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Sequential quadratic programming models for solving the OPF problem in DC grids

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ABSTRACT

In this paper, we address the optimal power flow problem in dc grids (OPF-DC). Our approach is based on sequential quadratic programming which solves the problem associated with non-convexity of the model. We propose two different linearizations and compare them to a non-linear algorithm. The first model is a Newton-based linearization which takes the Jacobian of the power flow as a linearization for the optimization stage, and the second model uses the nodal currents as auxiliary variables to linearize over the inequality constraints. Simulation results in radial and meshed grids demonstrate the efficiency of the proposed methodology and allow finding the same solution given by the exact nonlinear representation of the OPF-DC problem.

1. Introduction

DC-DISTRIBUTION and dc-microgrids are emerging concepts for low voltage installations [1,2]. These present advantages regarding efficiency and controllability since most of the new components of modern electrical grids, such as solar power, energy storage, and electric vehicles, are inherently dc [3]. These components are usually integrated by dc/dc converters (see Fig. 1) which can introduce a non-linear behavior to the dynamics of the grid [4].

Classical operation methodologies such as the economic dispatch [5], state estimation [6] and optimal power flow [7] require to be adapted to this new context. In this paper, we address the optimal power flow, henceforth OPF-DC, which is a non-linear/non-convex problem [7]. The fact that the OPF-DC is non-convex could be counterintuitive to some. However, take into account that most of the components in a dc grid are integrated by dc/dc converters using a constant power control [8]. Therefore, its nodal power is given by an equation of the form $p = vi$ which is, in general, non-convex. We propose a sequential quadratic programming algorithm which consists of sequential linearizations of the constraints maintaining the quadratic behavior of the objective function. Our approach is based on three main observations: First; the objective function in the OPF-DC, as well as the inequality constraints, are convex; the non-convexity appears on the equality constraints which are non-affine. Second, normal operative conditions of a dc-grid allow linearizations which are very accurate (see [4] for more details). Third, there is a very efficient solver for quadratic programming which permits an easy and fast implementation of a sequential quadratic programming approach [9]. It is important to notice that the OPF-DC constitutes the tertiary control in the hierarchical structure for automatic operation of dc-grids [1]. Therefore, it is crucial to have an efficient algorithm to solve the problem for real-time applications [4].

The linearization of the equality constraints can be performed in different ways. We propose two different linearizations and study their performance regarding speed and accuracy. The first approach is based on the Newton-Raphson method for dc power grids [8], via Taylor's series expansion and the Jacobian matrix representation; while the second approach is based on admittance matrix formulation using voltages and currents as decision variables [9]. Notice that in the second model nonlinearities correspond to the inequalities associated with distributed generators and constant power loads.

The OPF-DC problem has been addressed by several authors in the recent years [11,12]. Most of the proposed solutions are based on convex approximations such as semidefinite and second-order approximations (see for example the seminal work of Low et al. about these approximations [13]). These approximations guarantee the global solution of the approximated problem but say nothing about the original non-convex problem. Only under well-defined circumstances, the optimum of the convex approximation is equal to the optimum of the original problem [14,15]. Our approach is different since we use a

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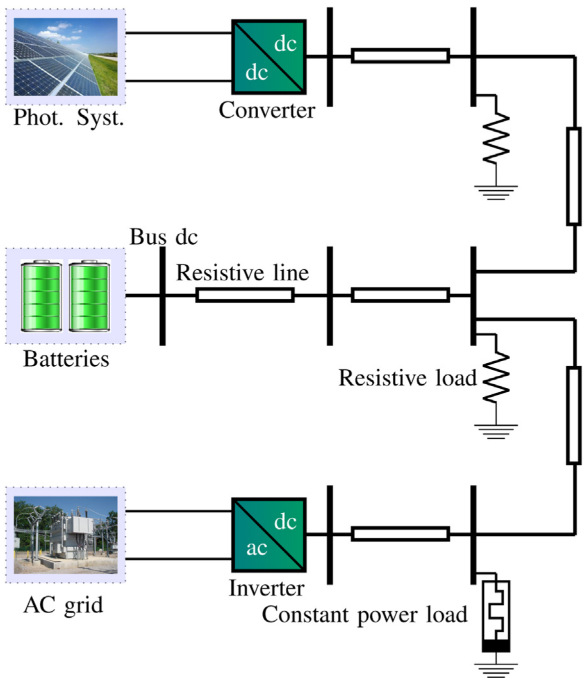

