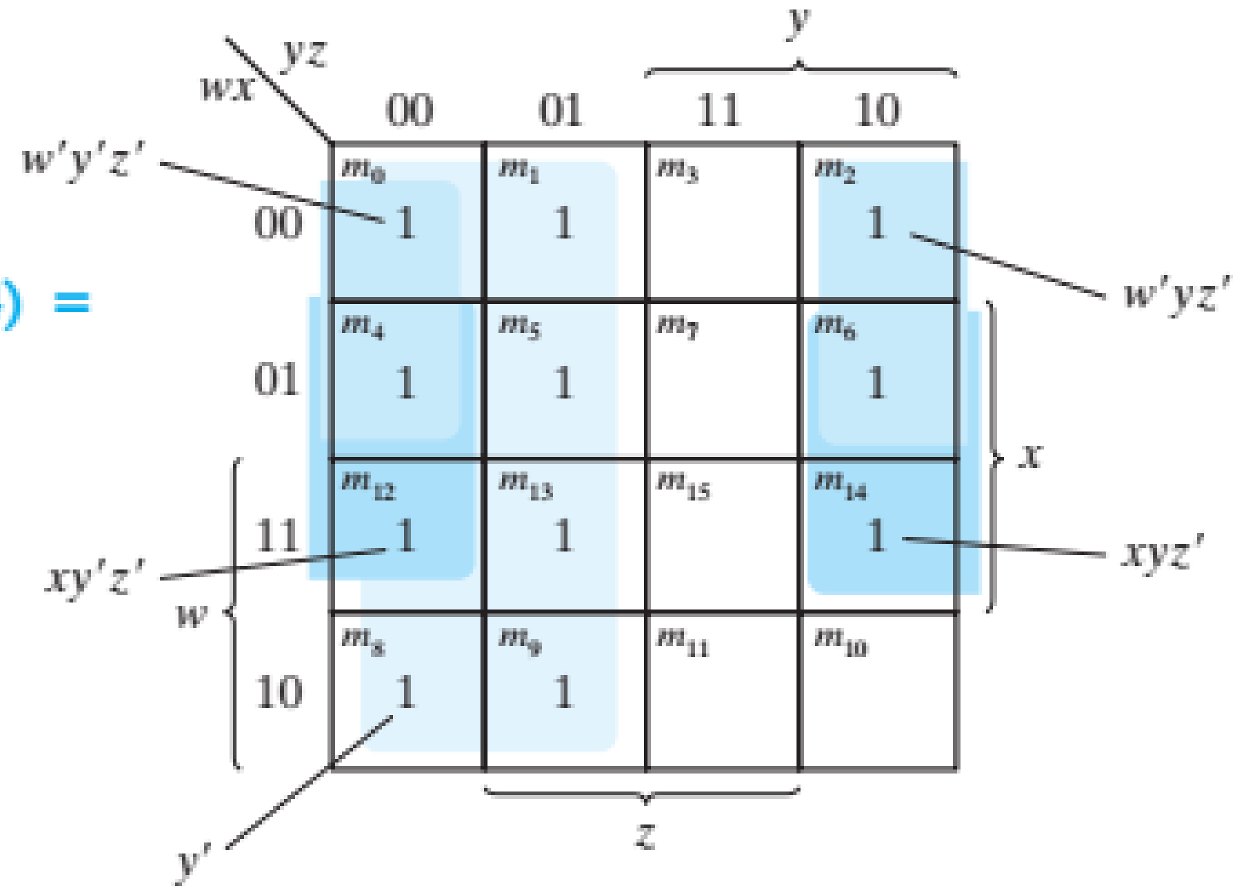
FOUR-VARIABLE K-MAP

Simplify the Boolean function

$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

