

# Digital

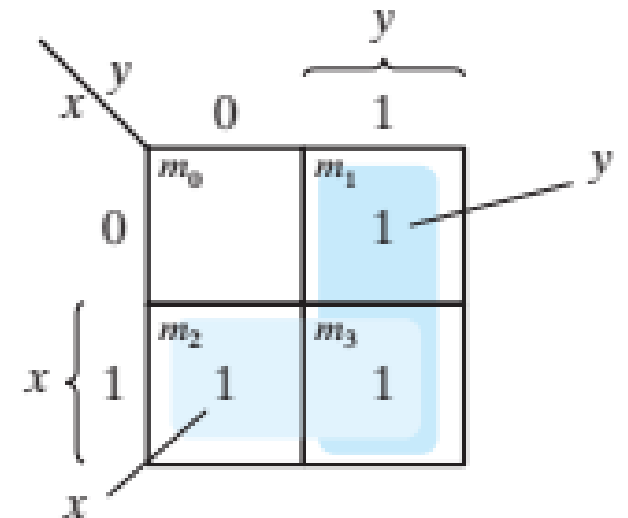
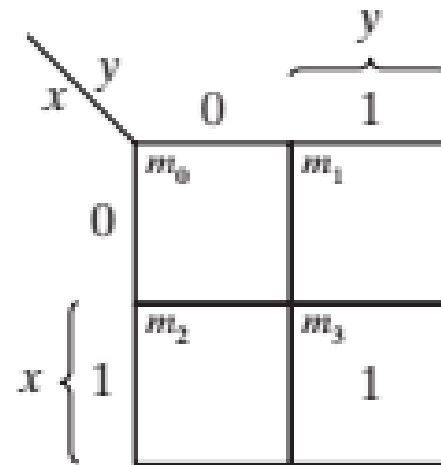
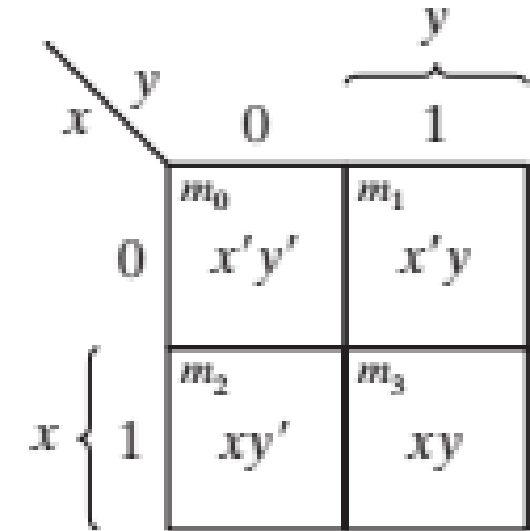
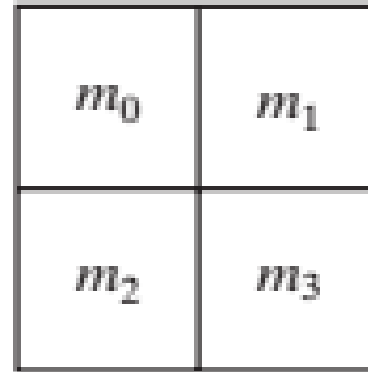
## **Gate-Level Minimization**

# THE MAP METHOD

## Two-Variable K-Map

$$m_1 + m_2 + m_3 = x'y + xy' + xy = x + y$$

$x$	$y$
0	0
0	1
1	0
1	1



# THE MAP METHOD

## Three-Variable K-Map

$m_0$	$m_1$	$m_3$	$m_2$
$m_4$	$m_5$	$m_7$	$m_6$

		$y$			
		00	01	11	10
$x$	0	$m_0$ $x'y'z'$	$m_1$ $x'y'z$	$m_3$ $x'yz$	$m_2$ $x'yz'$
	1	$m_4$ $xy'z'$	$m_5$ $xy'z$	$m_7$ $xyz$	$m_6$ $xyz'$
		$z$			

	$\bar{B}\bar{C}$	$\bar{B}C$	$BC$	$B\bar{C}$
$\bar{A}$	0	1	3	2
$A$	4	5	7	6

# THE MAP METHOD

## Three-Variable K-Map

		AB		A	
		00	01	11	10
C	0	0	2	6	4
	1	1	3	7	5
		B			

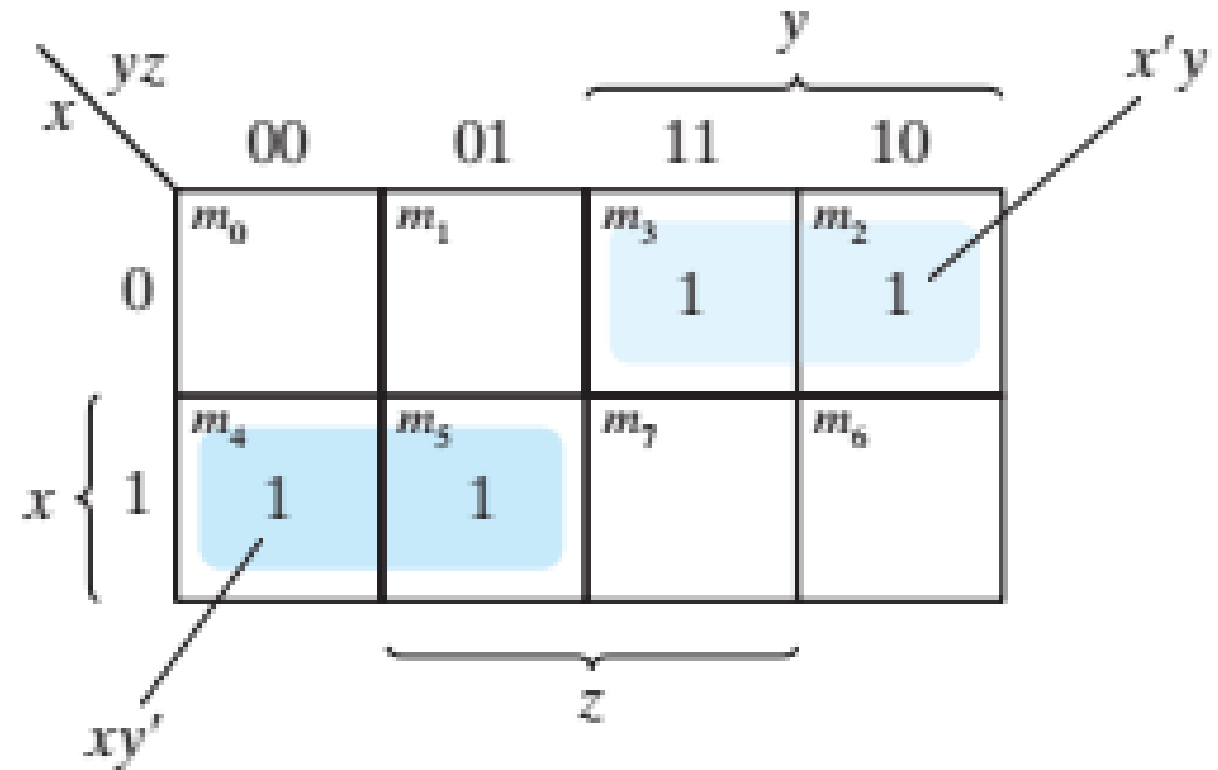
		BC		B	
		00	01	11	10
A	0	0	1	3	2
	1	4	5	7	6
		C			