Fig. 1. Possible configuration of a dc power grid including distributed energy resources and grid interconnection [10].

sequence of quadratic formulation in order to solve the original non-convex problem. Thus, the optimal solution is the optimum of the original problem. Being a non-convex problem, we cannot claim our optimum is global. However, the results demonstrate to be enough for most practical situations [16]. In addition, our approach can be faster since the quadratic optimization algorithms are very efficient and the algorithm requires only a few iterations (low processing times) to obtain an optimum.

The rest of the paper is organized as follows: Section 2 presents the mathematical formulation for the non-convex OPF-DC and the proposed linearizations; some comments about each of these linearizations are also presented. Next, Section 3 presents the sequential quadratic algorithm and some details about its geometry in the proposed linearization. After that, we study the performance of the algorithms and the linearizations on radial and meshed test systems in Sections 4 and 5. Finally, we give conclusions in Section 6.

2. Mathematical formulation

This section presents the nonlinear formulation of the optimal power flow problem in dc power grids as well as two equivalent quadratic convex approaches via Taylor's series expansion [15,16].

2.1. Exact nonlinear formulation

The optimal power flow in dc grids (OPF-DC) is a nonlinear non-convex constrained optimization model which exhibits the following structure [4]:

Model 1. Non convex OPF-DC

$$\text{Minimize} \;\; p_{\text{loss}} = v^T \mathbf{G}_L v, \tag{1}$$

$$\text{subjectto} \;\; p_g - p_d = \mathbf{D}(v)[\mathbf{G}_L + \mathbf{G}_N]v, \tag{2}$$

$$p_g^{\min} \le p_g \le p_g^{\max}, \tag{3}$$

$$v_{\min} \le v \le v_{\max}, \tag{4}$$

$$-\, i_{\text{km}}^{\max} \le \frac{v_k - v_m}{r_{\text{km}}} \le i_{\text{km}}^{\max}. \tag{5}$$

where $p_{\text{loss}} \in \mathbb{R}_+$ represents the total power losses of the dc grid associated to the resistive effects in line segments, $v \in \mathbb{R}^n$ corresponds to the nodal voltages and $\mathbf{G}_L \in \mathbb{R}^{n\times n}$ represents the admittance nodal matrix. The problem is constrained to technical requirements where $\mathbf{G}_N \in \mathbb{R}^{n\times n}$ corresponds to the conductance matrix associated to the constant resistive loads or shunt resistive elements in the network; $p_g \in \mathbb{R}^n$ corresponds to the voltage and power controlled nodes which include generation, energy storage and controlled loads; $p_d \in \mathbb{R}^n$ represents all constant power consumptions; $p_g^{\min} \in \mathbb{R}^n$ and $p_g^{\max} \in \mathbb{R}^n$ represents respectively the minimum and maximum nodal powers; while $v_{\min} \in \mathbb{R}^n$ and $v_{\max} \in \mathbb{R}^n$ corresponds to the minimum and maximum allowed voltage in each node, respectively; $i_{\text{km}}^{\max}$ represents the maximum permissible current for the driver associated to the nodes k and m with resistance value r_{km}. Finally, $\mathbf{D}(v) \in \mathbb{R}^{n\times n}$ is a diagonal positive definite matrix with the nodal voltages, e.g., $\mathbf{D}(v) = \text{diag}(v)$. It is also important to highlight that the decision variable corresponds to the power generation by each generator (slack or distributed generator).

The interpretation of the non-convex model given from (1) to (4) is presented as follows: Eq. (1) corresponds to the objective function associated to the total power losses minimization, (2) is the power balance in all nodes of the network, (3) represents the upper and lower bounds of the power generation and (4) represents the voltage regulation bounds; while Eq. (5) represents the thermal constrain associated to each distribution line.

Notice that the only non-convex constraint on Model 1 is the power balance given by (2), since it is an non-affine equality constraint that contains a multiplication of the voltages; Nevertheless, the rest of the model is convex.