$F(w, x, y, z) = \Sigma(0,1, 2, 4, 5, 6, 8, 9, 12, 13, 14) =$

$y' + w'z' + xz'$



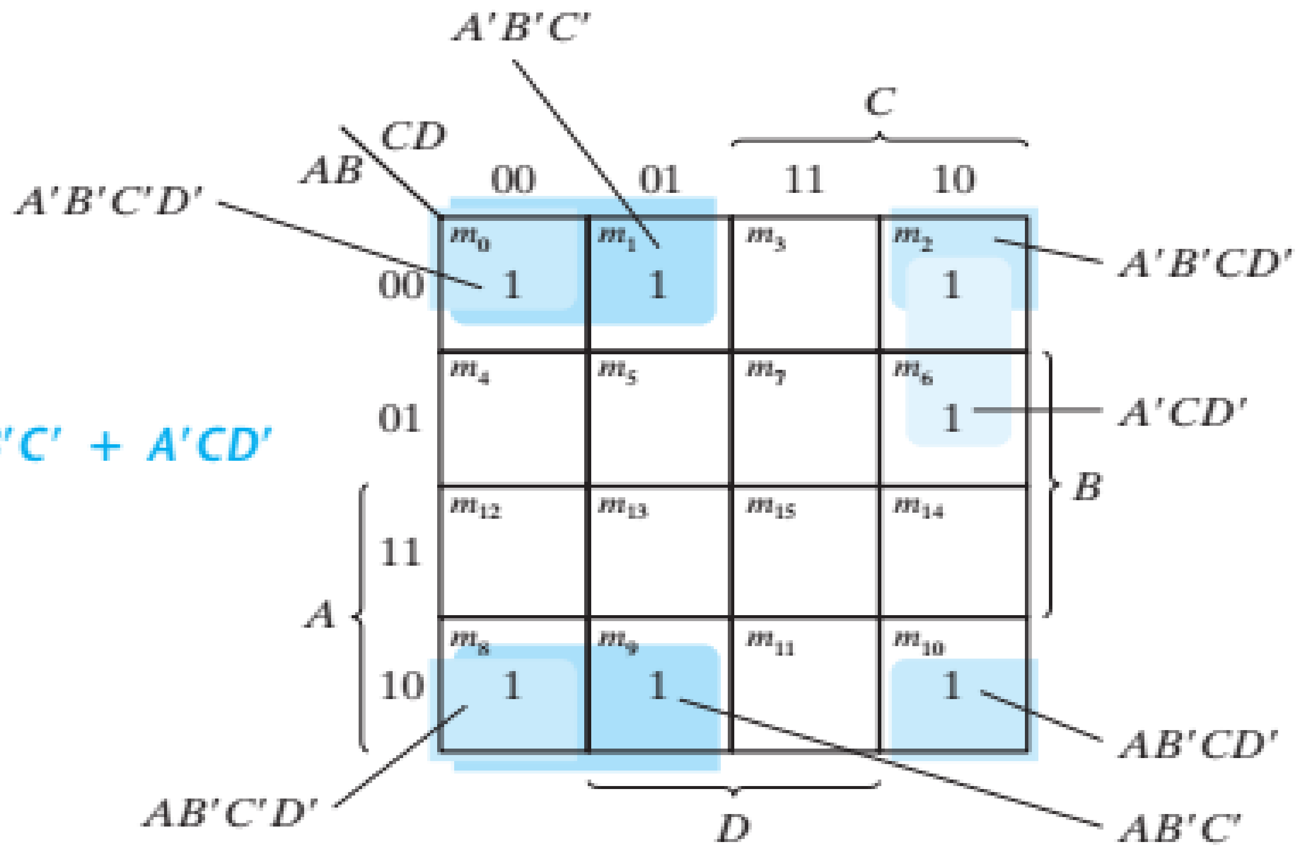
Note: $w'y'z' + w'yz' = w'z'$
$xy'z' + xyz' = xz'$

