Chapter 5: Equilibrium of a Rigid Body

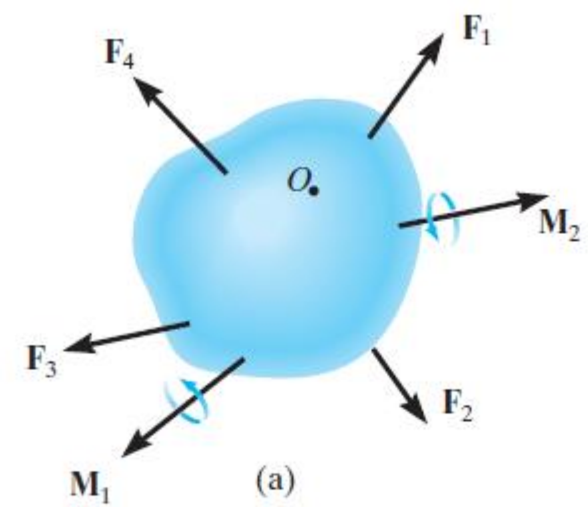
Equilibrium of a Rigid Body

CHAPTER OBJECTIVES

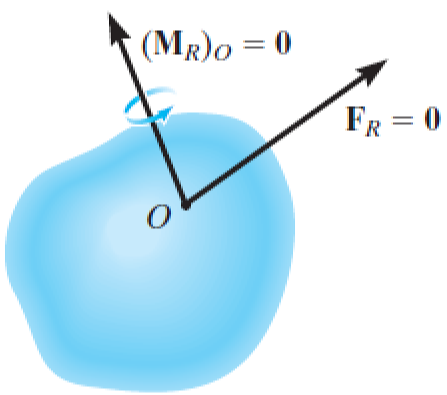
- To develop the equations of equilibrium for a rigid body.
- To introduce the concept of the free-body diagram for a rigid body.
- To show how to solve rigid-body equilibrium problems using the equations of equilibrium.

Equilibrium of a Rigid Body

Conditions for Rigid-Body Equilibrium



If this resultant force and couple moment are both equal to zero, then the body is said to be in ***equilibrium***. Mathematically, the equilibrium of a body is expressed as

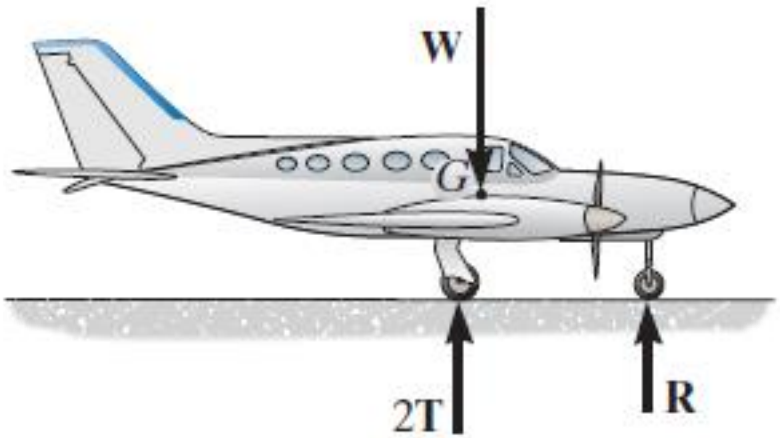


$$F_R = \Sigma F = 0$$

$$(M_R)_O = \Sigma M_O = 0$$

Equilibrium of a Rigid Body

EQUILIBRIUM IN TWO DIMENSIONS



Equilibrium of a Rigid Body

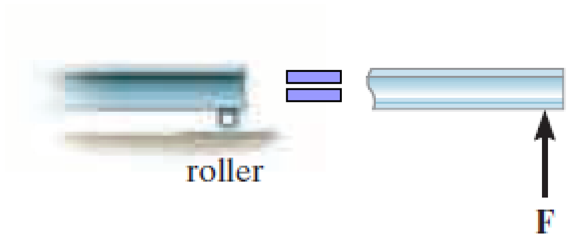
Free-Body Diagrams

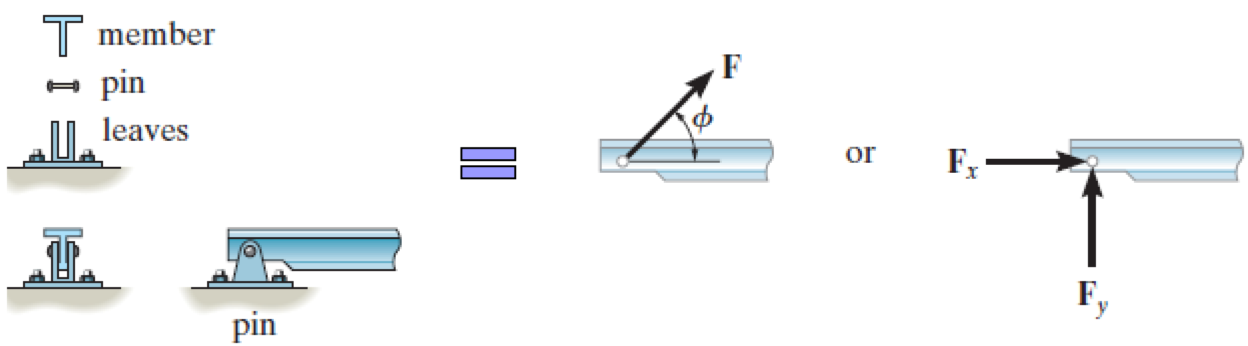
Successful application of the equations of equilibrium requires a complete specification of *all* the known and unknown external forces that act *on* the body. The best way to account for these forces is to draw a ***free-body diagram***. This diagram is a sketch of the outlined shape of the body, which represents it as being *isolated* or “free” from its surroundings, i.e., a “free body.” On this sketch it is necessary to show *all* the forces and couple moments that the surroundings exert *on the body* so that these effects can be accounted for when the equations of equilibrium are applied. *A thorough understanding of how to draw a free-body diagram is of primary importance for solving problems in mechanics*.