# Three-Variable K-Map

Simplify the Boolean function

$$F(x, y, z) = \Sigma(2, 3, 4, 5)$$

$x$	$y$	$z$
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



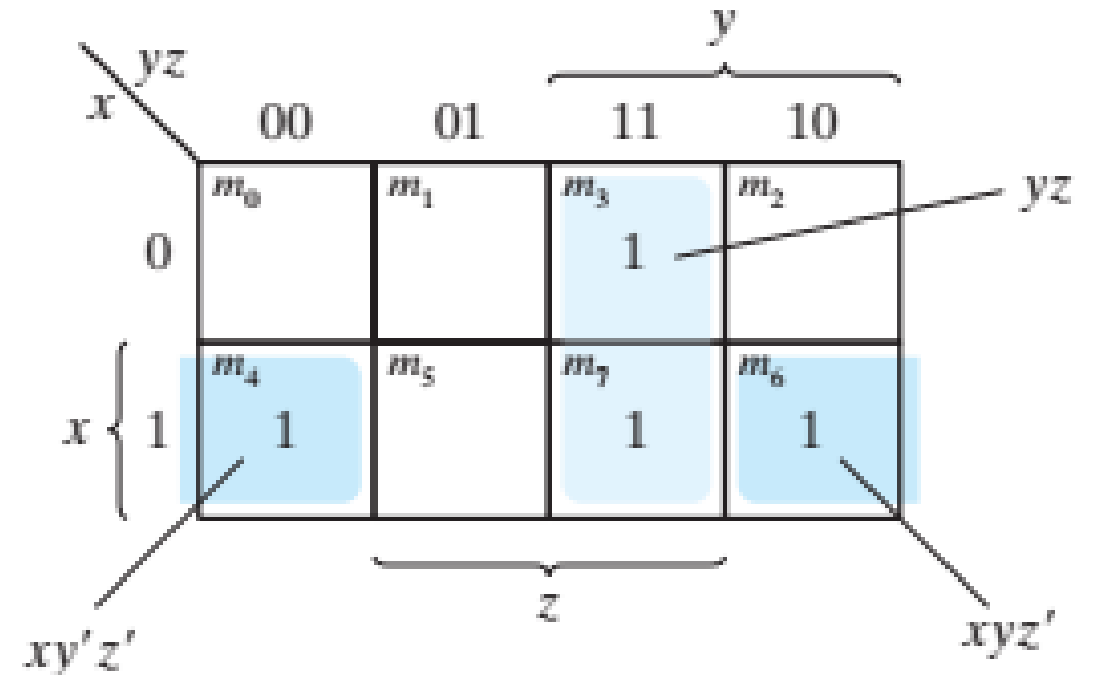
$$F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$$

# Three-Variable K-Map

Simplify the Boolean function

$$F(x, y, z) = \Sigma(3, 4, 6, 7)$$

$x$	$y$	$z$
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



Note:  $xy'z' + xyz' = xz'$

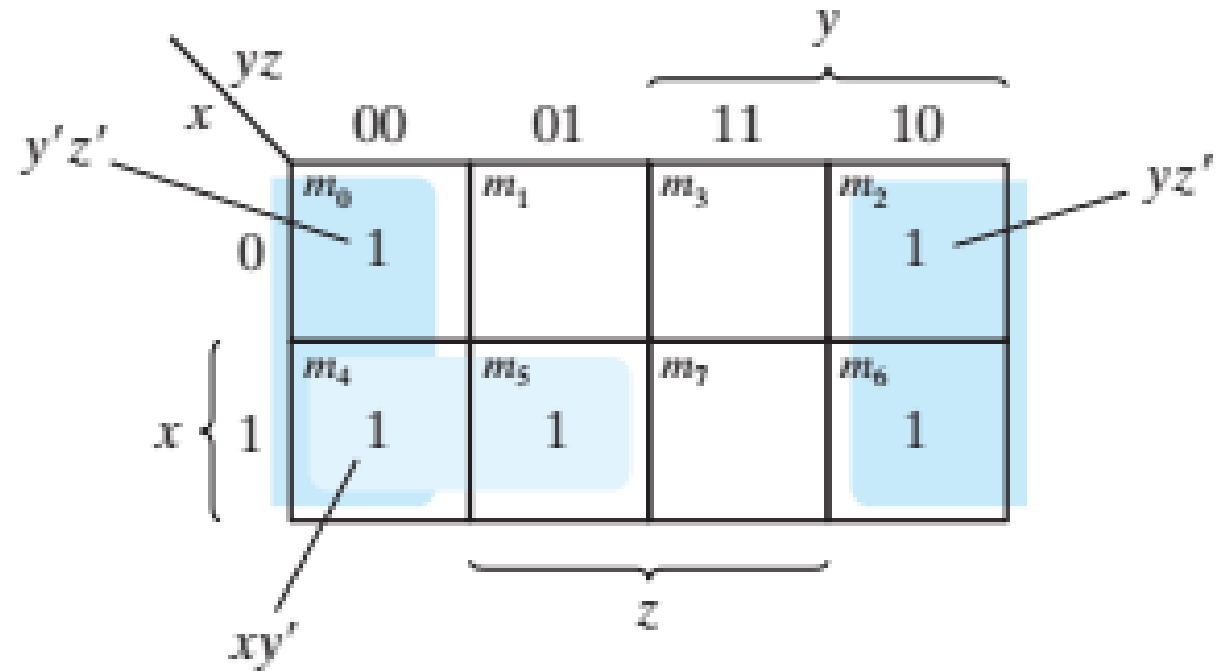
$$F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$$