FOUR-VARIABLE K-MAP

Simplify the Boolean function

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

$A'B'C' + B'CD' + A'BCD' + AB'C' = B'D' + B'C' + A'CD'$



Note: $A'B'C'D' + A'B'CD' = A'B'D'$

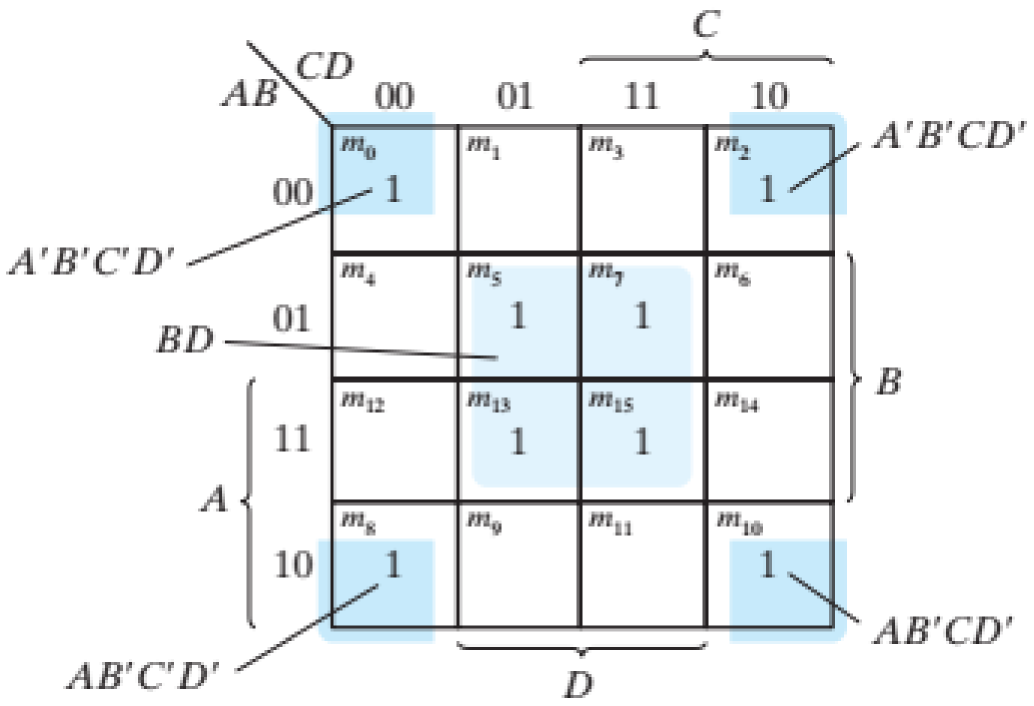
$AB'C'D' + AB'CD' = AB'D'$

$A'B'D' + AB'D' = B'D'$

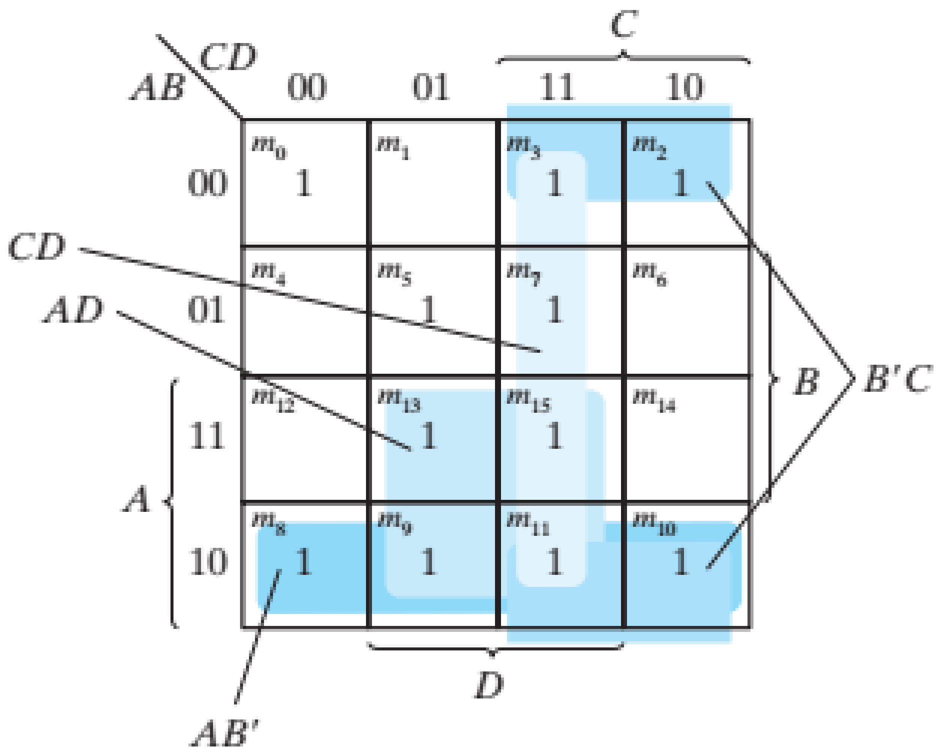
$A'B'C' + AB'C' = B'C'$

FOUR-VARIABLE K-MAP

$$F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$