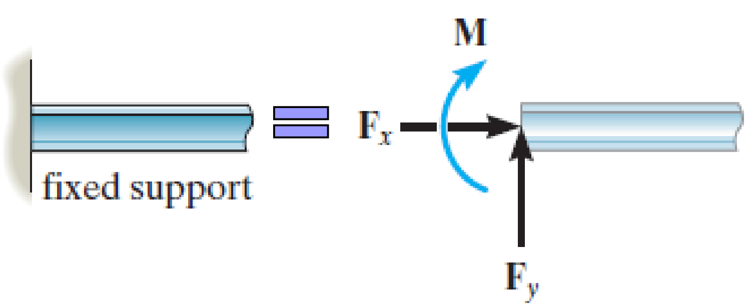
Equilibrium of a Rigid Body

Free-Body Diagrams

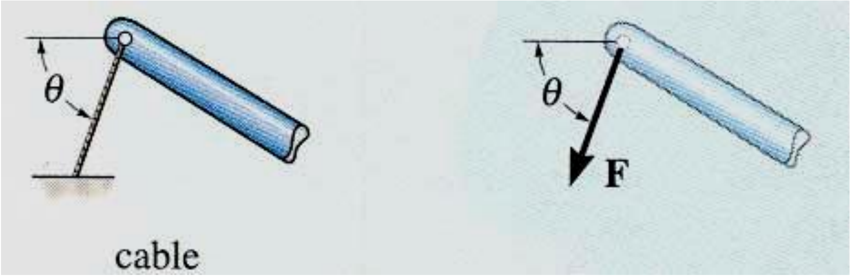
Support Reactions

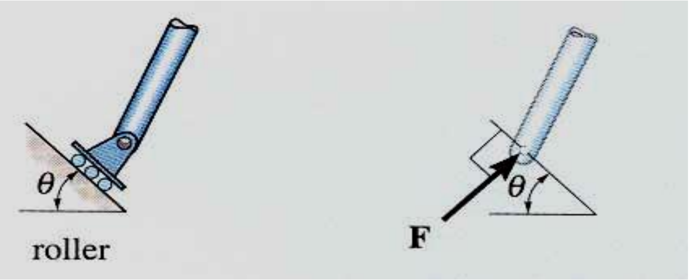


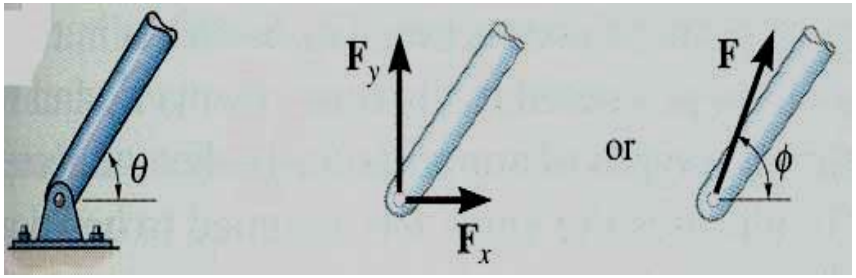




SUPPORT REACTIONS IN 2-D







Equilibrium of a Rigid Body

Free-Body Diagrams

Support Reactions

Equilibrium of a Rigid Body

Free-Body Diagrams

Support Reactions

Equilibrium of a Rigid Body

Free-Body Diagrams

Support Reactions

Equilibrium of a Rigid Body

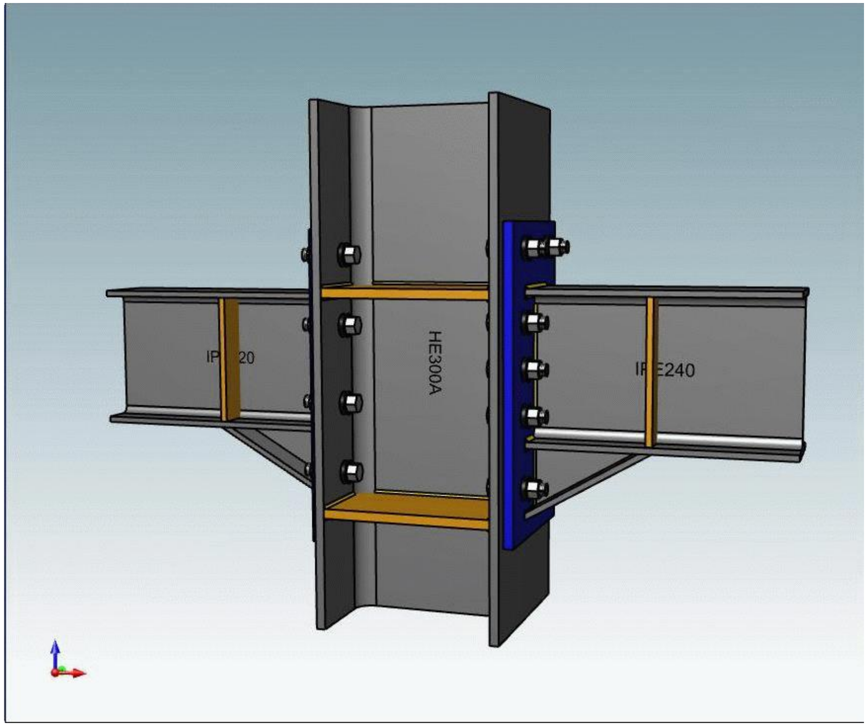
Free-Body Diagrams

Support Reactions

Equilibrium of a Rigid Body

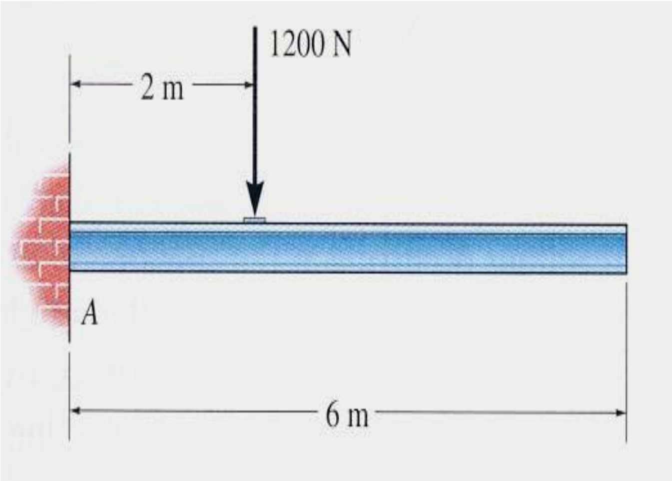
Free-Body Diagrams

Support Reactions

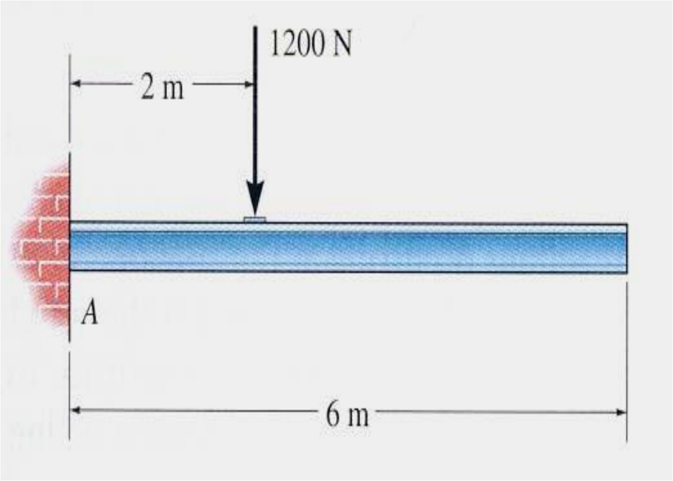


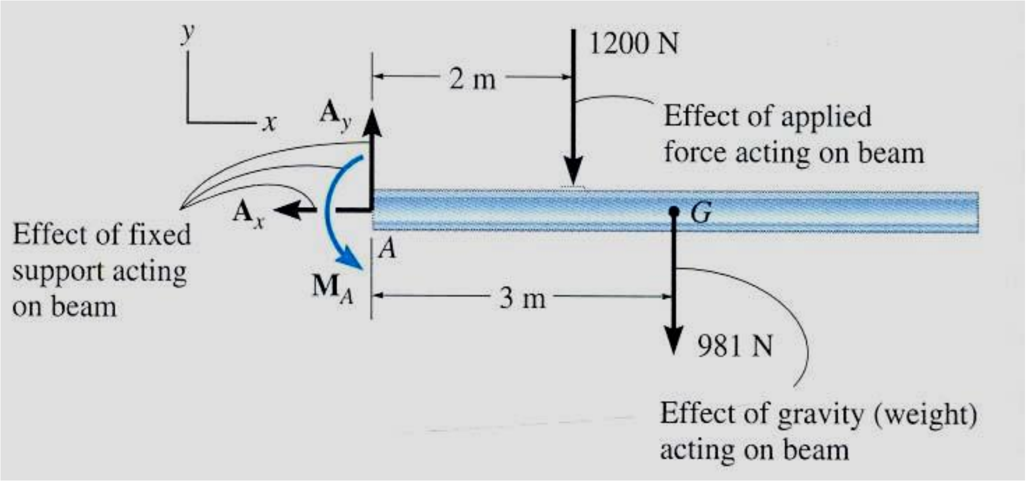
FREE-BODY DIAGRAMS

Draw the free-body diagram of the uniform beam shown in figure. The beam has a mass of 100 kg.



FREE-BODY DIAGRAMS
(continued)

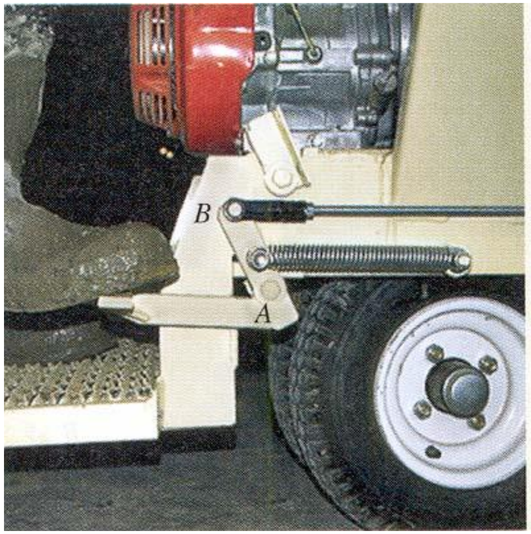




Idealized model

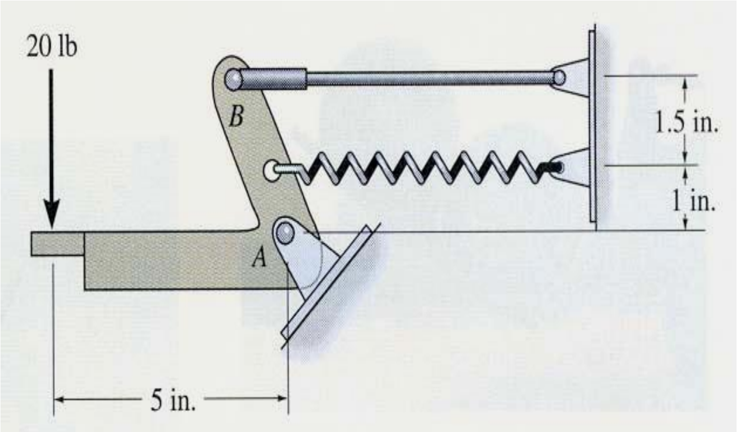
Free-body diagram

EXAMPLE

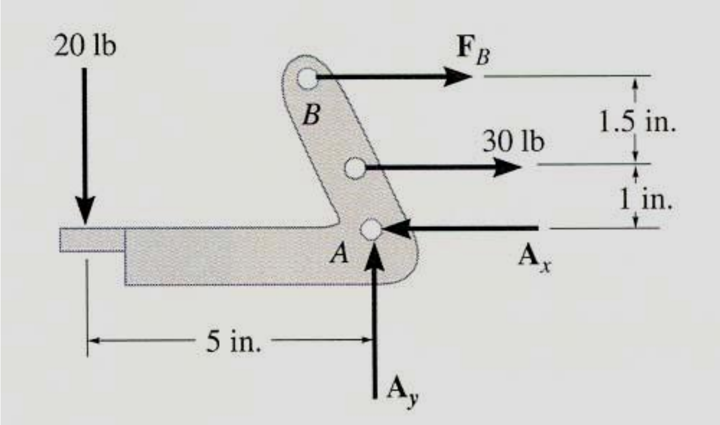


Given: An operator applies 20 lb to the foot pedal. A spring with k = 20 lb/in is stretched 1.5 in.

Draw: A free-body diagram of the foot pedal.

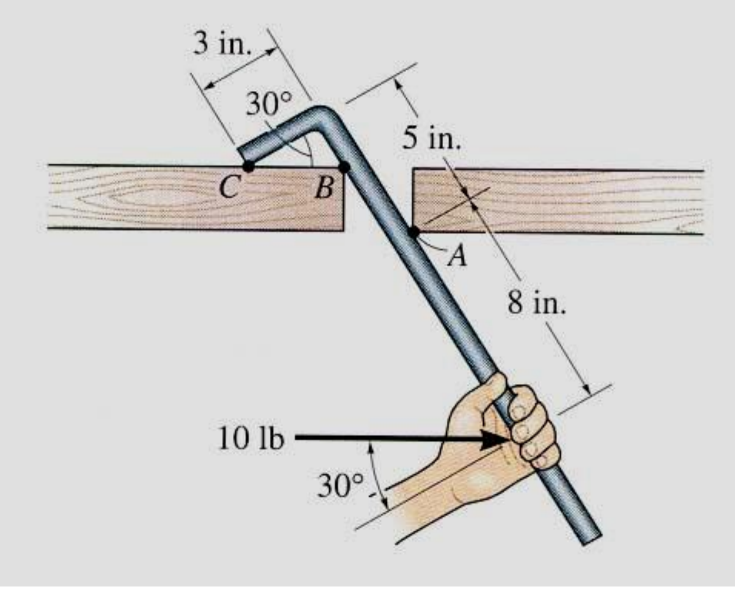


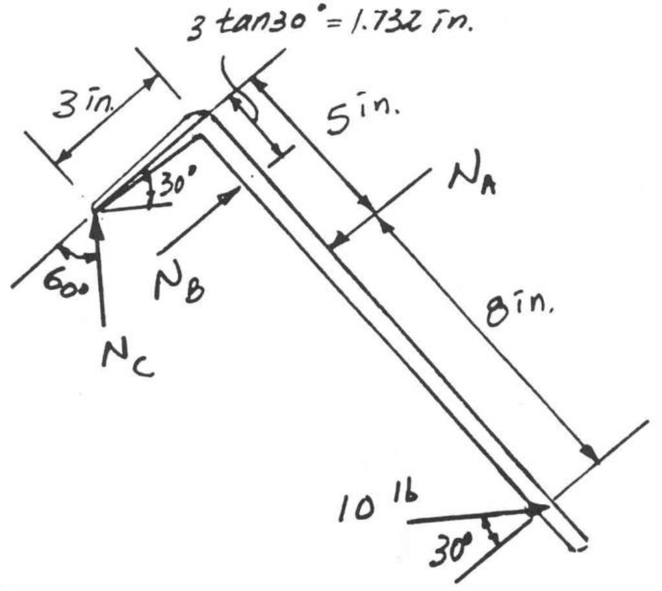
The idealized model



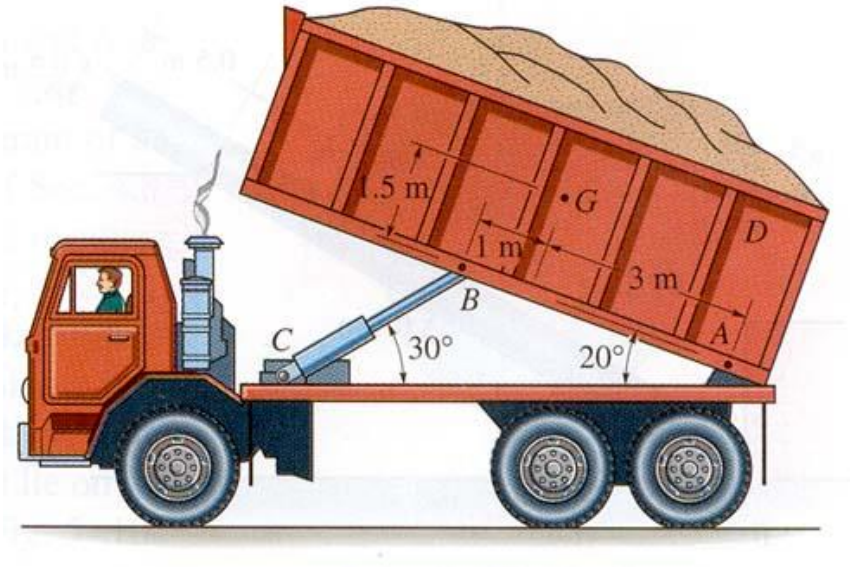
The free-body diagram

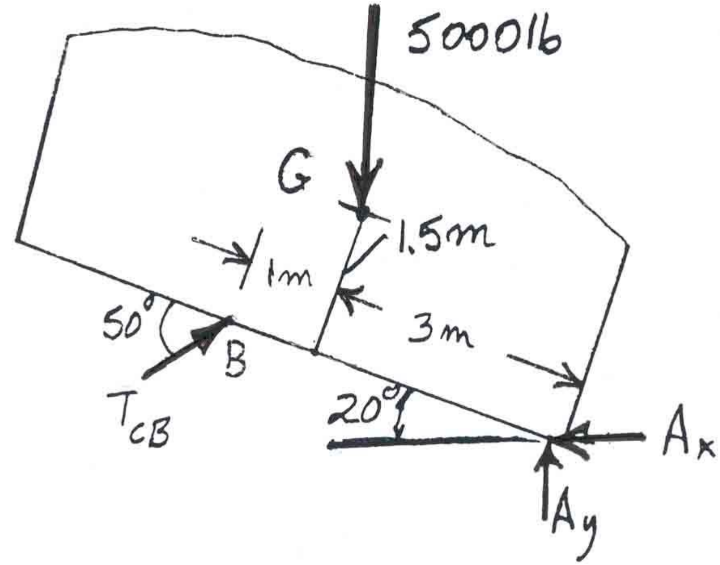
EXAMPLE





Draw a FBD of the bar, which has smooth points of contact at A, B, and C.





Draw a FBD of the 5000 lb dumpster (D). It is supported by a pin at A and the hydraulic cylinder BC (treat as a short link).

