

Advanced Algebra

Description: This course is specifically designed for students who have successfully completed Intermediate algebra. The advanced algebra is designed to take foundational algebraic concepts and build upon them by studying Categories Theorem, Module Theory and ring theory.

Prerequisites: Abstract Algebra

Course Objectives:

Part One. Category Theory

Chapter I. Category

- 1 Category and Examples of Categories
- 2 Constructions on Categories
- 3 Free Categories
- 4 Epis and monos; Initial and Terminal Objects
- 5 Products
- 6 Duality; Coproducts
- 7 Subobjects; Pullbacks
- 8 Cartesian Closed Categories
- 9 Natural Transformations

Chapter II. Functors

- 10 Functor Categories
- 11 Equivalence of Categories
- 12 Adjoint Functors
- 13 Adjoint Functor Theorem

Part Two. Module Theory

Chapter III. Modules

- 14 Basic Properties of Modules
- 15 Modules and Module Homomorphisms
- 16 Submodules and Quotient Modules
- 17 Operations on Submodules, Direct Sum and Product
- 18 Finitely Generated Modules
- 19 Exact Sequences
- 20 Free Modules, Projective Modules and Injective Modules

Chapter IV. Tensor Product

- 21 Properties of Tensor Product
- 22 Restriction and Extension of Scalars
- 23 Exactness Properties of the Tensor Product, Flat Modules

24 Tensor Product of Algebras

Part Three. Ring Theory

Chapter V. Rings

- 25 Review of Rings and Their Homomorphisms
- 26 Commutative Rings and Examples
- 27 Chain Conditions, Noetherian Rings and Modules
- 28 Artinian Rings and Modules
- 29 The Krull Intersection Theorem
- 30 Nakayama's Lemma
- 31 Hilbert Basis Theorem

REFERENCES

- [1] Hungerford, Thomas W. Algebra, Graduate Text in Mathematics, 73, Springer-Verlag, New York-Berlin, 2003.
- [2] Rotman, Joseph J. Advanced Modern Algebra, Third Edition, Parts 1 and 2,"Graduate Studies in Mathematics, American Mathematical Society, 2015.