# Three-Variable K-Map

Simplify the Boolean function

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

x	y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



Note:  $y'z' + yz' = z'$

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6) = z' + xy'$$

# Three-Variable K-Map

For the Boolean function

$$F = A'C + A'B + AB'C + BC$$

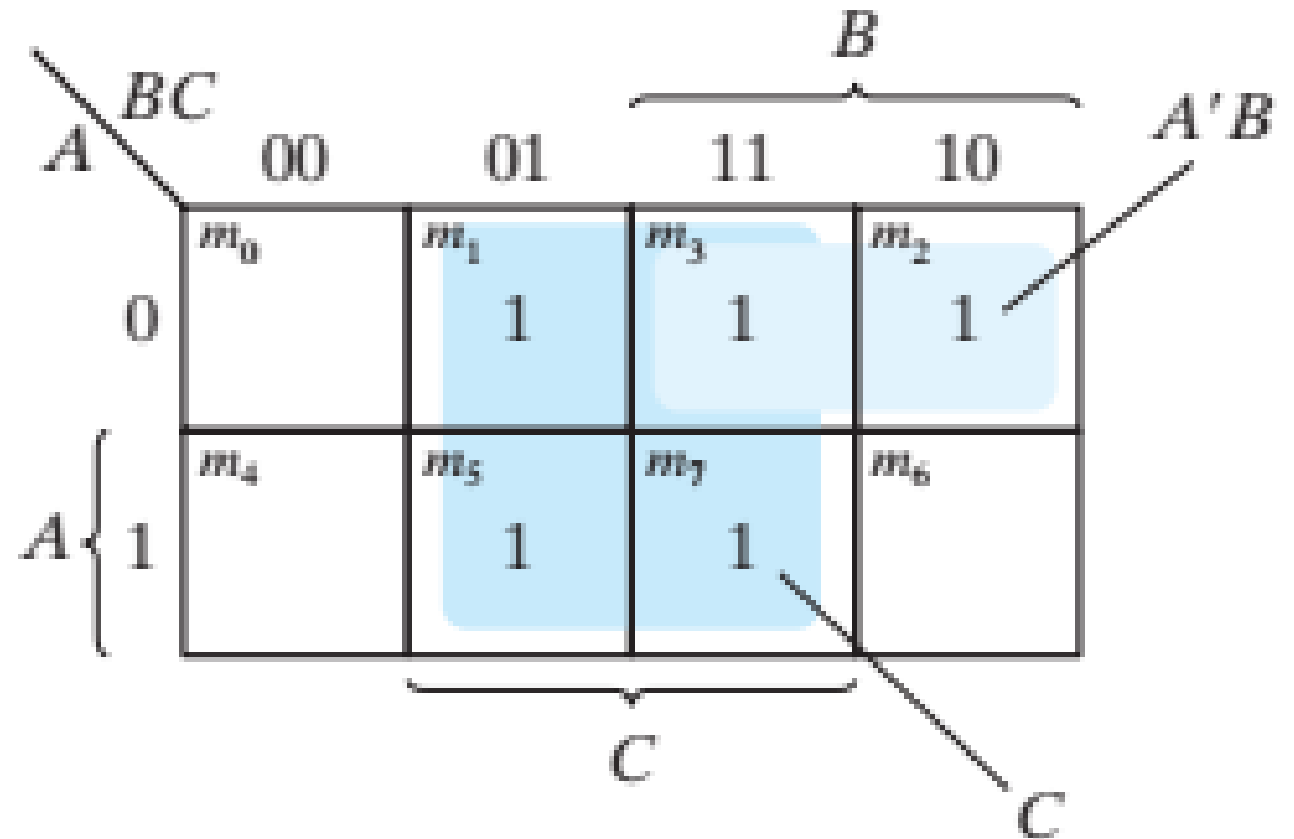
- (a) Express this function as a sum of minterms.
- (b) Find the minimal sum-of-products expression.

$$\begin{aligned} F &= A'C(B + B') + A'B(C + C') + AB'C + BC(A + A') \\ &= (A'BC + A'B'C) + (A'BC + A'BC') + AB'C + (ABC + A'BC) \\ &= m_3 + m_1 + m_2 + m_5 + m_7 \end{aligned}$$