An alternative formulation of the exact nonlinear model can be obtained by separating (2) in two equations as follows

$$\mathbf{A}_g p_g - \mathbf{D}_g(v_g)[\mathbf{G}_{\text{gg}} v_g + \mathbf{G}_{\text{gd}} v_d] = 0, \tag{6}$$

$$\mathbf{A}_d[p_{\text{gd}} - p_d] = \mathbf{D}_d(v_d)[\mathbf{G}_{\text{dg}} v_g + \mathbf{G}_{\text{dd}} v_d], \tag{7}$$

where $\mathbf{A}_g \in \mathbb{R}^{s\times n}$ activates the generation variables associated only with the ideal generators, $\mathbf{A}_d \in \mathbb{R}^{(n-s)\times n}$ activates the power consumptions associated only with the demand, step nodes or distributed generators; $v_d \in \mathbb{R}^{n-s}$ represents the unknown voltages in the demand, step and distributed generator nodes, $v_s \in \mathbb{R}^s$ represents the constant voltage profiles at the ideal generator nodes (known and well-defined voltages), additionally $\mathbf{D}_g(v_g) \in \mathbb{R}^{s\times s}$ and $\mathbf{D}_v(v_d) \in \mathbb{R}^{(n-s)\times(n-s)}$ have the same interpretation of $\mathbf{D}(v)$, and

$$\mathbf{G} = \begin{pmatrix} \mathbf{G}_{\text{gg}} & \mathbf{G}_{\text{gd}} \\ \mathbf{G}_{\text{dg}} & \mathbf{G}_{\text{dd}} \end{pmatrix}, \quad \mathbf{D}(v) = \begin{pmatrix} \mathbf{D}_g(v_g) & 0 \\ 0 & \mathbf{D}_d(v_d) \end{pmatrix}$$

Recall that (6) is and affine set of equations, while (7) continues being a set of nonlinear non-convex constraints.

2.2. Quadratic model based on Newton-Raphson method

The Newton–Raphson method is the most classical nonlinear numerical method for power flow analysis in both ac and dc grids (see [8] for a formal demonstration of the superconvergence of the algorithm in dc grids) and its computational effort is lower than successive approximation methods [17]. Additionally, The Newton-Raphson method for power flow analysis only focuses on the nonlinear constraint (7), since it contains all unknown voltage variables, supposing that the demand and distributed generators have well-defined their values.

The application of the Newton-Raphson method is based on the Taylor series expansion for multiple nonlinear set of equations with multiple variables. In this sense, the right-hand side of (7) can be linearized around an operating point v_{d0} as follows

$$\mathbf{D}_d(v_d)[\mathbf{G}_{dg}v_g + \mathbf{G}_{dd}v_d] \approx \mathbf{D}_d(v_{d0})[\mathbf{G}_{dg}v_g + \mathbf{G}_{dd}v_{d0}] + \mathbf{J}_d(v_{d0})[v_d - v_{d0}] \quad (8)$$

where $\mathbf{J}_d(v_{d0}) \in \mathbb{R}^{(n-s)\times(n-s)}$ is the Jacobian matrix [8].

Notice that combining (8) and (7) the set of affine constraints are obtained around the operational point v_{d0}; Additionally, if we combine these expressions the next quadratic convex optimization model is obtained for optimal power flow analysis in direct current power grids.

Model 2. Newton-based Convex Quadratic OPF-DC

$$\begin{aligned}
&\text{Minimize } p_{\text{loss}} = v^T\mathbf{G}_L v,\\
&\mathbf{A}_g p_g - \mathbf{D}_g(v_g)[\mathbf{G}_{gg}v_g + \mathbf{G}_{gd}v_d] = 0,\\
&\mathbf{A}_d[p_{\text{gd}} - p_d] = \mathbf{D}_d(v_{d0})[\mathbf{G}_{dg}v_g + \mathbf{G}_{dd}v_{d0}] + \mathbf{J}_d(v_{d0})[v_d - v_{d0}],\\
&p_g^{\min} \le p_g \le p_g^{\max}\\
&p_{\text{gd}}^{\min} \le p_{\text{gd}} \le p_{\text{gd}}^{\max}\\
&v_{\min} \le v \le v_{\max},\\
&-i_{\text{km}}^{\max} \le \frac{v_k - v_m}{r_{\text{km}}} \le i_{\text{km}}^{\max}.
\end{aligned} \quad (9)$$

Recall that the effectiveness and accurateness of the proposed quadratic convex model based on Newton-Raphson method depends exclusively of the linearization point, i.e., the selection of v_{d0}; nevertheless, in section 3 we will be present a recursively methodology for solving (9), enhancing the same solution obtained when nonlinear exact formulation is solved.