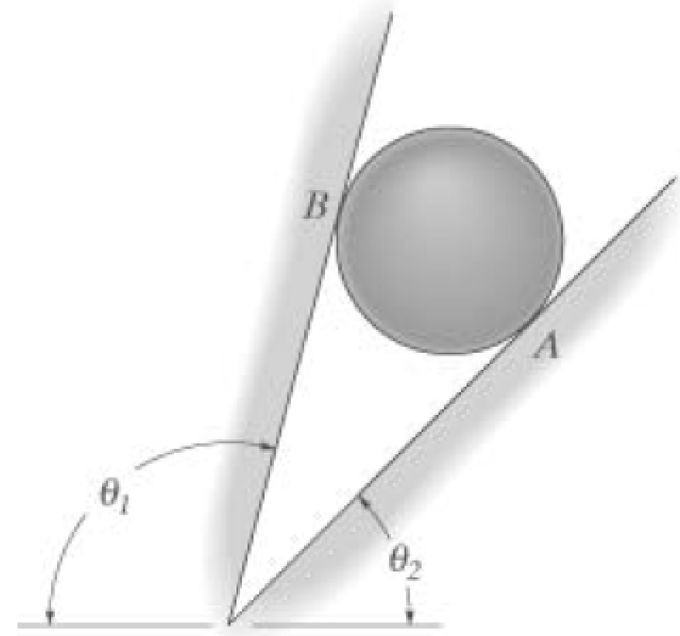
Draw the free-body diagram of the sphere of weight W resting between the smooth inclined planes. Explain the significance of each force on the diagram.

Given:

$W = 10$ lb

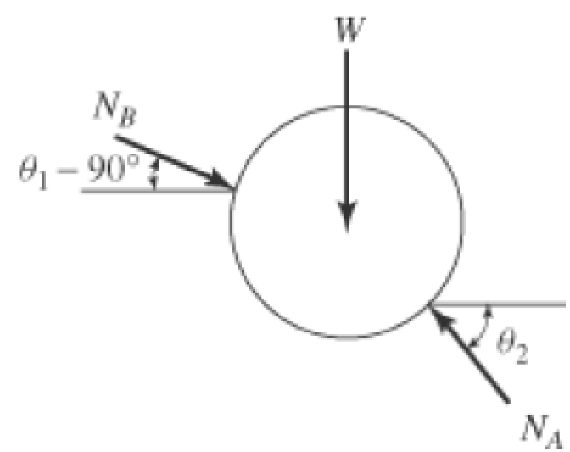
$\theta_1 = 105$ deg

$\theta_2 = 45$ deg



N_A, N_B force of plane on sphere.

W force of gravity on sphere.



Draw the free-body diagram of the hand punch, which is pinned at A and bears down on the smooth surface at B.

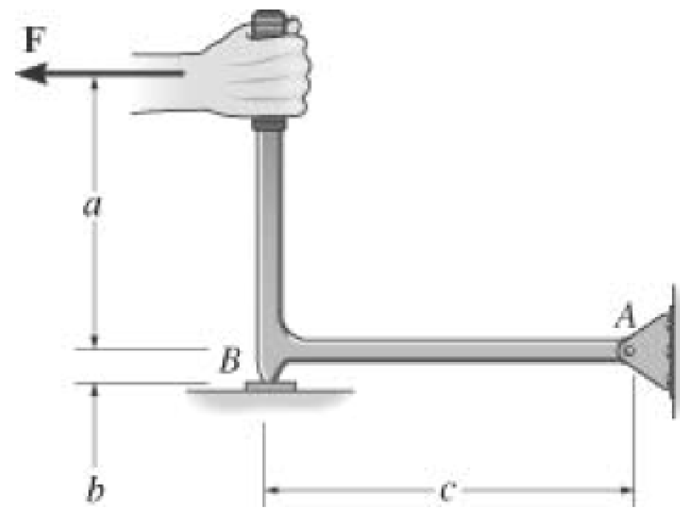
Given:

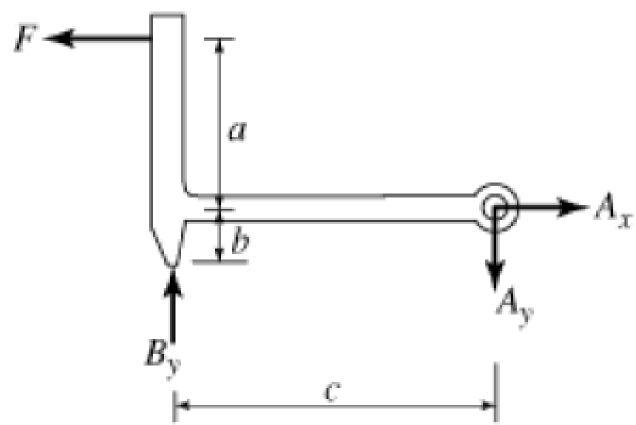
$F = 8$ lb

$a = 1.5$ ft

$b = 0.2$ ft

$c = 2$ ft





Draw the free-body diagram of the beam supported at A by a fixed support and at B by a roller. Explain the significance of each force on the diagram.

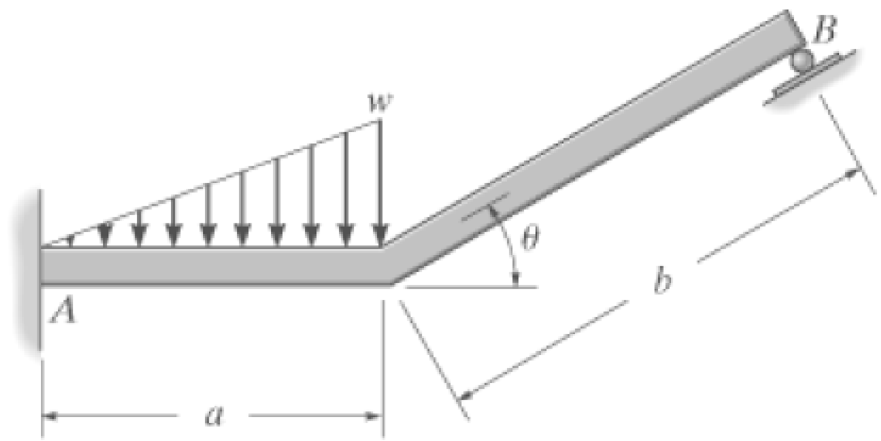
Given:

$$w = 40 \frac{\text{lb}}{\text{ft}}$$

$$a = 3 \text{ ft}$$

$$b = 4 \text{ ft}$$

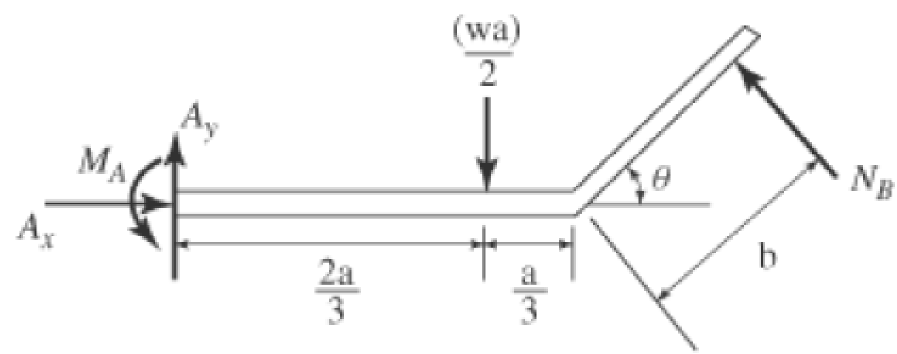
$$\theta = 30 \text{ deg}$$



A_x, A_y, M_A effect of wall on beam.

N_B force of roller on beam.

$\frac{wa}{2}$ resultant force of distributed load on beam.



Draw the free-body diagram of the jib crane *AB*, which is pin-connected at *A* and supported by member (link) *BC*.

Units Used:

$kN = 10^3\ N$

Given:

$F = 8\ kN$

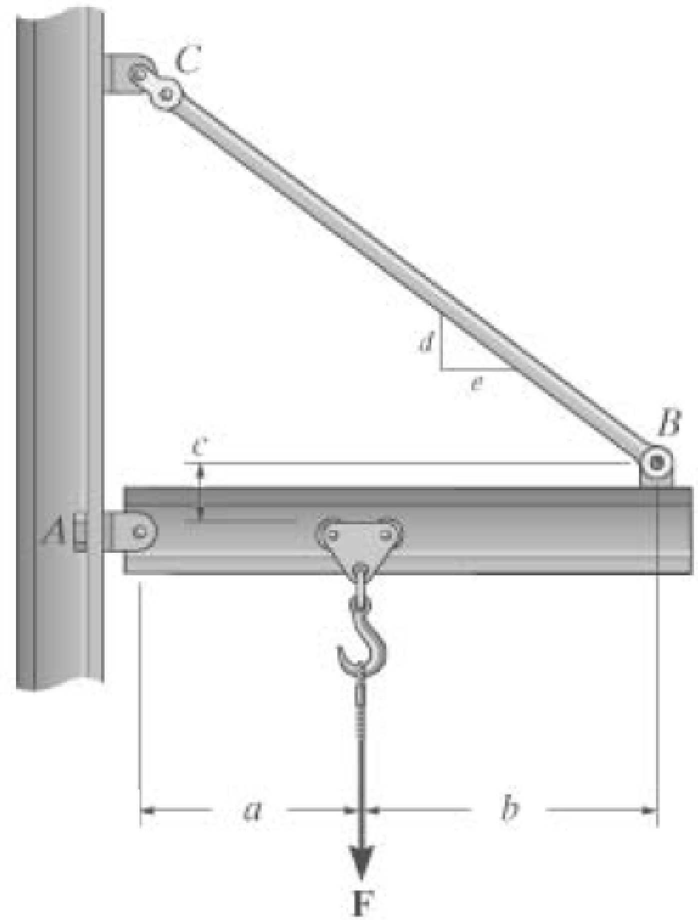
$a = 3\ m$

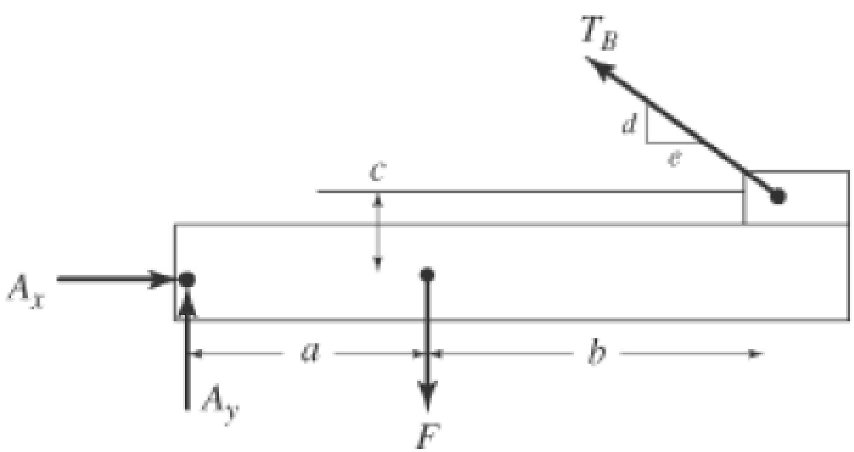
$b = 4\ m$

$c = 0.4\ m$

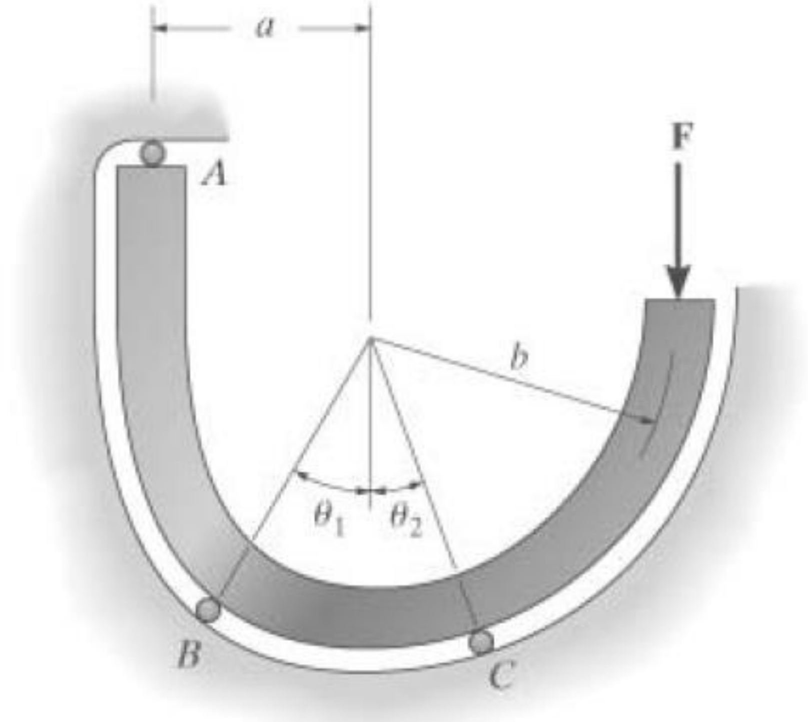
$d = 3$

$e = 4$





Draw the free-body diagram of the *C*-bracket supported at *A*, *B*, and *C* by rollers. Explain the significance of each forcce on the diagram.