# Three-Variable K-Map

$$F(A, B, C) = \Sigma(1, 2, 3, 5, 7)$$



$$A'C + A'B + AB'C + BC = C + A'B$$

# FOUR-VARIABLE K-MAP

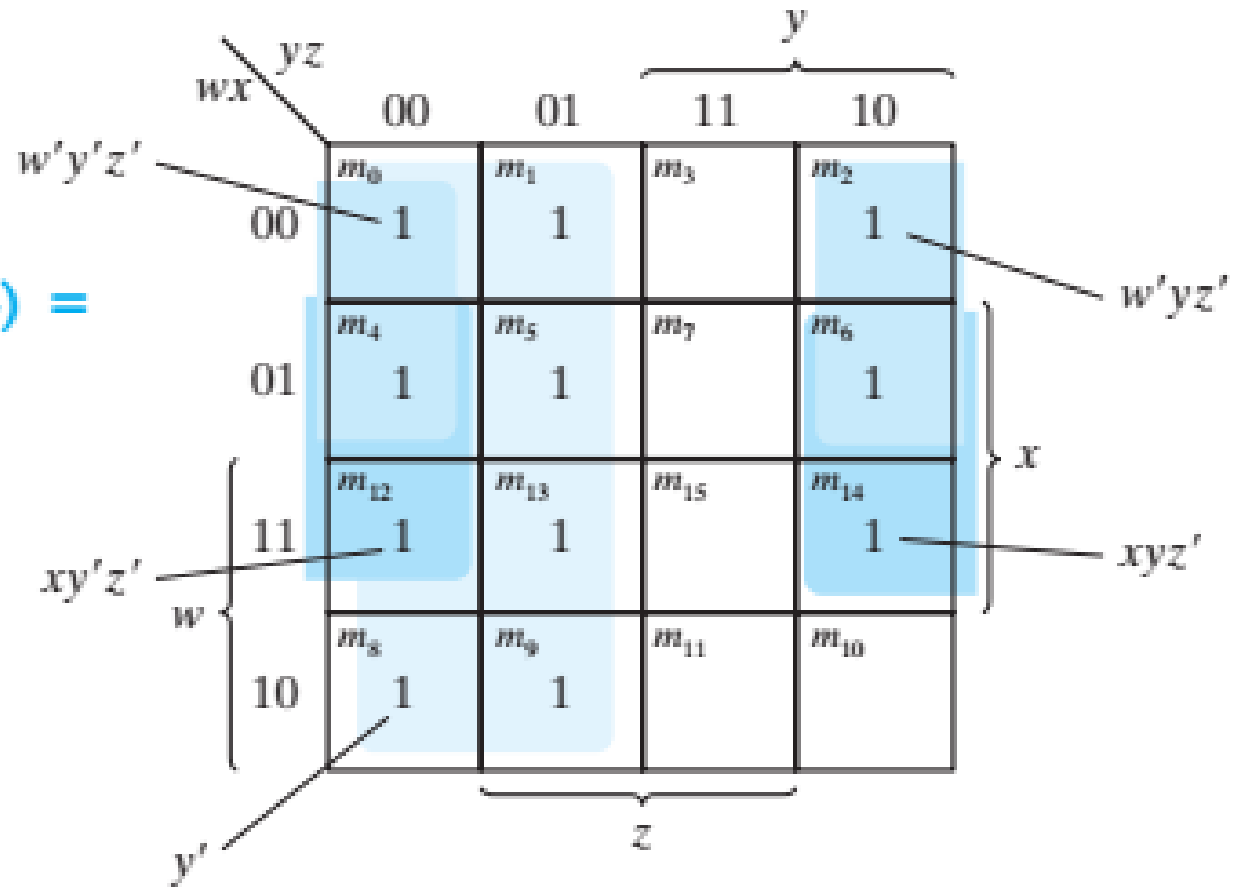
$m_0$	$m_1$	$m_3$	$m_2$
$m_4$	$m_5$	$m_7$	$m_6$
$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$
$m_8$	$m_9$	$m_{11}$	$m_{10}$

		$y$				
		$yz$	$00$	$01$	$11$	$10$
$w$	$wx$	$00$	$m_0$ $w'x'y'z'$	$m_1$ $w'x'y'z$	$m_3$ $w'x'yz$	$m_2$ $w'x'yz'$
	$01$	$m_4$ $w'xy'z'$	$m_5$ $w'xy'z$	$m_7$ $w'xyz$	$m_6$ $w'xyz'$	
	$11$	$m_{12}$ $wxy'z'$	$m_{13}$ $wxy'z$	$m_{15}$ $wxyz$	$m_{14}$ $wxyz'$	
	$10$	$m_8$ $wx'y'z'$	$m_9$ $wx'y'z$	$m_{11}$ $wx'yz$	$m_{10}$ $wx'yz'$	