2.3. Third model: quadratic model based on current-voltage representation

An alternative formulation of the OPF-DC is given considering the currents as follows

Non-convex OPF-DC with Nodal Currents

$$\begin{aligned}
&\min p_{\text{loss}} = v^T\mathbf{G}_L v,\\
&\mathbf{A}_g i_g - [\mathbf{G}_{gg}v_g + \mathbf{G}_{gd}v_d] = 0,\\
&\mathbf{A}_d i_{\text{gd}} - [\mathbf{G}_{dg}v_g + \mathbf{G}_{dd}v_d] = 0,\\
&p_g^{\min} \le \mathbf{D}_d(v_g)i_g \le p_g^{\max}\\
&p_{\text{gd}}^{\min} \le \mathbf{D}_d(v_d)i_{\text{gd}} \le p_{\text{gd}}^{\max}\\
&v_{\min} \le v \le v_{\max},\\
&-i_{\text{km}}^{\max} \le \frac{v_k - v_m}{r_{\text{km}}} \le i_{\text{km}}^{\max}.
\end{aligned} \quad (10)$$

where i_g represents the current delivered by the ideal generators, and i_{gd} the current provided by the distributed generators and consumed by the constant power loads.

Notice that the model (10) is exactly equivalent to the nonlinear non-convex optimizing model defined from (1) to (4); nevertheless, it can also became into a convex quadratic model if constraint associated to the distributed generation and constant power loads.

By applying the same Taylor's series expansion presented to second proposed model, the following quadratic optimization model is achieved

Model 3. Convex Quadratic OPF-DC with Nodal Currents

$$\begin{aligned}
&\text{Minimize } p_{\text{loss}} = v^T\mathbf{G}_L v,\\
&\mathbf{A}_g i_g - [\mathbf{G}_{gg}v_g + \mathbf{G}_{gd}v_d] = 0,\\
&\mathbf{A}_d i_{\text{gd}} - [\mathbf{G}_{dg}v_g + \mathbf{G}_{dd}v_d] = 0,\\
&p_g^{\min} \le \mathbf{D}_d(v_g)i_g \le p_g^{\max}\\
&[2\mathbf{D}_d^{-1}(v_{d0}) - \mathbf{D}_d^{-2}(v_{d0})\mathbf{D}_d(v_d)]p_{\text{gd}}^{\min} \le i_{\text{gd}}\\
&i_{\text{gd}} \le [2\mathbf{D}_d^{-1}(v_{d0}) - \mathbf{D}_d^{-2}(v_{d0})\mathbf{D}_d(v_d)]p_{\text{gd}}^{\max}\\
&v_{\min} \le v \le v_{\max},\\
&-i_{\text{km}}^{\max} \le \frac{v_k - v_m}{r_{\text{km}}} \le i_{\text{km}}^{\max}.
\end{aligned} \quad (11)$$

Likewise Model 2, this is a convex quadratic model, the main difference lies in the additional variables given by the currents i_g. Hence, the linearization appears in the inequality constrained variables.

2.4. General comments

Models 2 and 3 correspond to convex approximations of the optimal power flow problem in dc power grids with a quadratic objective function and linear constraints. These models have been developed by applying Taylor's series expansion over the non-linear set of equations given in (2).

Model 3 corresponds to an alternative formulation of the optimal power flow problem by using voltage and currents as variables, which is common-less in specialized literature [9], where power balance equations (see nonlinear exact modeling) are extensively used [4,11,14].

The main advantage of the proposed convex models respect to the existing formulations in specialized literature (i.e., semidefinite programming [12] and second-order cone programming models [7,14]), lies that our formulations the number of variables remain constant, nevertheless the aforementioned existing model, the variables increase in square-form [18].

To obtain the exact solution given by the exact nonlinear model, a recursive programming model can be easily adapted for each one of the proposed convex model as presented in next section.

3. Proposed recursive solution methodology

The proposed quadratic convex models in the previous section estimate the solution of the exact nonlinear non-convex model for the optimal power flow problem in dc power grids if we use an iterative process that modifies the linearization point v_{d0}, then, the approximation gap between the exact model and its corresponding convex reformulations are reduced.