Given:

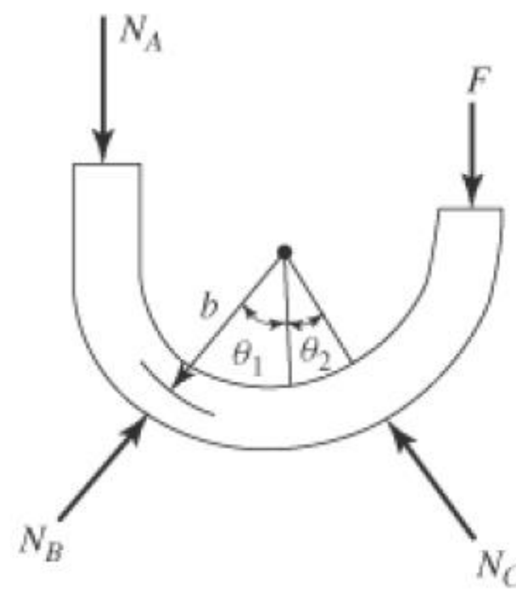
$a = 3$ ft

$b = 4$ ft

$\theta_1 = 30$ deg

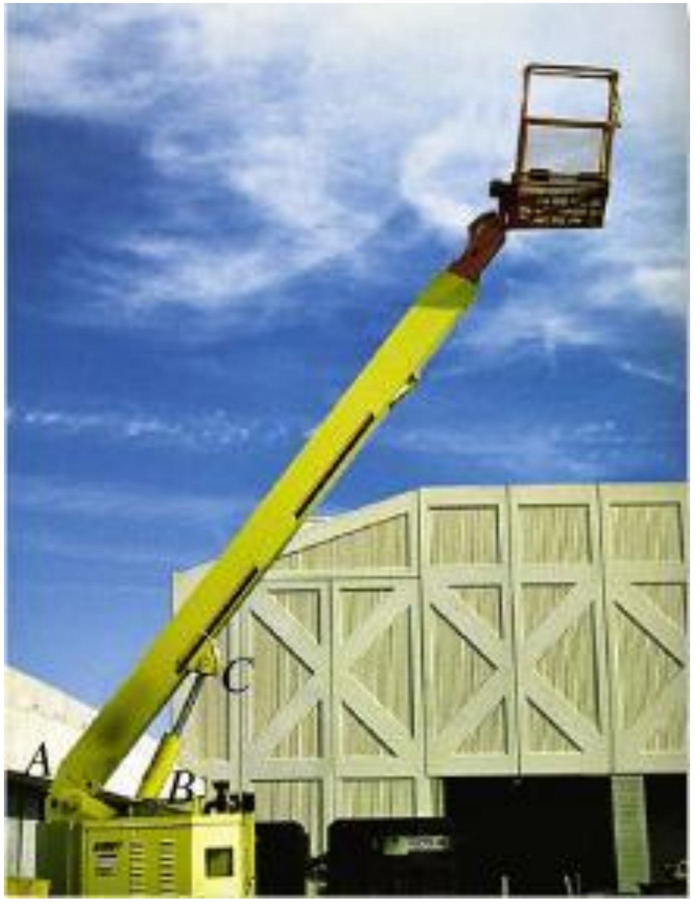
$\theta_2 = 20$ deg

$F = 200$ lb



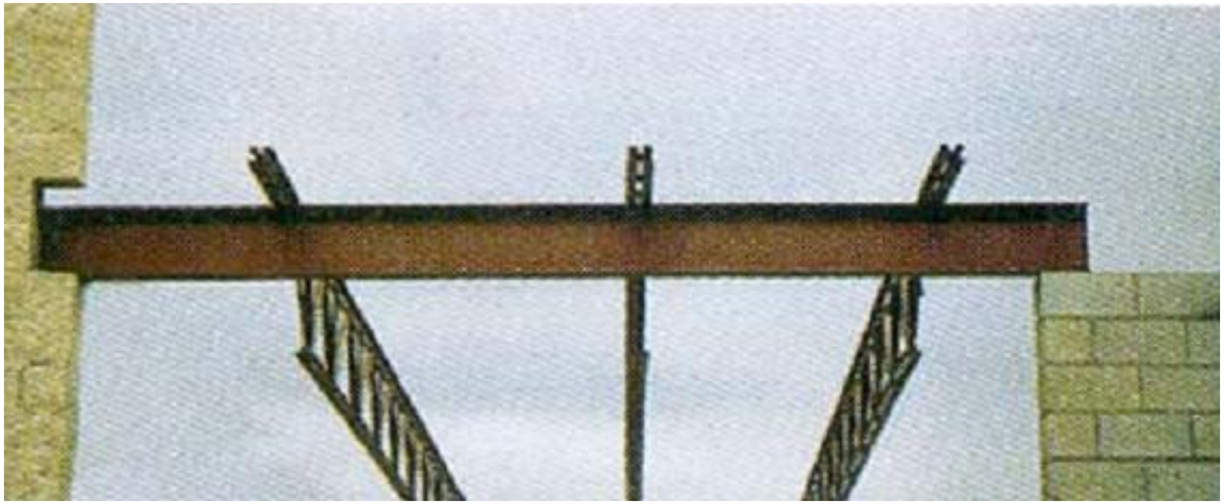
N_A, N_B, N_C force of rollers on beam.

EQUATIONS OF EQUILIBRIUM & TWO- AND THREE-FORCE MEMBERS



For a given load on the platform, how can we determine the forces at the joint A and the force in the link (cylinder) BC?

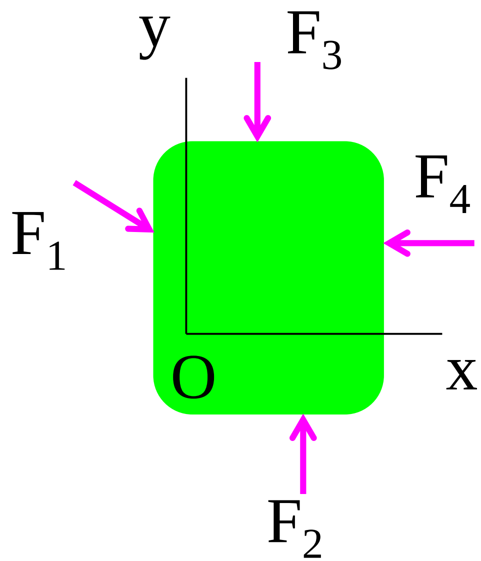
APPLICATIONS
(continued)



A steel beam is used to support roof joists.   How can we determine the support reactions at each end of the beam?

EQUATIONS OF EQUILIBRIUM

A body is subjected to a system of forces that lie in the x-y plane. When in equilibrium, the net force and net moment acting on the body are zero. This 2-D condition can be represented by the three scalar equations:



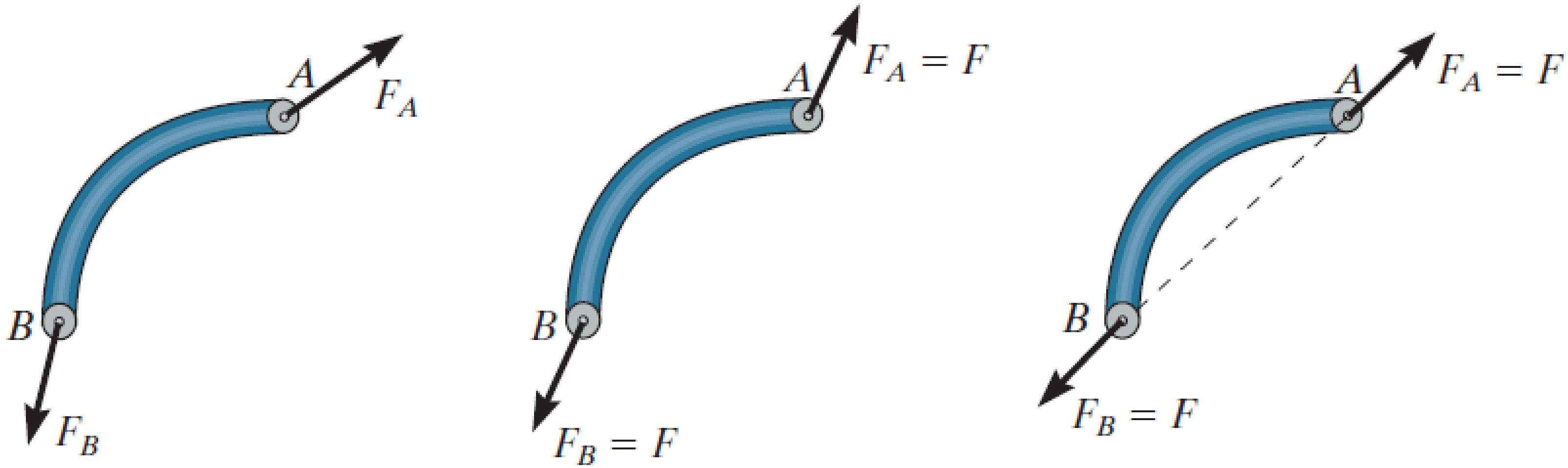
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_O = 0$$

Where point O is any arbitrary point.

Please note that these equations are the ones most commonly used for solving 2-D equilibrium problems. There are two other sets of equilibrium equations that are rarely used.

TWO-FORCE MEMBERS & THREE FORCE-MEMBERS

Two-Force Members



Therefore, for any two-force member to be in equilibrium, the two forces acting on the member *must have the same magnitude, act in opposite directions, and have the same line of action, directed along the line joining the two points where these forces act.*

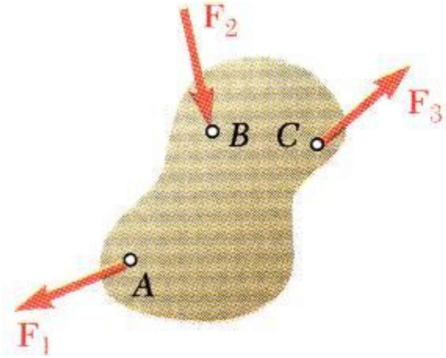
TWO-FORCE MEMBERS & THREE FORCE-MEMBERS

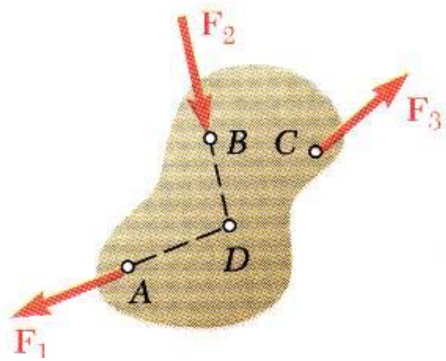
- Consider a rigid body subjected to forces acting at only 3 points.

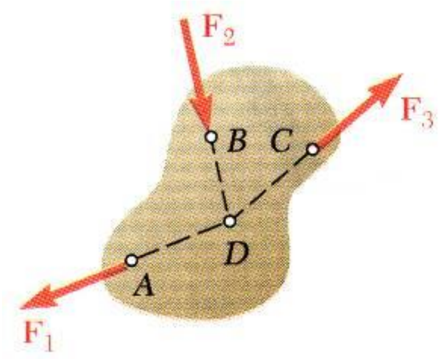
- Assuming that their lines of action intersect, the moment of $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ about the point of intersection represented by *D* is zero.

- Since the rigid body is in equilibrium, the sum of the moments of $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, and $\boldsymbol{F_3}$ about any axis must be zero. It follows that the moment of $\boldsymbol{F_3}$ about *D* must be zero as well and that the line of action of $\boldsymbol{F_3}$ must pass through *D*.