Note: $A'B'C'D' + A'B'CD' = A'B'D'$
$AB'C'D' + AB'CD' = AB'D'$
$A'B'D' + AB'D' = B'D'$



$$\begin{aligned} F &= BD + B'D' + CD + AD \\ &= BD + B'D' + CD + AB' \\ &= BD + B'D' + B'C + AD \\ &= BD + B'D' + B'C + AB' \end{aligned}$$

FOUR-VARIABLE K-MAP

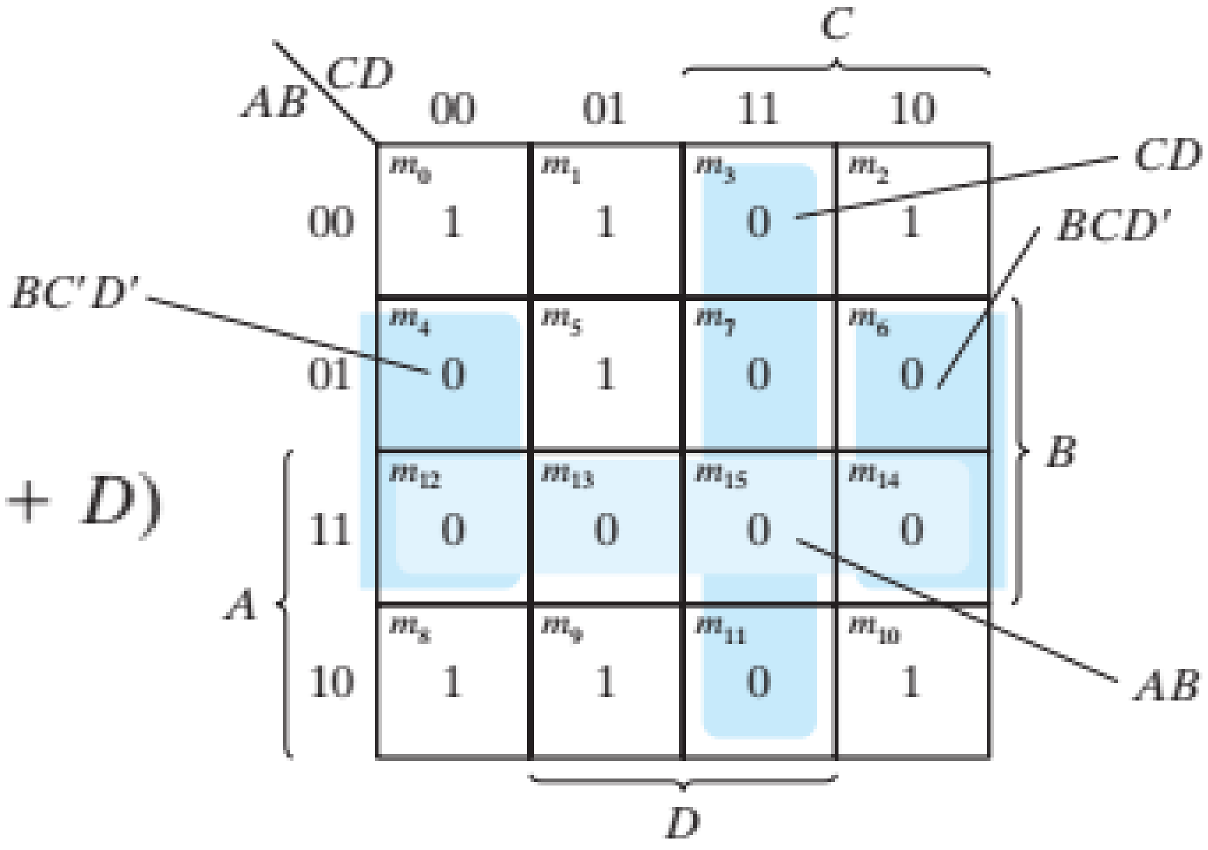
Simplify the following Boolean function into (a) sum-of-products form and (b) product-of-sums form:

$$F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10)$$

(a) $F = B'D' + B'C' + A'C'D$

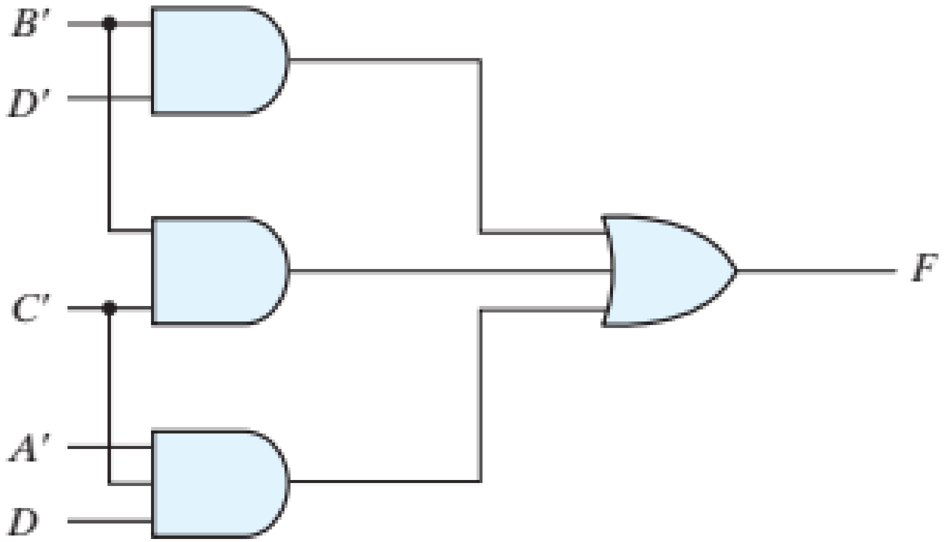
$F' = AB + CD + BD'$

(b) $F = (A' + B')(C' + D')(B' + D)$

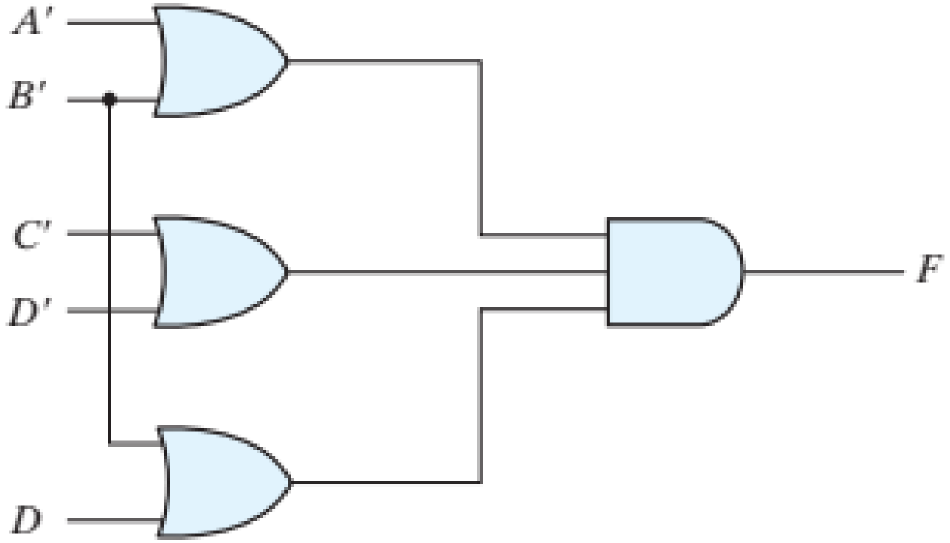


Note: $BC'D' + BCD' = BD'$

FOUR-VARIABLE K-MAP



(a) $F = B'D' + B'C' + A'C'D$



(b) $F = (A' + B')(C' + D')(B' + D)$

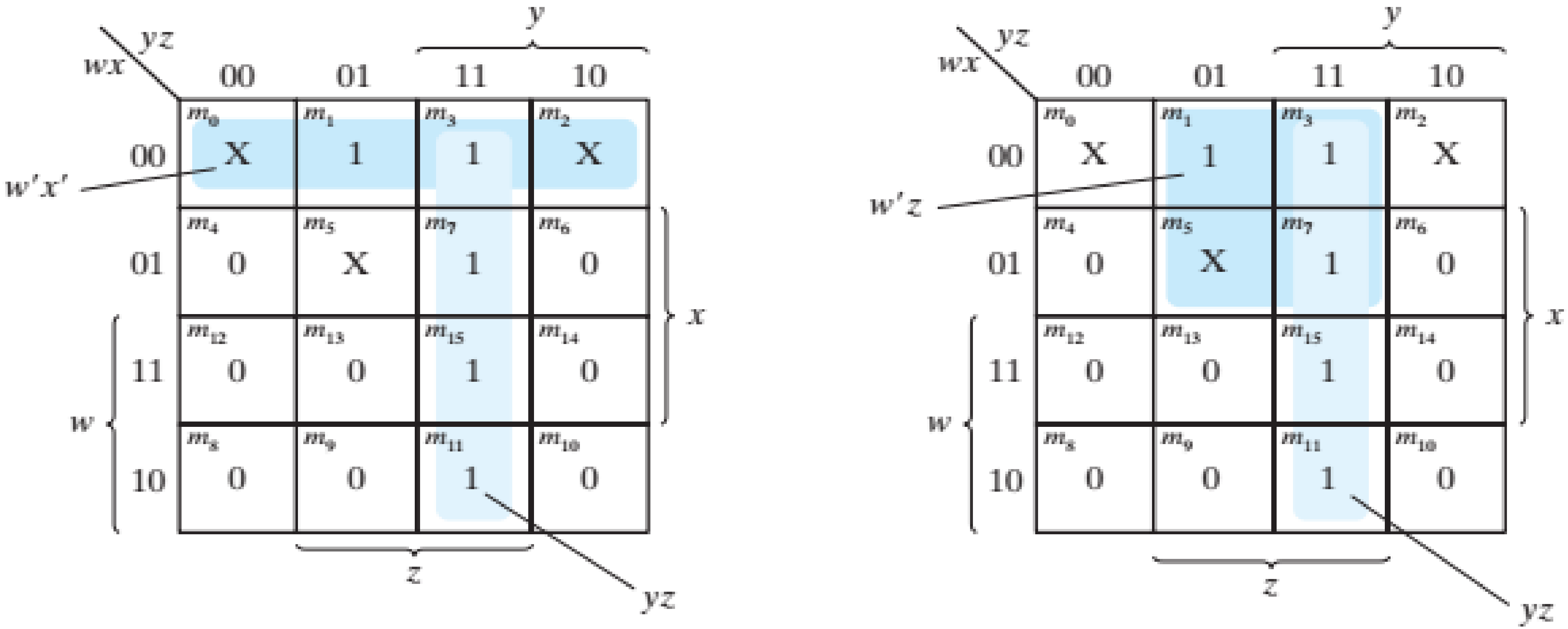
DON'T-CARE CONDITIONS