# FOUR-VARIABLE K-MAP

Simplify the Boolean function

$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$



$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14) =$$

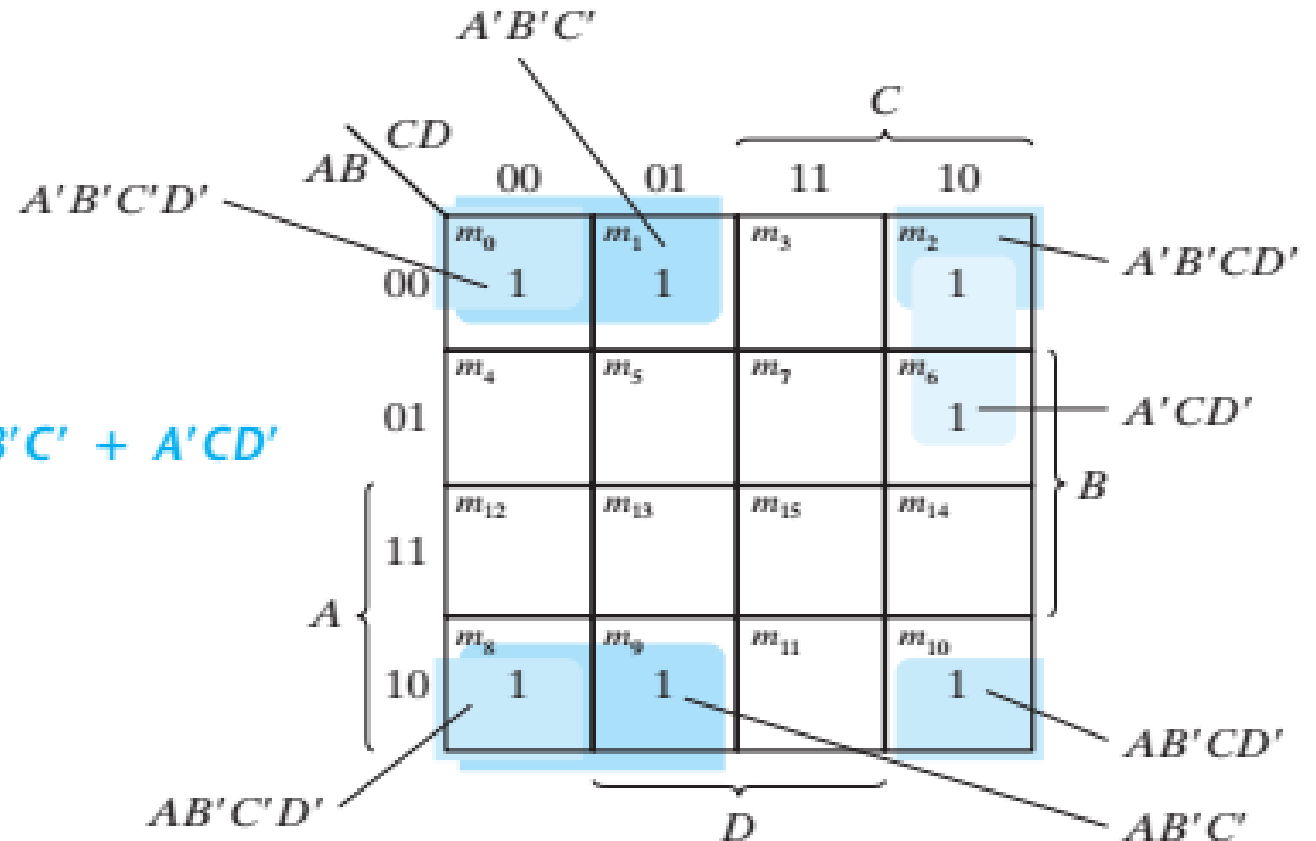
$$y' + w'z' + xz'$$

Note:  $w'y'z' + w'yz' = w'z'$   
 $xy'z' + xyz' = xz'$

# FOUR-VARIABLE K-MAP

Simplify the Boolean function

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

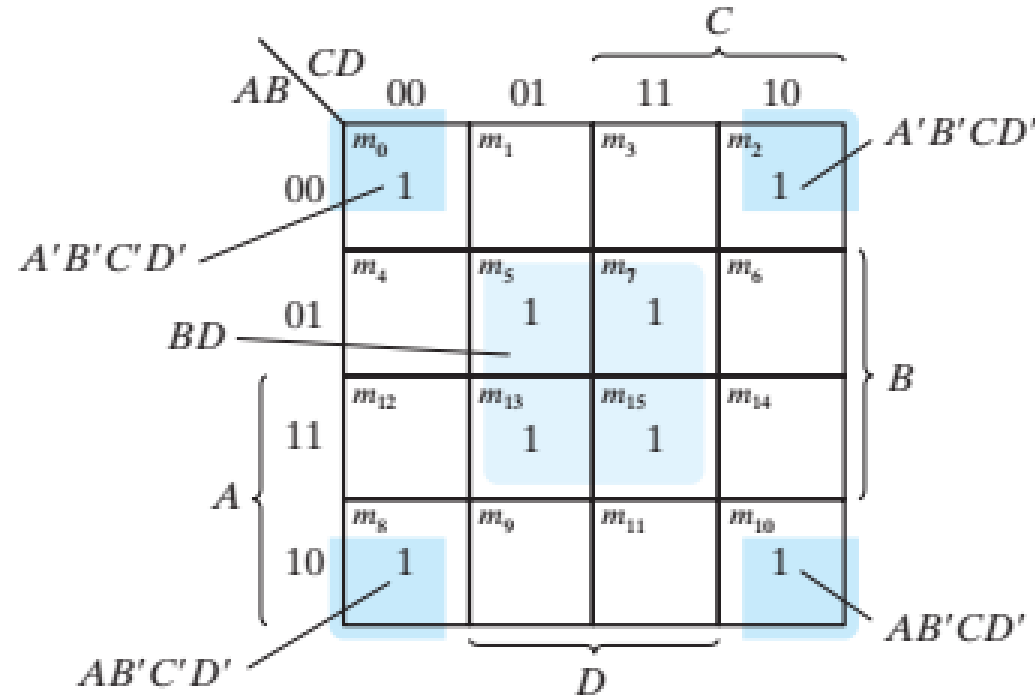


$$A'B'C' + B'CD' + A'BCD' + AB'C' = B'D' + B'C' + A'CD'$$

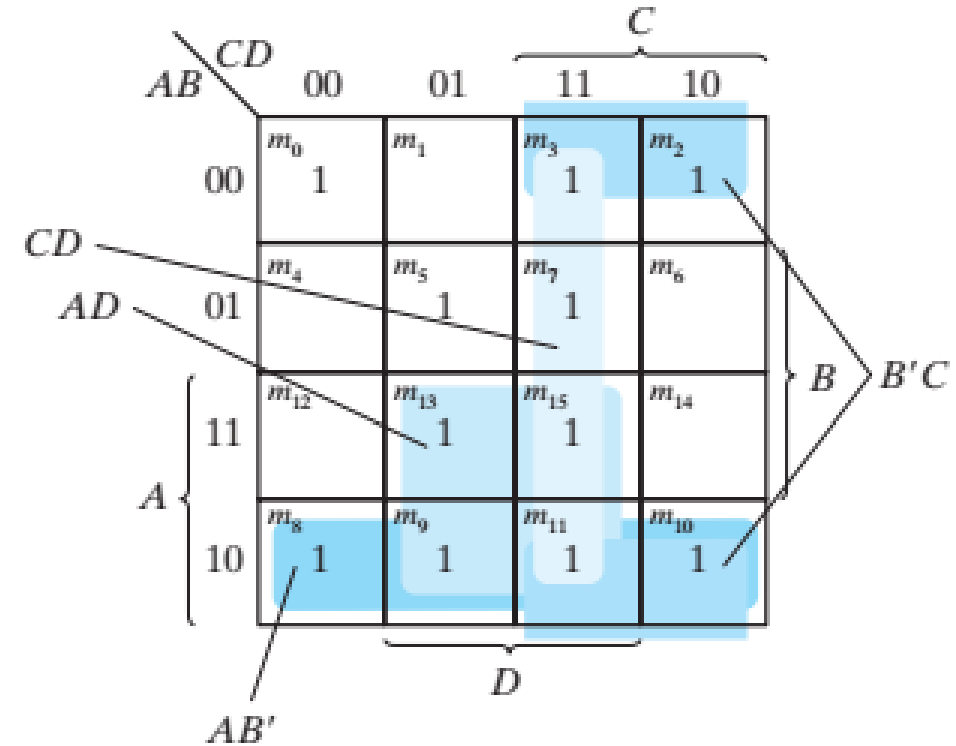
Note:  $A'B'C'D' + A'B'CD' = A'B'D'$   
 $AB'C'D' + AB'CD' = AB'D'$   
 $A'B'D' + AB'D' = B'D'$   
 $A'B'C' + AB'C' = B'C'$