- ***The lines of action of the three forces must be concurrent.***



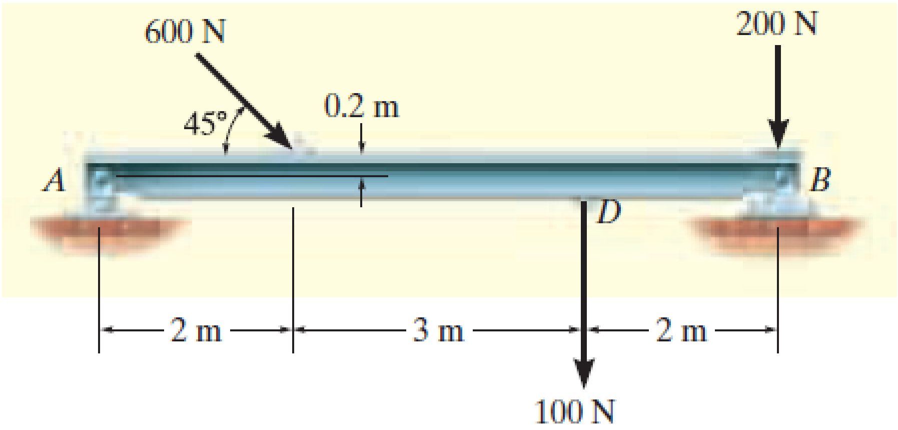




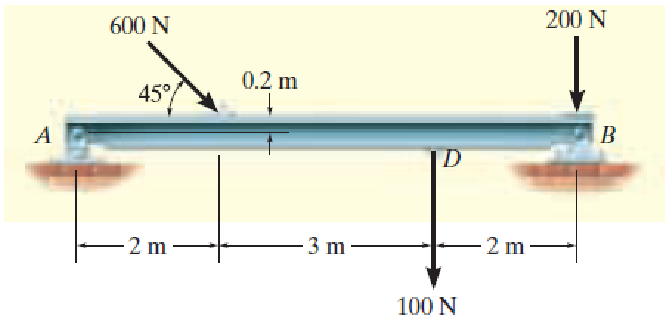
EQUATIONS OF EQUILIBRIUM

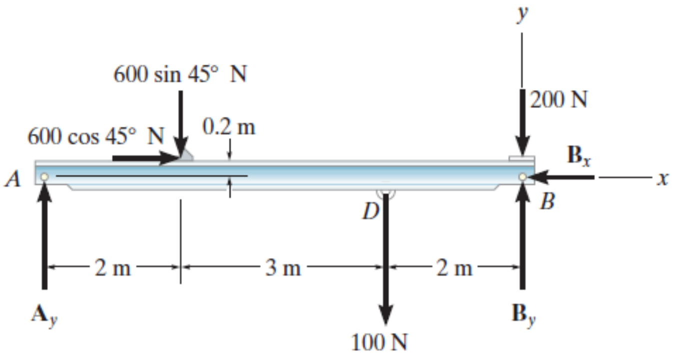
Example

Determine the horizontal and vertical components of reaction on the beam caused by the pin at *B* and the rocker at *A* as shown in Fig. 5–12*a*. Neglect the weight of the beam.



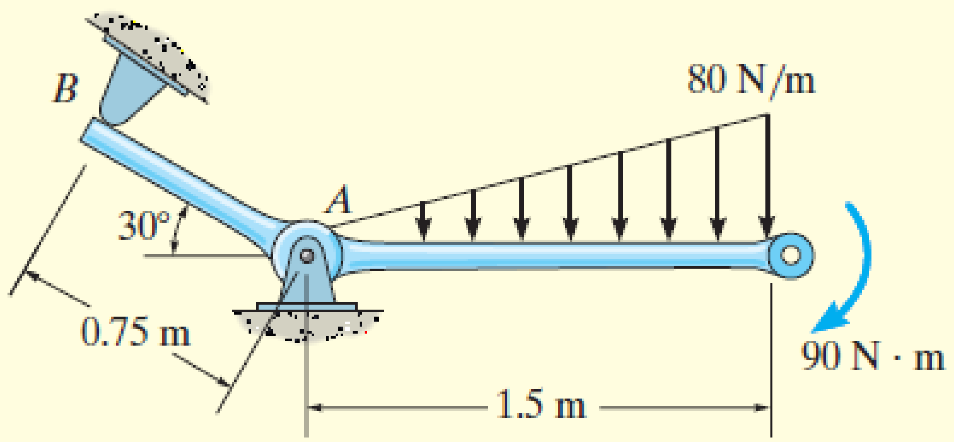
EQUATIONS OF EQUILIBRIUM
Example



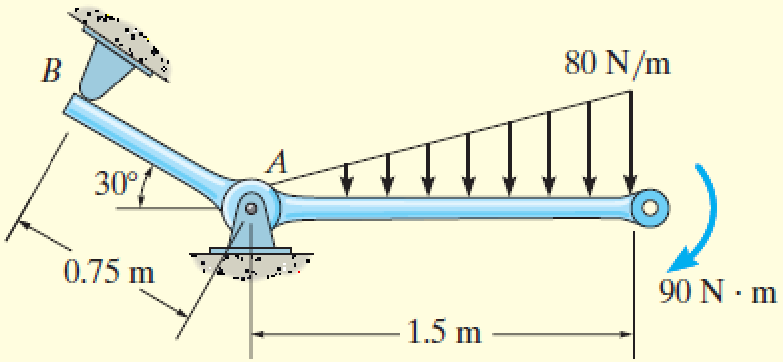


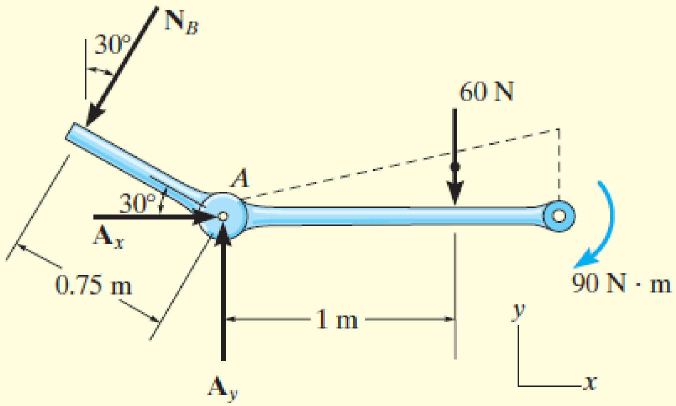
EQUATIONS OF EQUILIBRIUM
Example

The member shown in Fig. 5–14*a* is pin connected at *A* and rests against a smooth support at *B*. Determine the horizontal and vertical components of reaction at the pin *A*.



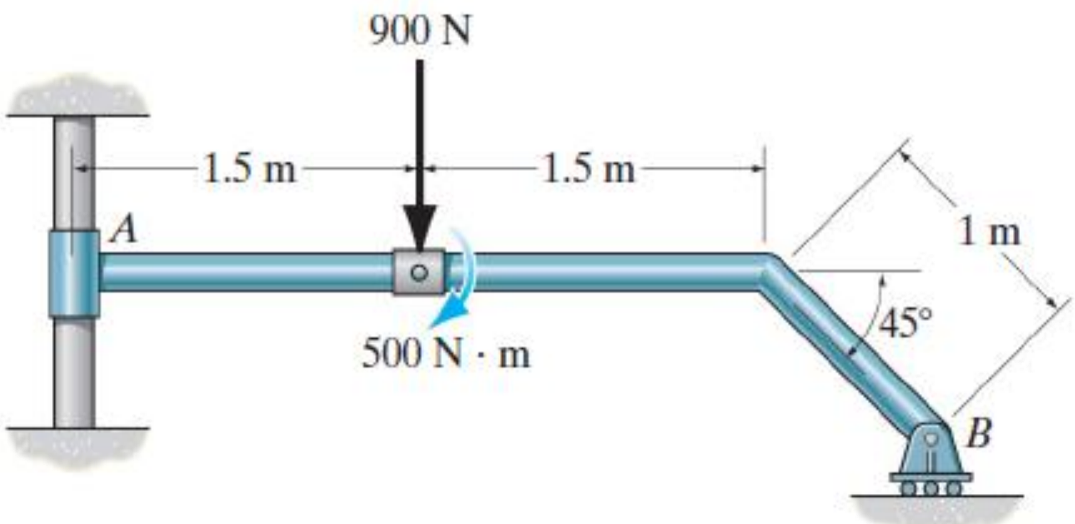
EQUATIONS OF EQUILIBRIUM
Example





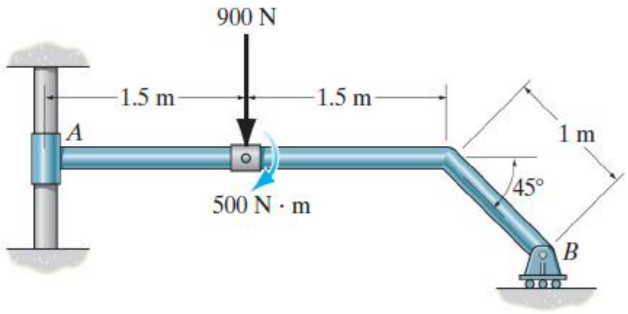
EQUATIONS OF EQUILIBRIUM
Example

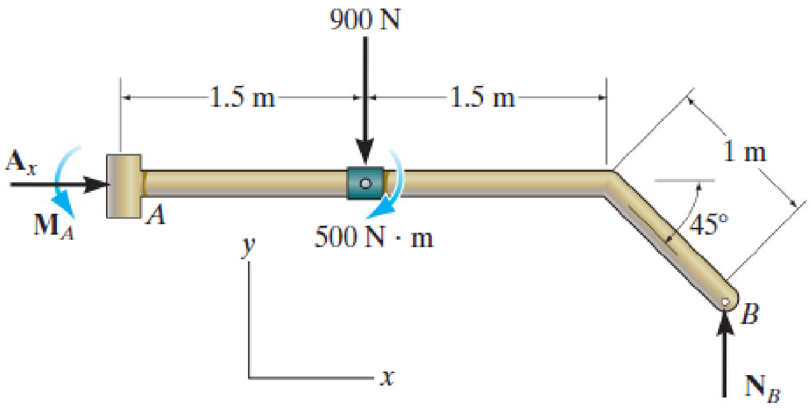
Determine the support reactions on the member in Fig. 5–19*a*. The collar at *A* is fixed to the member and can slide vertically along the vertical shaft.



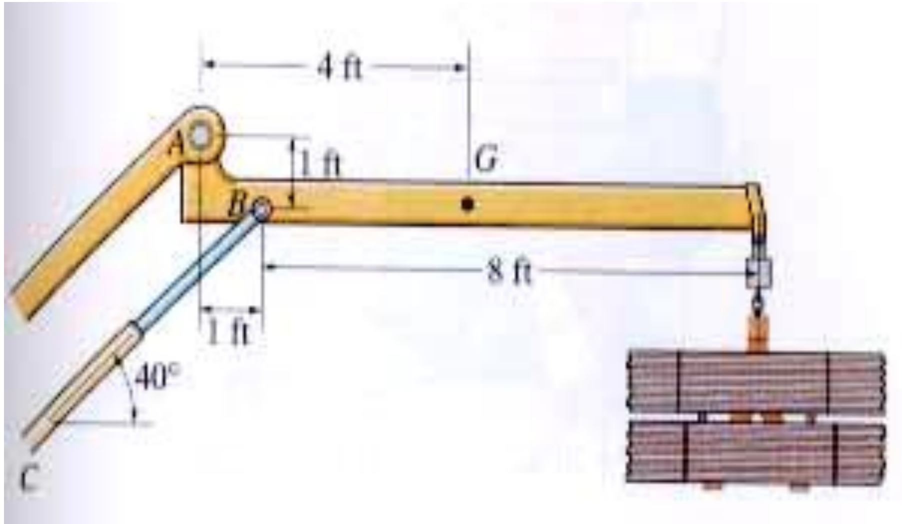
EQUATIONS OF EQUILIBRIUM

Example





EXAMPLE



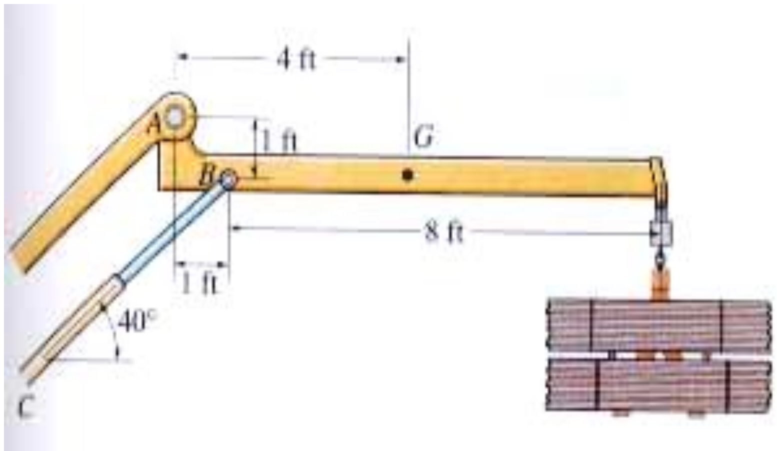
Given: Weight of the boom = 125 lb, the center of mass is at G, and the load = 600 lb.