Algorithm 1 describes the iterative process that solves the proposed quadratic optimization models. Observe that if the numerical error between two continuous iterative calculations is lower than a predefined tolerance, we say that the proposed convex models achieve the same solution generated by the exact nonlinear formulation; otherwise, the proposed convex models obtain approximated solutions.

Algorithm 1. Recursive solution for the convex proposed models to achieve the exact nonlinear solution.

Data: Define dc grid, stopping criteria and maximum interation parameters.
$v_0 = \mathbf{1}$;
for $t = 1 : t_{max}$ **do**
  Solve (9) or (11) using a quadratic optimizer package;
  Actualize v_t;
  if $|v_t - v_0| \le$ error **then**
    Exact solution achieved;
    Result: Return v_t and p_{loss}.
    break;
  else
    $v_0 = v_t$;
    if $t == t_{max}$ **then**
      An approximate solution was reached;
      Result: Return v_t and p_{loss}.
    end
  end
end

On the other hand, Fig. 2 depicts the evolution of the proposed convex models under the solution space given in terms of voltage and current. Notice that solution space is constrained by the maximum and minimum voltage values $v_{\max}$ and $v_{\min}$; additionally is constrained in the lower case by a convex function $p_{\min}/v$, while the upper case the non-linear function $p_{\max}/v$ is non-convex. Now observe that in the general

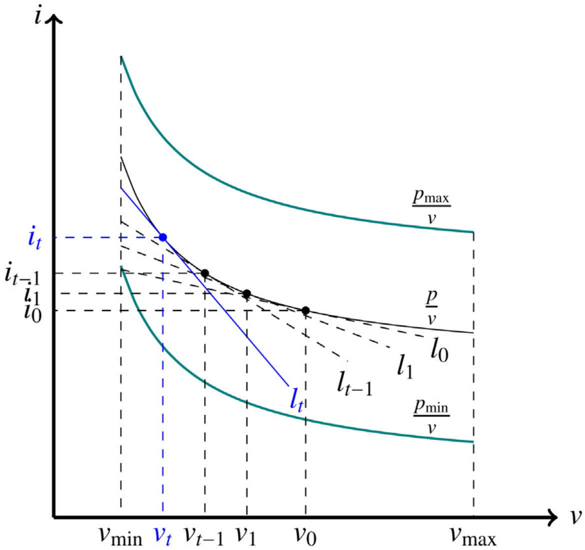

Fig. 2. Iterative behavior of the recursive quadratic models.

case, the solution of the power flow equations is located over the nonlinear function p/v, which entails that the proposed approximations are obligated to be tangent lines on that point. In this sense, if the solution of the optimal power flow problem in dc power grid is given by (v_t, i_t), besides the starting point is given by (v_0, i_0), then, l_0 represents the first linear approximation of the problem, which after solving the convex approximation models produce the solution set (v_1, i_1). Now, if this point is used as the new starting point, then, l_1 represents the new linear approximation. Notice that if this procedure is followed recursively, then after t iterations, the proposed convex models achieve the same solution as the original nonlinear non-convex model.

4. Test systems and comparison methods

Two dc power grids are employed to validate the proposed convex models for solving the optimal power flow problem considering multiple distributed generators or slack nodes. The first test system has 10 nodes and originally was proposed in [17] for power flow convergence analysis by using Gauss-Seidel approximations. Second test system was proposed by [8] for analyzing the power flow convergence in dc power grids via Newton-Raphson methods. Next sections present the detailed information of each test system. The information of these test systems are given in p.u. and it was calculated using 1 kV and 100 kW as voltage and power bases, respectively.

4.1. 10-Nodes test feeder

The 10-nodes test system presented in [17] was initially built under a radial configuration. Nevertheless, to demonstrate the application of the proposed convex models to mesh configurations, we transform this test system into a mesh configuration as depicted in Fig. 3 depicts the 10-nodes test feeder with the parameters listed in Tables 1 and 2.