# FOUR-VARIABLE K-MAP

$$F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$



Note:  $A'B'C'D' + A'B'CD' = A'B'D'$   
 $AB'C'D' + AB'CD' = AB'D'$   
 $A'B'D' + AB'D' = B'D'$



$$\begin{aligned} F &= BD + B'D' + CD + AD \\ &= BD + B'D' + CD + AB' \\ &= BD + B'D' + B'C + AD \\ &= BD + B'D' + B'C + AB' \end{aligned}$$

# FOUR-VARIABLE K-MAP

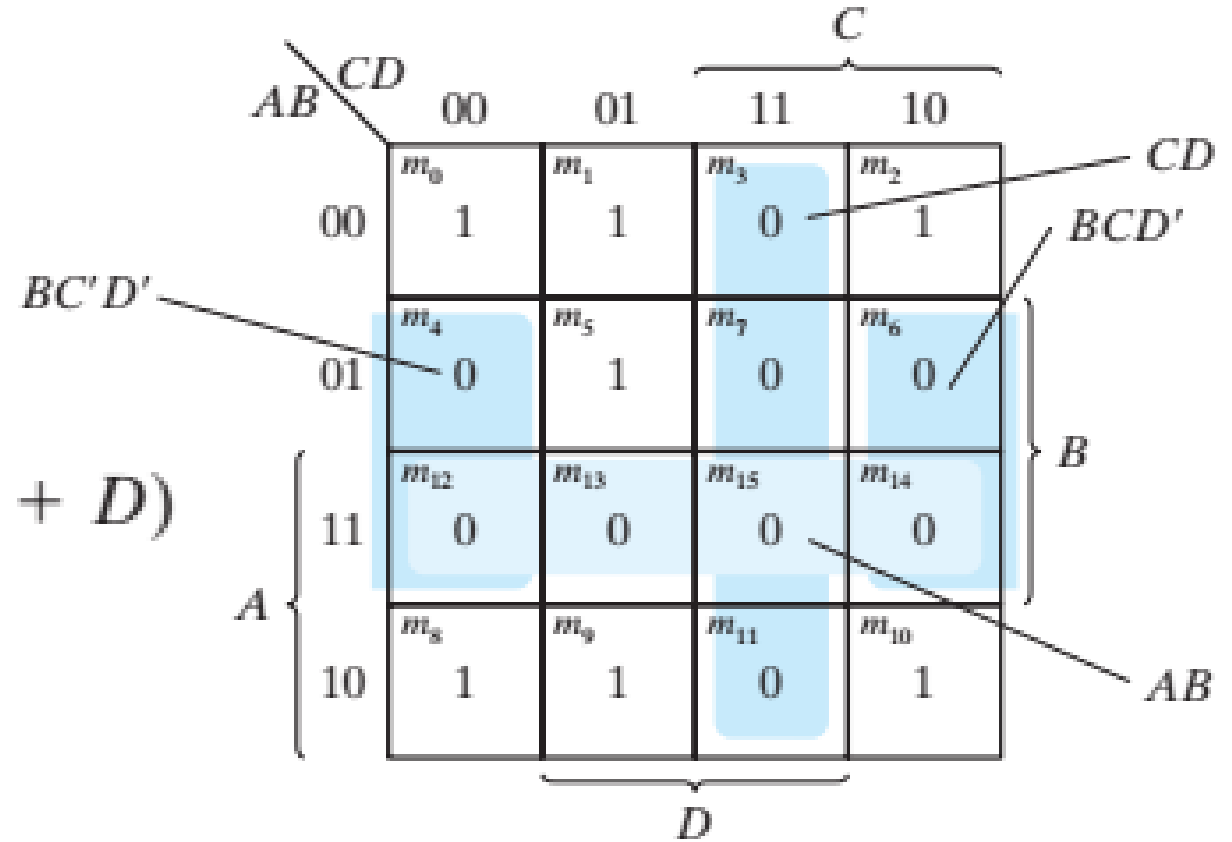
Simplify the following Boolean function into (a) sum-of-products form and (b) product-of-sums form:

$$F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10)$$

(a)  $F = B'D' + B'C' + A'C'D$

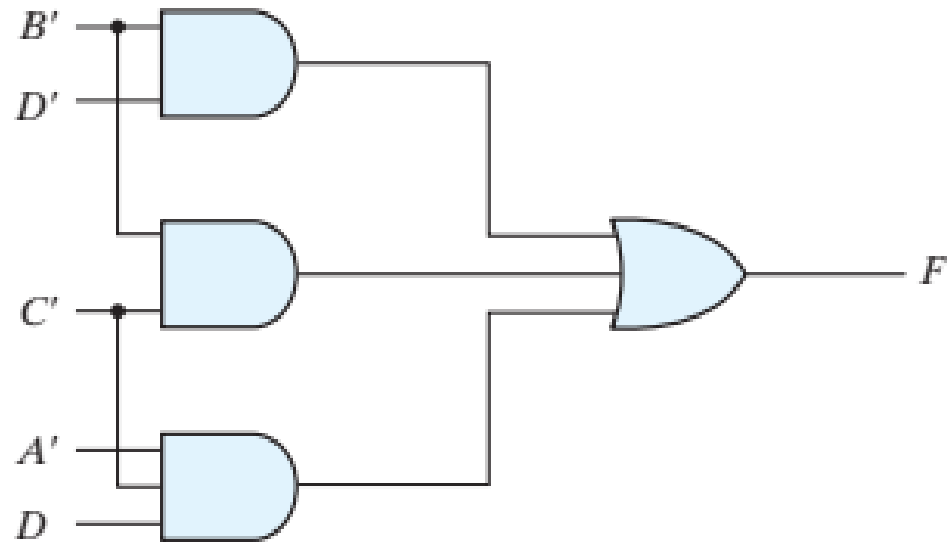
$$F' = AB + CD + BD'$$

(b)  $F = (A' + B')(C' + D')(B' + D)$

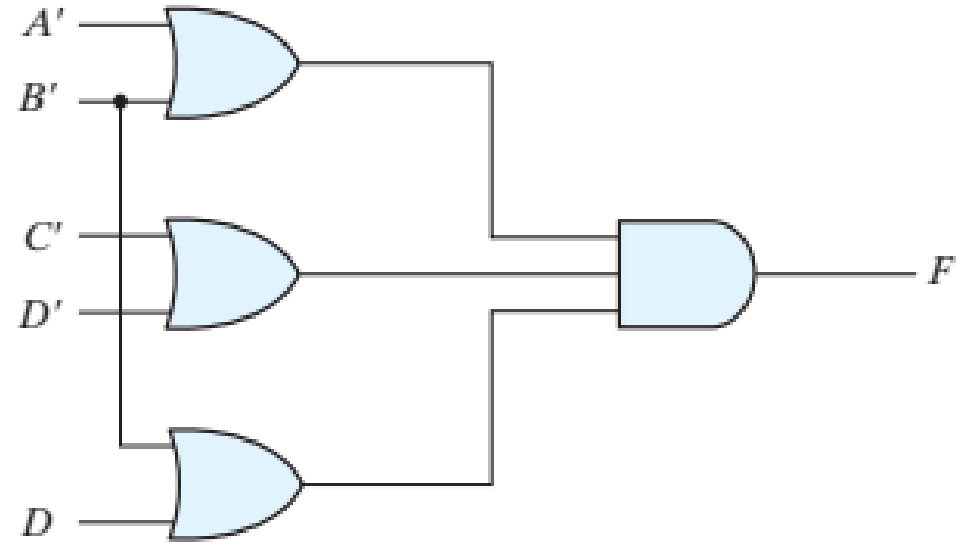


Note:  $BC'D' + BCD' = BD'$

# FOUR-VARIABLE K-MAP



(a)  $F = B'D' + B'C' + A'C'D$



(b)  $F = (A' + B')(C' + D')(B' + D)$

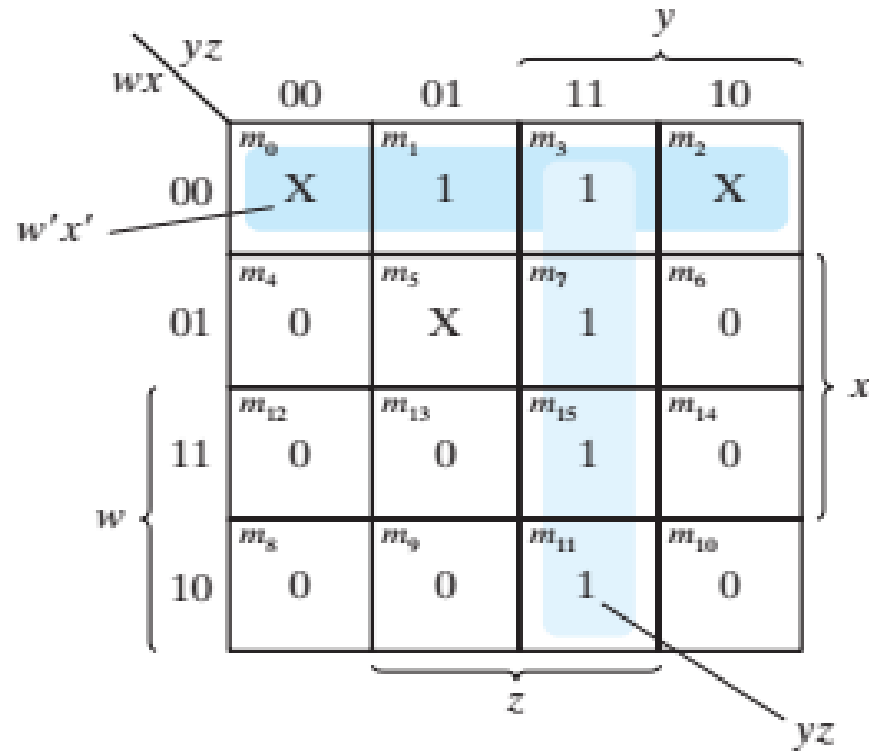
# DON'T-CARE CONDITIONS

Simplify the Boolean function

$$F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$$

which has the don't-care conditions

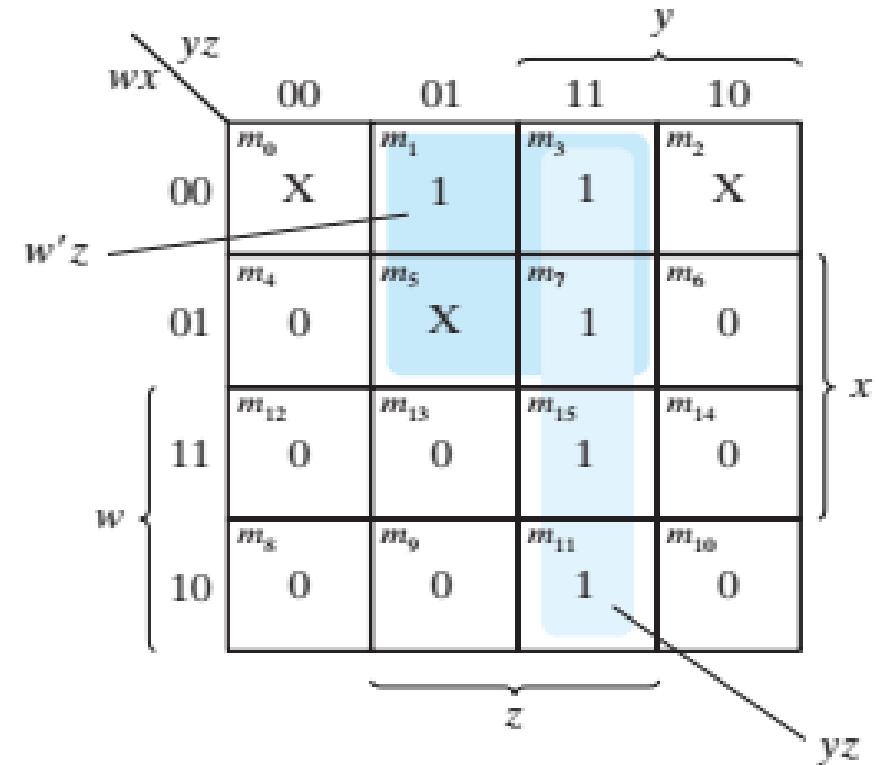
$$d(w, x, y, z) = \Sigma(0, 2, 5)$$



A 4x4 Karnaugh map for the function F(w, x, y, z) = yz + w'x'. The vertical axis is labeled 'w' with values 00, 01, 11, 10. The horizontal axis is labeled 'yz' with values 00, 01, 11, 10. The top-left corner is labeled 'wx' and the top-right corner is labeled 'yz'. The map is divided into four quadrants by 'w' and 'yz'. The cells are labeled m0 through m15. The function value is 1 for m1, m3, m7, m11, and m15. Don't-care conditions (X) are present in m0, m2, and m5. The expression F = yz + w'x' is indicated by a bracket under the 'yz' axis and a bracket to the left of the 'w' axis.