Find: Support reactions at A and B.

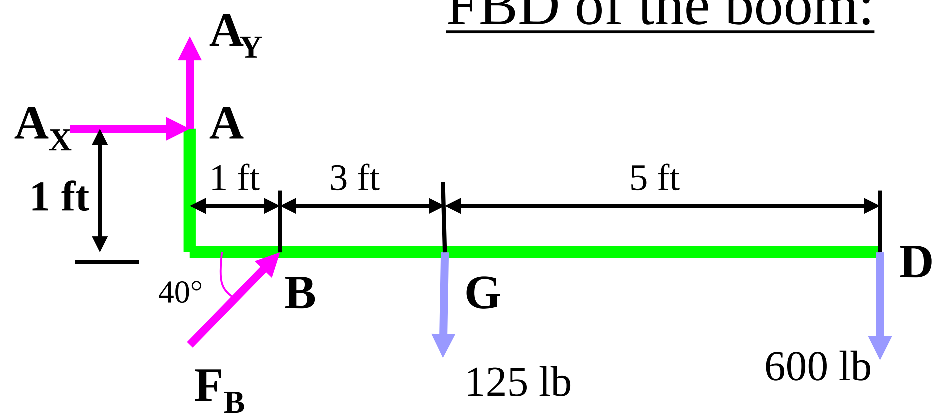
<u>Plan:</u>

1. Put the x and y axes in the horizontal and vertical directions, respectively.
2. Determine if there are any two-force members.
3. Draw a complete FBD of the boom.
4. Apply the E-of-E to solve for the unknowns.

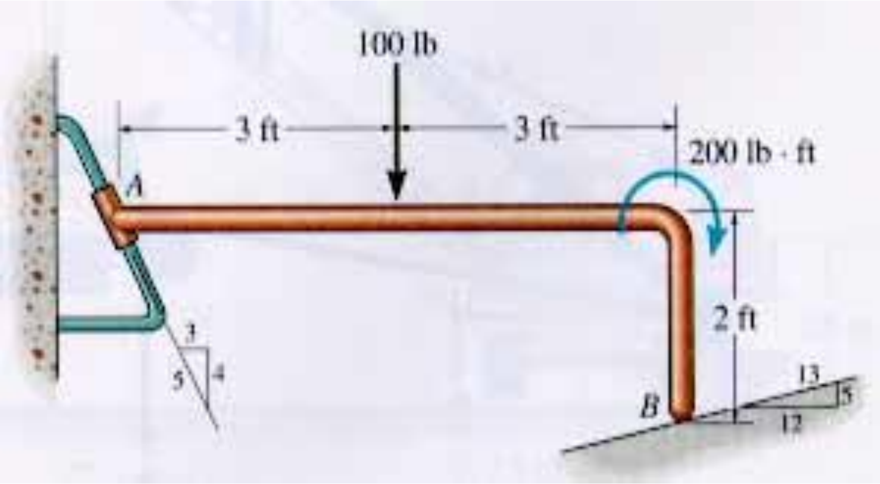
EXAMPLE
(Continued)



<u>FBD of the boom:</u>



<u>**Note:**</u> Upon recognizing CB as a two-force member, the number of unknowns at B are reduced from two to one. Now, using Eof E, we get,



Given: The load on the bent rod is supported by a smooth inclined surface at B and a collar at A. The collar is free to slide over the fixed inclined rod.

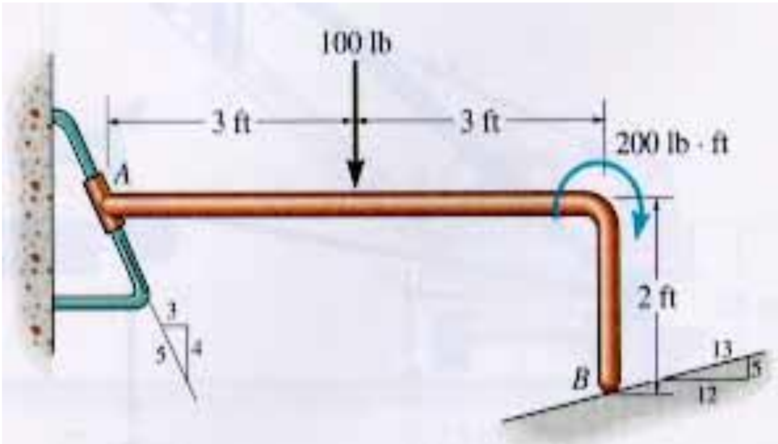
Find: Support reactions at A and B.

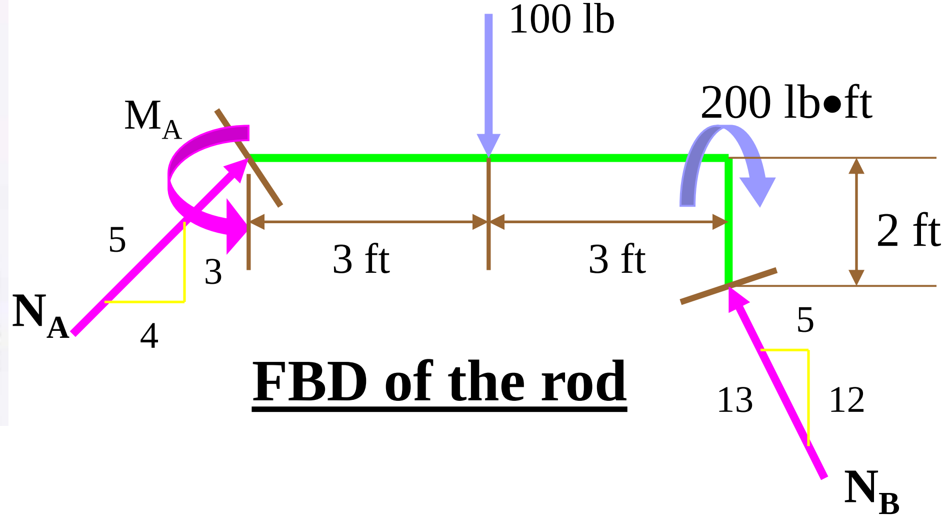
Plan:

a) Establish the x – y axes.

b) Draw a complete FBD of the bent rod.

c) Apply the E-of-E to solve for the unknowns.



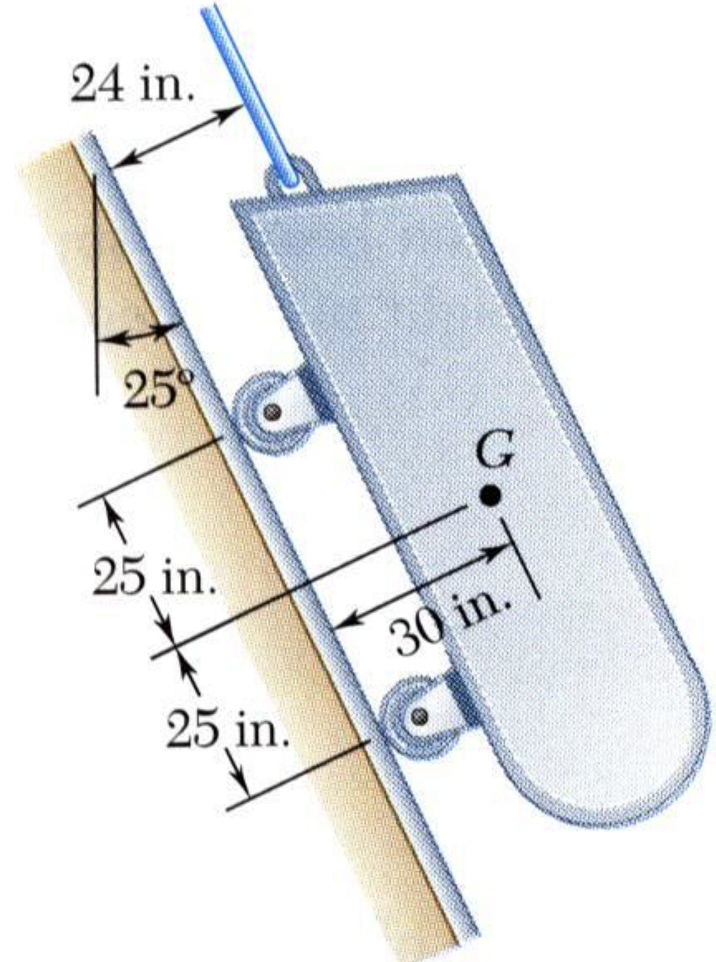


FBD of the rod

Sample Problem

A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb, and it is applied at at *G*. The cart is held in position by the cable.

Determine the tension in the cable and the reaction at each pair of wheels.

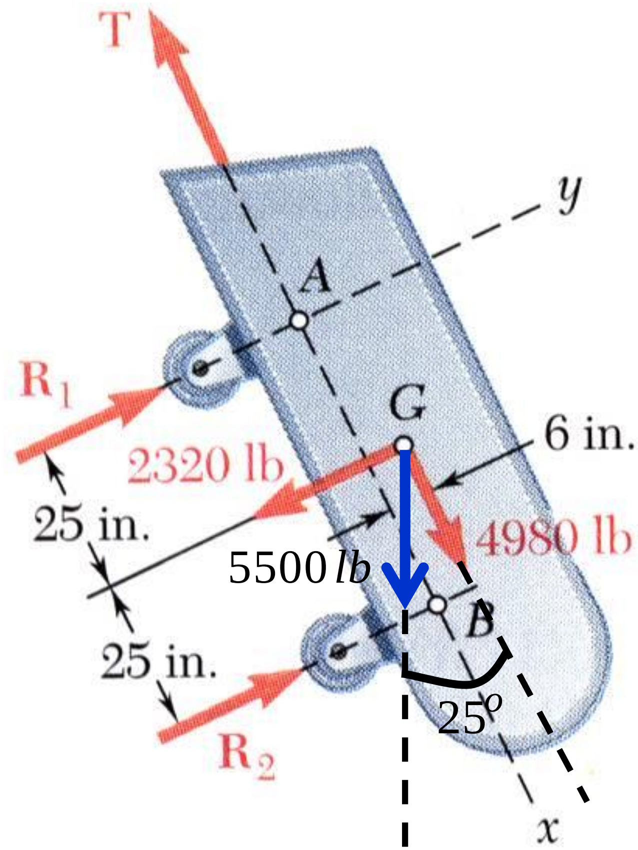


❑ Sample Problem

SOLUTION:

- Create a free-body diagram

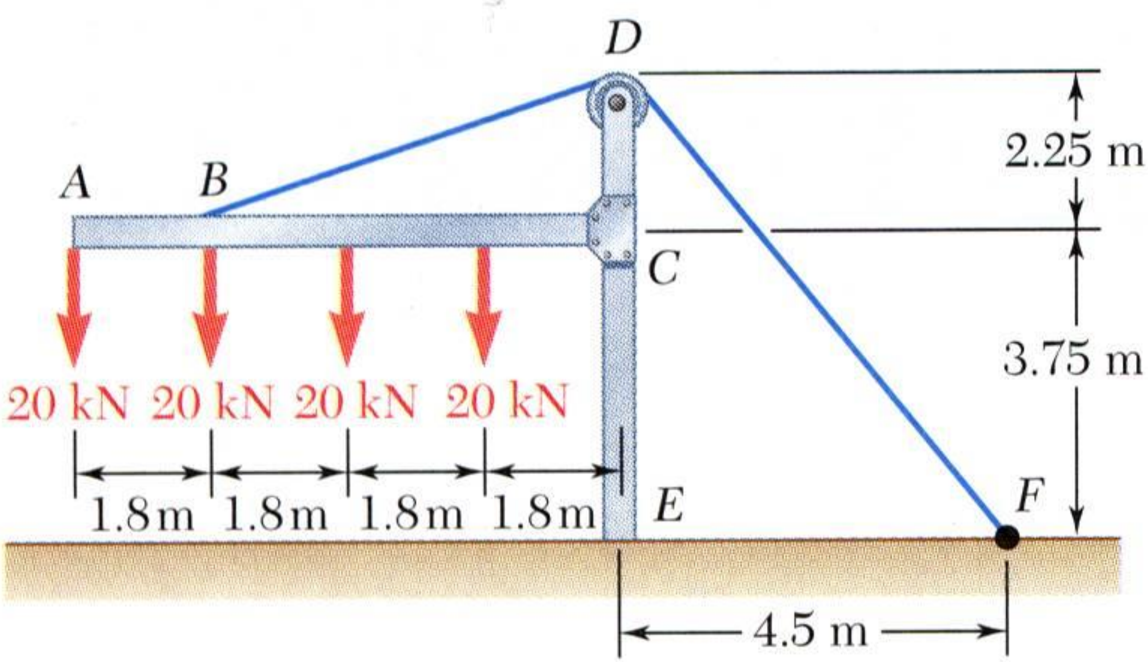
- Determine the reactions at the wheels.