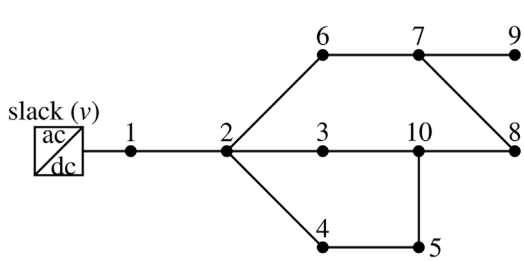

Fig. 3. Electrical configuration for the 10-node test system.

Table 1
Electrical parameters of the 10-nodes test system

From	To	R_{line} [pu]	Type of node	P [pu] – R [pu]
1 (slack)	2	0.0050	Step-node	–
2	3	0.0015	P	−0.8
2	4	0.0020	P	−1.3
4	5	0.0018	P	0.0–2.5
2	6	0.0023	R	2.0
6	7	0.0017	Step-node	–
7	8	0.0021	P	0.0–2.5
7	9	0.0013	P	−0.7
3	10	0.0015	R	1.25

Table 2
Proposed connections becoming the grid into a mesh topology.

From	To	R [pu]	From	To	R [pu]
5	10	0.0035	9	10	0.0025

For optimization purposes, we consider that this test system the possibility to dispatch two distributed generators located at nodes 5 and 8 with generation capacities from 0 to 2.5 p.u.

4.2. 21-Node test system

This test system is conformed by 21 nodes and 20 lines with multiple constant power loads, an adaptation of the test system originally presented in [8]. Besides, we include two ideal voltage generators at nodes 1 and 21. The electrical configuration of this test system is illustrated in Fig. 3, while its power consumption are listed in Table 3.

For this system, we consider that the three distributed generators, located at nodes 9, 12 and 18, respectively. They can be dispatched from 0 to 1.5 p.u. For simulation purposes, we also consider that initially, the generators do not produce energy (Fig. 4).

4.3. Comparison methods

To validate the accuracy and efficiency of the proposed sequential quadratic programming models the third proposed models are compared with the exact nonlinear formulation as well as a semidefinite programming model existing in the specialized literature.

These computational validations are carried out in a desk-computer with an INTEL(R) Core(TM) $i5-3550$ processor at 3.50 GHz, 8 GB RAM, running a 64-bits Windows 7 Professional operating system by using a MATLAB programming language. In this sense, the exact nonlinear model (Model 1) is solve via interior point methods available for **fmincon** nonlinear optimization package; for the proposed convex quadratic approximations (Models 2 and 3) the **quadprog** optimization package is employed; while a semidefinite programming (SDP) model developed [12] is implemented under **CVX** environment for MATLAB.

Table 3
Electrical parameters for the 21-nodes test system.

From	To	R_{line} [pu]	P [pu]	From	To	R_{line} [pu]	P [pu]
1	2	0.0053	0.70	11	12	0.0079	0.68
1	3	0.0054	0.00	11	13	0.0078	0.10
3	4	0.0054	0.36	10	14	0.0083	0.00
4	5	0.0063	0.04	14	15	0.0065	0.22
4	6	0.0051	0.36	15	16	0.0064	0.23
3	7	0.0037	0.00	16	17	0.0074	0.43
7	8	0.0079	0.32	16	18	0.0081	0.34
7	9	0.0072	0.80	14	19	0.0078	0.09
3	10	0.0053	0.00	19	20	0.0084	0.21
10	11	0.0038	0.45	19	21	0.0082	0.21

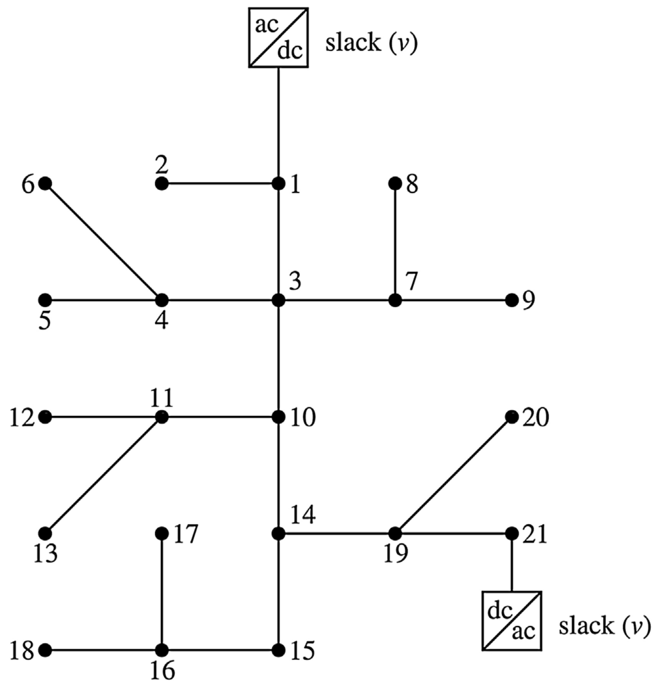

Fig. 4. Electrical configuration for the 21-nodes test system.