Simplify the Boolean function

$$F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$$

which has the don't-care conditions

$$d(w, x, y, z) = \Sigma(0, 2, 5)$$



(a) $F = yz + w'x'$

(b) $F = yz + w'z$

XOR FUNCTIONS

	00	01	11	10
0		1		1
1	1		1	

$$X=(B'C+BC')$$

$$X'=(B'C+BC')'=(B'C)'\bullet(BC')'$$

$$X'=(B+C')(B'+C)$$

$$X'=BB'+BC+B'C'+CC'$$

$$X'=BC+B'C'$$

$$F = m_1 + m_2 + m_4 + m_7$$

$$= A'B'C + A'BC' + AB'C' + ABC$$

$$= A'(B'C+BC') + A(B'C'+BC)$$

$$= A'X + AX' = A \oplus X = A \oplus (B \oplus C)$$

XOR FUNCTIONS

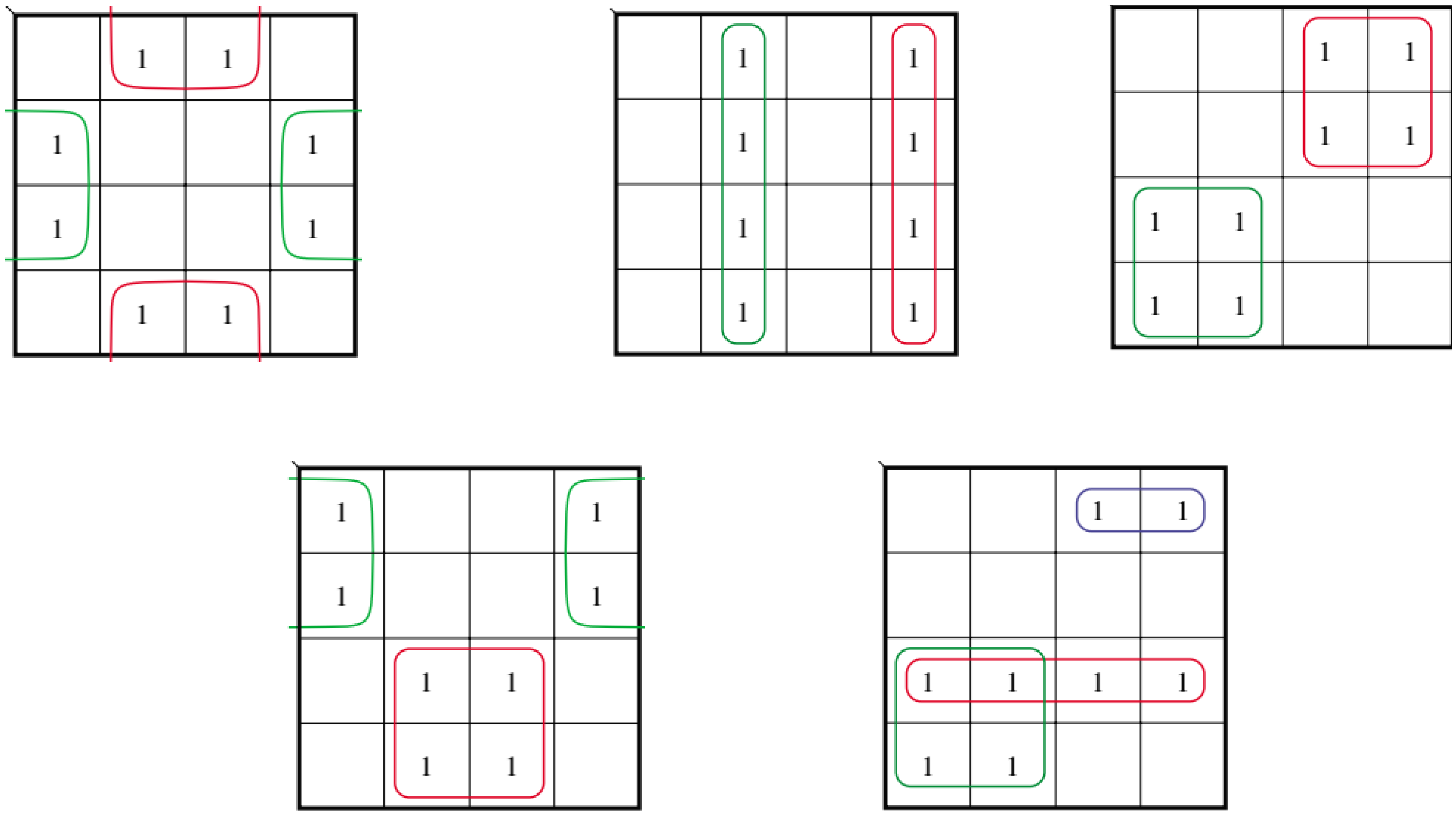
	00	01	11	10
0		1		1
1	1		1	

$$F_a = A \oplus B \oplus C$$

	00	01	11	10
0	1		1	
1		1		1

$$F_b = \overline{A \oplus B \oplus C}$$

XOR FUNCTIONS



XOR FUNCTIONS

			1
		1	
			1
		1	

	1		1
1		1	
	1		1
1		1	

	1		1
	1		1
1		1	
1		1	

		1	
	1		
1			
			1

		1	
			1
1			
	1		