❑ Sample Problem

The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

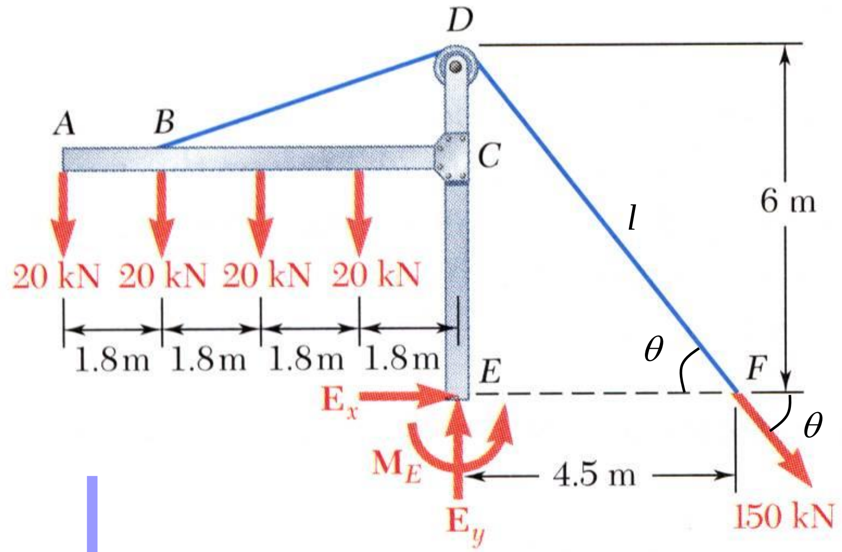
Determine the reaction at the fixed end *E*.



❑ Sample Problem 03

SOLUTION:

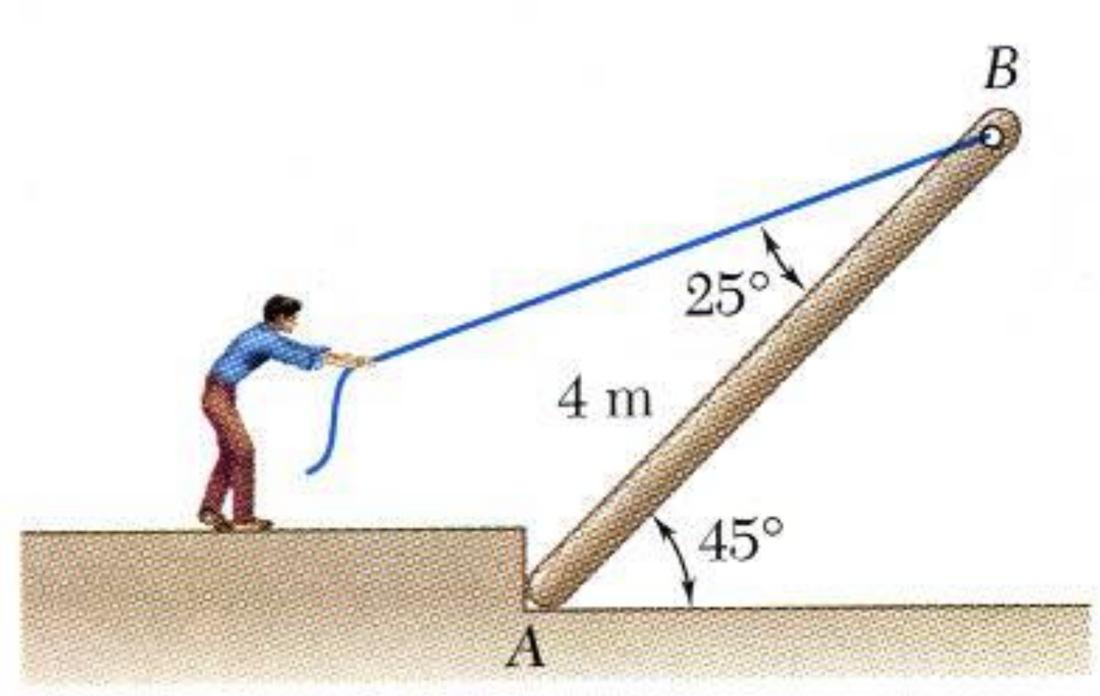
- Create a free-body diagram for the frame and cable.



❑ Sample Problem 04

A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

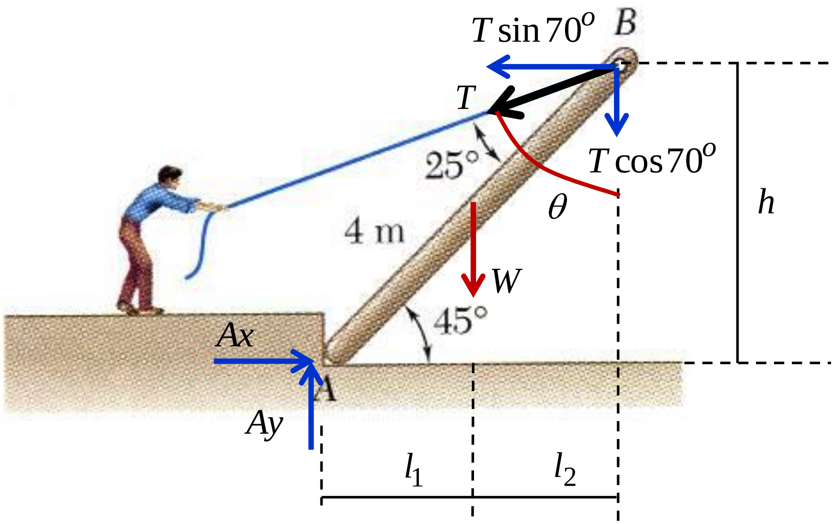
Find the tension in the rope and the reaction at *A*.



Sample Problem 04

SOLUTION:

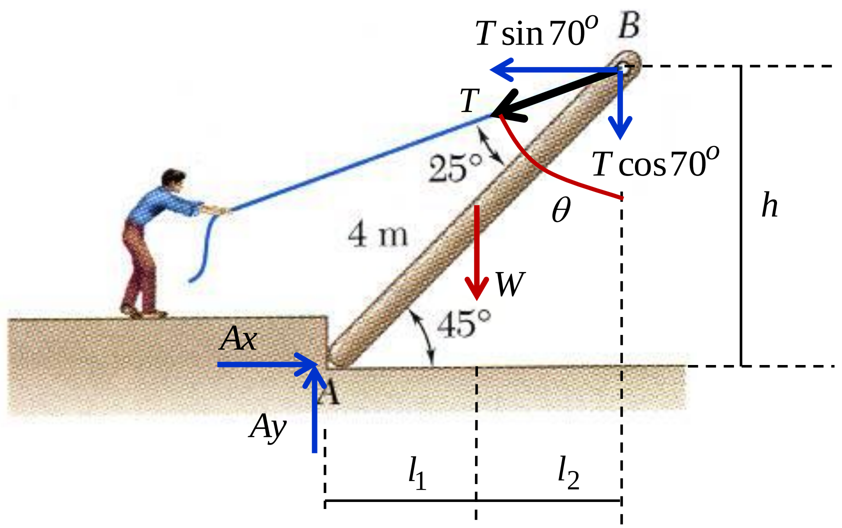
- Create a free-body diagram of the joist.

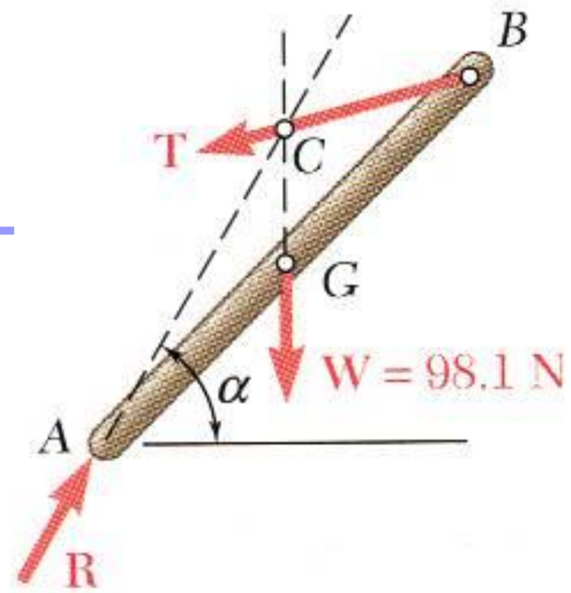


❑ Sample Problem 04

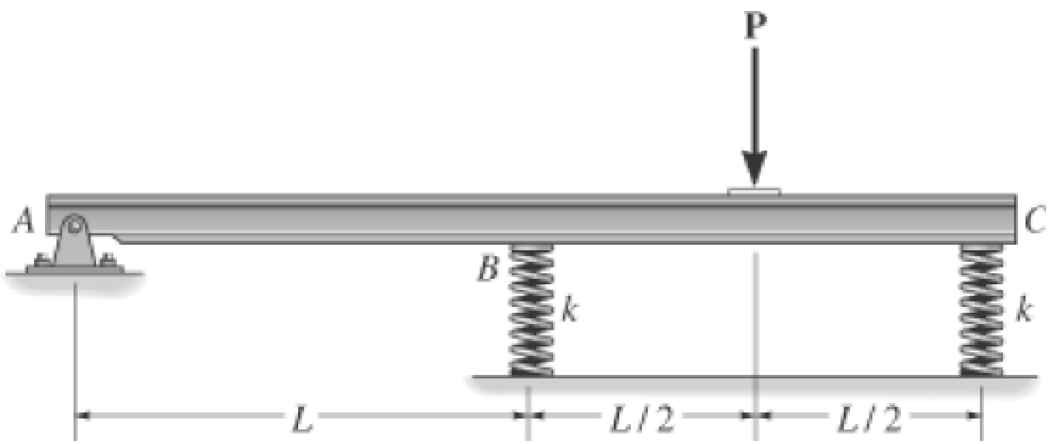
SOLUTION:

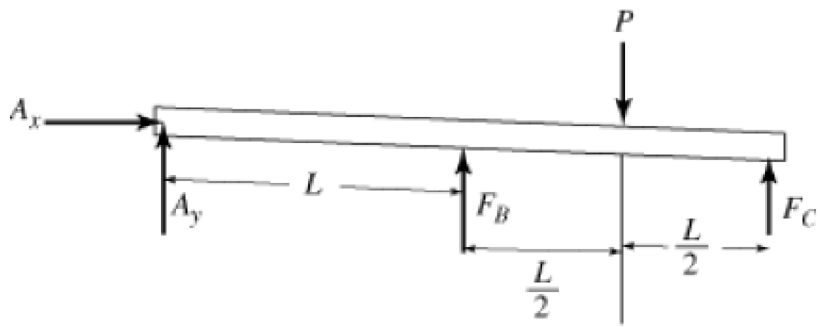
- Create a free-body diagram of the joist.

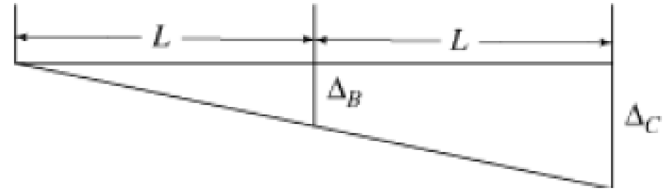




The rigid beam of negligible weight is supported horizontally by two springs and a pin. If the springs are uncompressed when the load is removed, determine the force in each spring when the load **P** is applied. Also, compute the vertical deflection of end *C*. Assume the spring stiffness *k* is large enough so that only small deflections occur. *Hint*: The beam rotates about *A* so the deflections in the springs can be related.