w \ yz	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

(a)  $F = yz + w'x'$



A 4x4 Karnaugh map for the function F(w, x, y, z) = yz + w'z'. The vertical axis is labeled 'w' with values 00, 01, 11, 10. The horizontal axis is labeled 'yz' with values 00, 01, 11, 10. The top-left corner is labeled 'wx' and the top-right corner is labeled 'yz'. The map is divided into four quadrants by 'w' and 'yz'. The cells are labeled m0 through m15. The function value is 1 for m1, m3, m7, m11, and m15. Don't-care conditions (X) are present in m0, m2, and m5. The expression F = yz + w'z' is indicated by a bracket under the 'yz' axis and a bracket to the left of the 'w' axis.

w \ yz	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

(b)  $F = yz + w'z'$



# XOR FUNCTIONS

	00	01	11	10
0		1		1
1	1		1	

$$X = (B'C + BC')$$

$$X' = (B'C + BC')' = (B'C)' \cdot (BC')'$$

$$X' = (B + C')(B' + C)$$

$$X' = BB' + BC + B'C' + CC'$$

$$X' = BC + B'C'$$

$$F = m_1 + m_2 + m_4 + m_7$$

$$= A'B'C + A'BC' + AB'C' + ABC$$

$$= A'(B'C + BC') + A(B'C' + BC)$$

$$= A'X + AX' = A \oplus X = A \oplus (B \oplus C)$$

# XOR FUNCTIONS

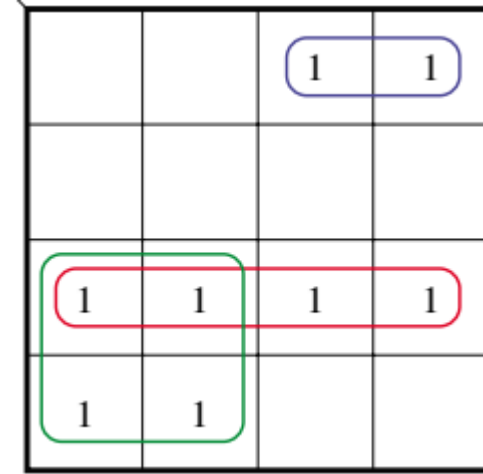
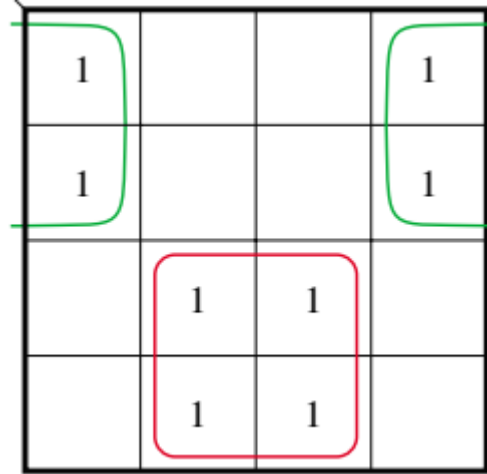
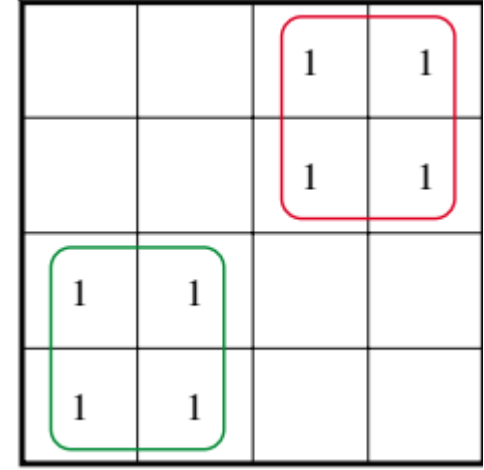
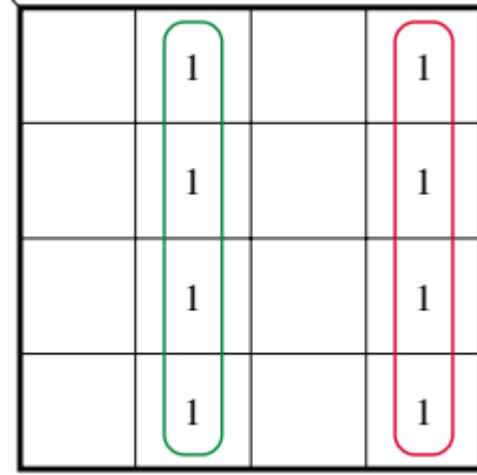
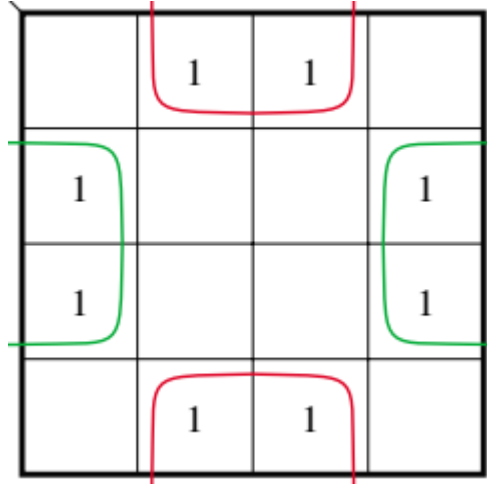
	00	01	11	10
0		1		1
1	1		1	

$$F_a = A \oplus B \oplus C$$

	00	01	11	10
0	1		1	
1		1		1

$$F_b = \overline{A \oplus B \oplus C}$$

# XOR FUNCTIONS



# XOR FUNCTIONS

			1
		1	
			1
		1	

	1		1
1		1	
	1		1
1		1	

	1		1
	1		1
1		1	
1		1	

		1	
	1		
1			
			1

		1	
			1
1			
	1		