5. Numerical results

This section presents the results obtained with the proposed convex models as well as the solutions obtained with the comparison methods. Additionally, a complete discussion is provided.

5.1. 10-Nodes test feeder

In this test system the admissible region for the voltage profile are considered from 0.95 p.u to 1.00 p.u. All simulation are carry-out using per-unit representation considering that the starting point for all unknown voltage corresponds to 1.00 p.u.

Table 4 shows the objective function achieved for the proposed convex methods as well as the comparison methodologies. Observe that the proposed convex models achieve the same objective function, which corresponds to 1.2931 kW and represents a reduction around of 86.8618% when compared to the base case (9.8423 kW without distributed generation). On the other hand, when processing time is observed, in this case, model 1 exhibits the maximum computation effort (100%) followed by the semidefinite programming model which takes the 70.9961% the total simulation time. Additionally, models 2 and 3 take 4.2482% and 3.1806% of the total time expended by the exact model, respectively. These results evidence the speed convergence of the proposed convex models is better when compared with classical models as well as the accuracy and efficiency in terms of power losses estimation with errors lower than 1×10^{-3} %.

Table 5 lists the power generation obtained by the solution of the comparison methods as well as the proposes quadratic models; in this sense, we can observe that the convex models included the semidefinite programming model have an estimation error in the power generation at node 5 around 1.4533×10^{-3} %, while in the power generation at node 8 the estimation error is 2.7441×10^{-3} %. These results confirm the accuracy and efficiency of the proposed convex models in terms of power generation estimation.

Table 4
Power losses and processing times for each mathematical model at the 10-node test feeder.

Mathematical model	Solver	Power losses [kW]	Processing time [s]
Model 1	fmincon	1.2931	0.4496
SDP model	CVX	1.2931	0.3192
Model 2	quadprog	1.2931	0.0191
Model 3	quadprog	1.2931	0.0143

Table 5
Power generation for each mathematical model at the 10-node test feeder.

Mathematical model	Solver	Node 5 [kW]	Node 8 [kW]
Model 1	fmincon	240.8301	113.3808
SDP model	CVX	240.8336	113.3839
Model 2	quadprog	240.8336	113.3839
Model 3	quadprog	240.8336	113.3839

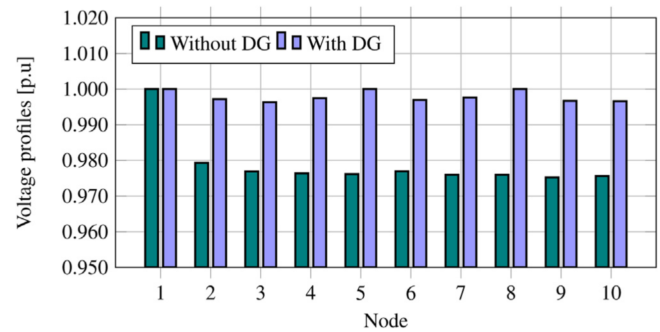

Fig. 5. Voltage profile at load nodes before and after dispatching the distributed generators.

Fig. 5 presents the voltage behavior in all nodes of the 10-node test feeder before and after the dispatch of the distributed generators. Notice that the operation of the distributed generators help to increase the voltage performance in the whole microgrids, which implies that when the voltage droop in terminal of any line is reduced, then the current through the line decrease, which consequently minimizes the power losses on the whole power grid. From Fig. 5 is evidenced that before dispatching distributed generators the voltage droop is 2.403% at node 9, nevertheless, when distributed generators are dispatched the maximum voltage droop is presented at node 3 with 0.370%.

5.2. 21-Nodes test feeder

In this test system the admissible region for the voltage profile are considered from 0.95 p.u to 1.05 p.u. The first voltage controlled source (node 1) is operated under 1.00 p.u, while second voltage controlled source (node 21) is assigned to 1.05 p.u.

Table 6 lists the objective function behavior for the comparison methods as well as the proposed convex formulations. Notice that model 1 and the semidefinite programming obtain the same objective function, while the three convex models exhibit an estimation error around 2.5981×10^{-2}. Nevertheless, in terms of processing time the semidefinite programming model takes 64.471% of the time expended by the exact nonlinear model; while models 2 and 3 take 3.3534% and 2.7284% of the exact model, respectively.

Table 6
Power losses and processing times for each mathematical model at the 21-node test feeder.

Mathematical model	Solver	Power losses [kW]	Processing time [s]
Model 1	fmincon	9.2374	0.8320
SDP model	CVX	9.2374	0.5364
Model 2	quadprog	9.2350	0.0279
Model 3	quadprog	9.2350	0.0227

Table 7
Power generation for each mathematical model at the 21-node test feeder.

Mathematical model	Solver	Node 9 [kW]	Node 12 [kW]	Node 18 [kW]
Model 1	fmincon	122.8905	120.2313	103.7837
SDP model	CVX	122.8898	120.2223	103.7841
Model 2	quadprog	125.9069	121.8379	104.2622
Model 3	quadprog	125.9069	121.8379	104.2622

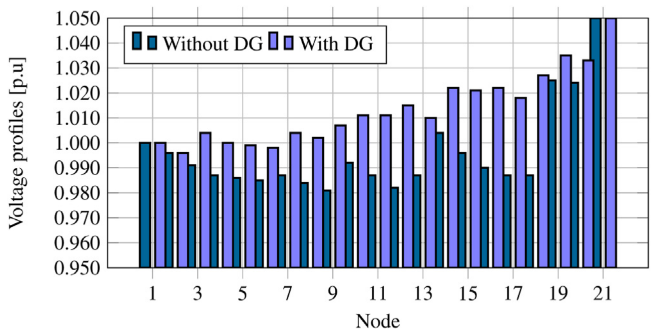

Fig. 6. Voltage profile at load nodes before and after dispatching the distributed generators.

Table 7 presents the power generation at each node for the proposed convex modes as well as the comparison methods; observe that the maximum estimation error between the exact nonlinear formulation or the semidefinite programming model and the two proposed convex models is 2.4545% in terms of power losses estimation. Additionally, when is compared the base case (without distributed generation) to the power losses after the dispatch of the distributed generators the power loss reductions reach to 56.1210%, which evidence the positive effect of the distributed generation performance of the electrical network.

On the other hand, Fig. 6 shows the voltage profile performance before and after the distributed generators are dispatched.

Notice that the voltage profile starts to increase uniformly when distributed generators begin producing power since the active power losses are correlated to the voltage profile performance; nevertheless, it is not possible to determine a linear behavior between both variables due to its nonlinear relation evidenced in the model 1. Finally, when voltage profile obtained by the exact model and the proposed convex approximations for the 14 node, an estimation error around 9.785×10^{-2} is observed, which validates the proposed quadratic programming model in terms of accuracy and efficiency.

6. Conclusions and future works

Two sequential quadratic programming models for solving the optimal power flow problem in dc grids were proposed, in this paper. Each proposed model was based on Taylor's linearization methods and compared them to the conventional non-linear algorithm. The first model was a Newton-based linearization which uses a Jacobian matrix to obtain a linear representation of the OPF problem, which has as advantage the well-known convergence speed of Newton methods in power flow applications. The second model employed admittance matrix as linear affine constraint as well as a Taylor's linearization method over the inequality constraints set, which allows obtaining an alternative power flow model based using voltages and currents as decision variables.

The simulation results showed that two models presented were able to reach the same solutions of the non-convex (Model 1) and convex (SDP model) model and even arriving at better solutions than these. These results verified the high efficiency and performance of the algorithms that required fewer iterations (low processing times) to obtain an optimum.

For the proposed convex reformulations can be adapted hourly representations that allow including renewable energy resources' stochasticity and battery energy storage variables for economic dispatch analyzes. It is also possible to use this formulations for dc planning studies, i.e., optimal location and sizing of distributed generators, by including those representations as slave OPF solution methods inside of master metaheuristic algorithms.

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