P
A_x
A_y
L
F_B
$\frac{L}{2}$
$\frac{L}{2}$
F_C


L
L
Δ_B
Δ_C

The horizontal beam is supported by springs at its ends. If the stiffness of the spring at A is k_A, determine the required stiffness of the spring at B so that if the beam is loaded with the force **F**, it remains in the horizontal position both before and after loading.

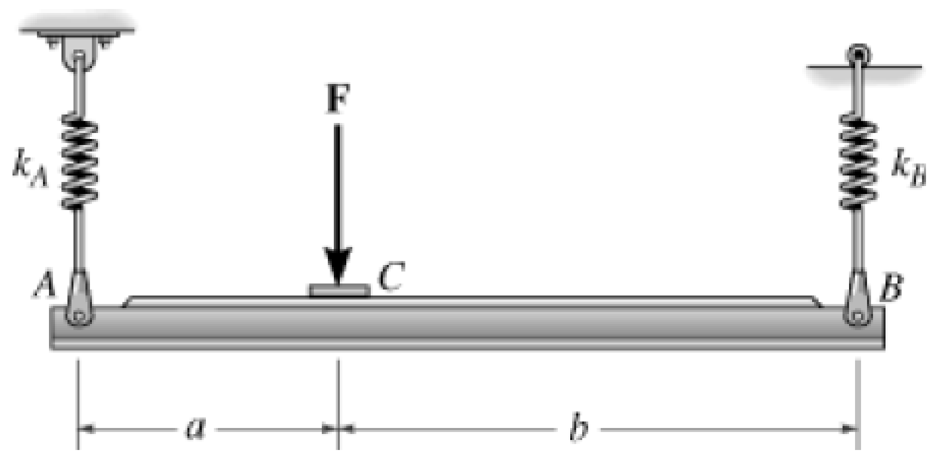
Units Used:

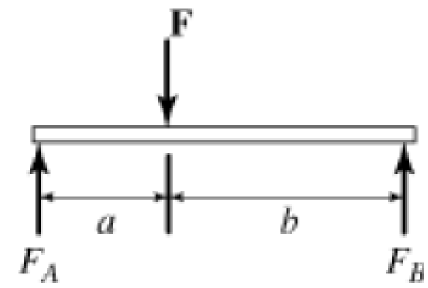
$$kN = 10^3 \ N$$

Given:

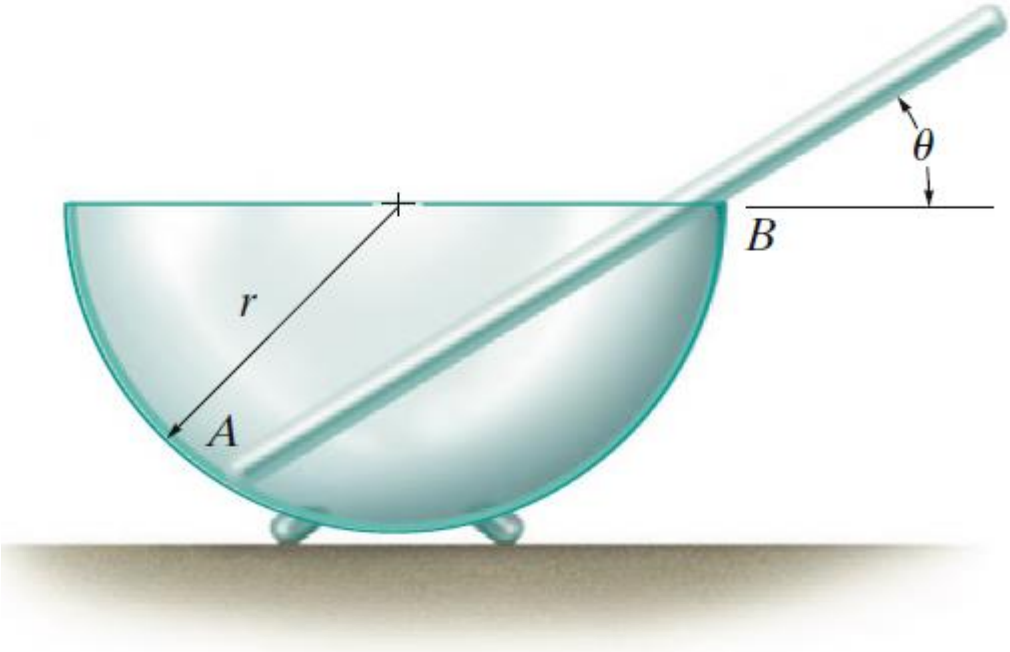
$$k_A = 5 \frac{kN}{m} \qquad a = 1 \ m$$

$$F = 800 \ N \qquad b = 2 \ m$$




F
a
b
F_A
F_B

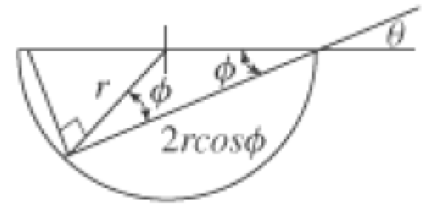
A uniform glass rod having a length *L* is placed in the smooth hemispherical bowl having a radius *r*. Determine the angle of inclination ө for equilibrium.



By observation $\phi = \theta$.

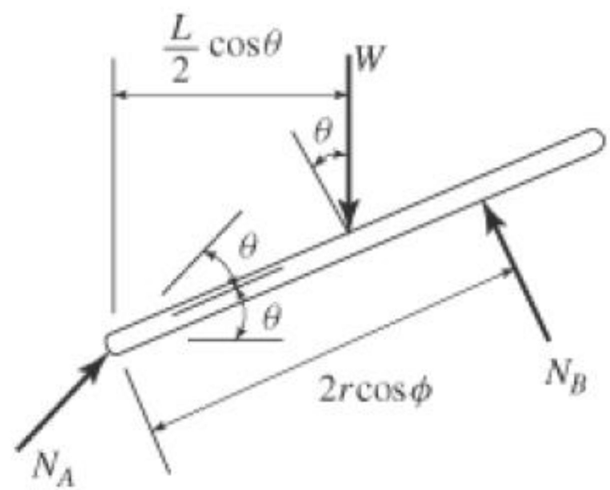
Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(2r\cos\theta) - W\left(\frac{L}{2}\cos\theta\right) = 0 \quad N_B = \frac{WL}{4r}$$



$$+\nearrow \Sigma F_x = 0; \quad N_A\cos\theta - W\sin\theta = 0 \quad N_A = W\tan\theta$$

$$+\nwarrow \Sigma F_y = 0; \quad (W\tan\theta)\sin\theta + \frac{WL}{4r} - W\cos\theta = 0$$



$$\sin^2\theta - \cos^2\theta + \frac{L}{4r}\cos\theta = 0$$

$$(1 - \cos^2\theta) - \cos^2\theta + \frac{L}{4r}\cos\theta = 0$$

$$2\cos^2\theta - \frac{L}{4r}\cos\theta - 1 = 0$$

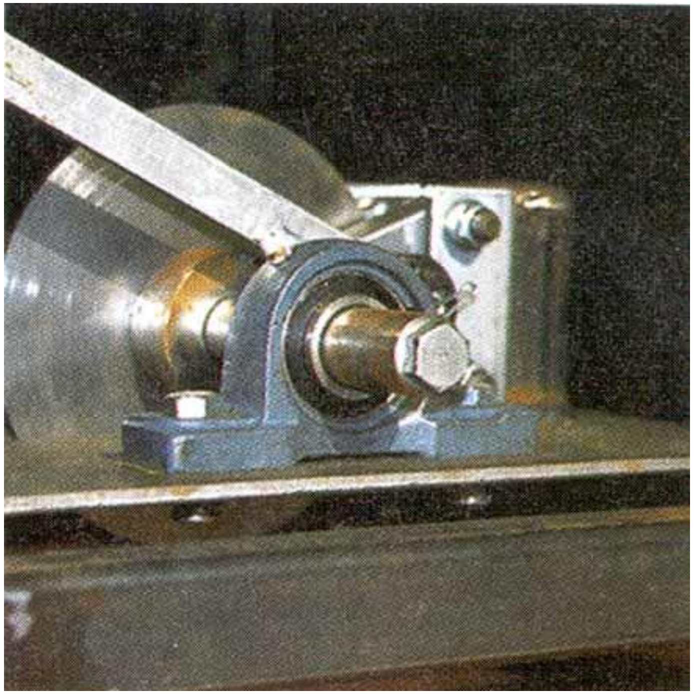
$$\cos\theta = \frac{L \pm \sqrt{L^2 + 128r^2}}{16r}$$

Take the positive root

$$\cos\theta = \frac{L + \sqrt{L^2 + 128r^2}}{16r}$$

$$\theta = \cos^{-1}\left(\frac{L + \sqrt{L^2 + 128r^2}}{16r}\right)$$

FREE-BODY DIAGRAMS, EQUATIONS OF EQUILIBRIUM & CONSTRAINTS FOR A RIGID BODY



Ball-and-socket joints and journal bearings are often used in mechanical systems.

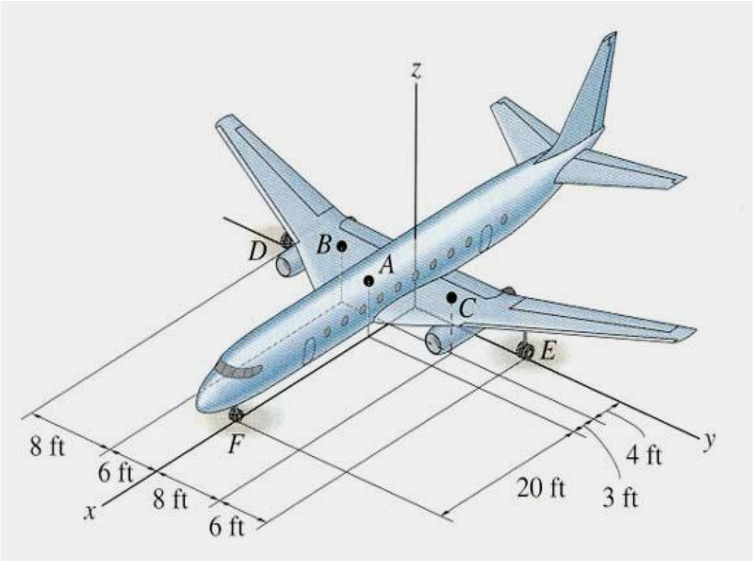
How can we determine the support reactions at these joints for a given loading?

FREE-BODY DIAGRAMS, EQUATIONS OF EQUILIBRIUM & CONSTRAINTS FOR A RIGID BODY

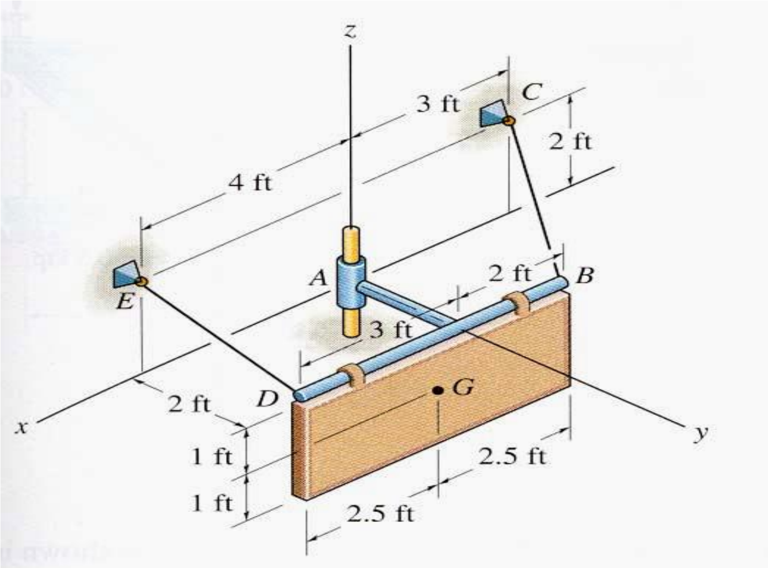
Ball-and-socket joints

journal bearings

APPLICATIONS (continued)

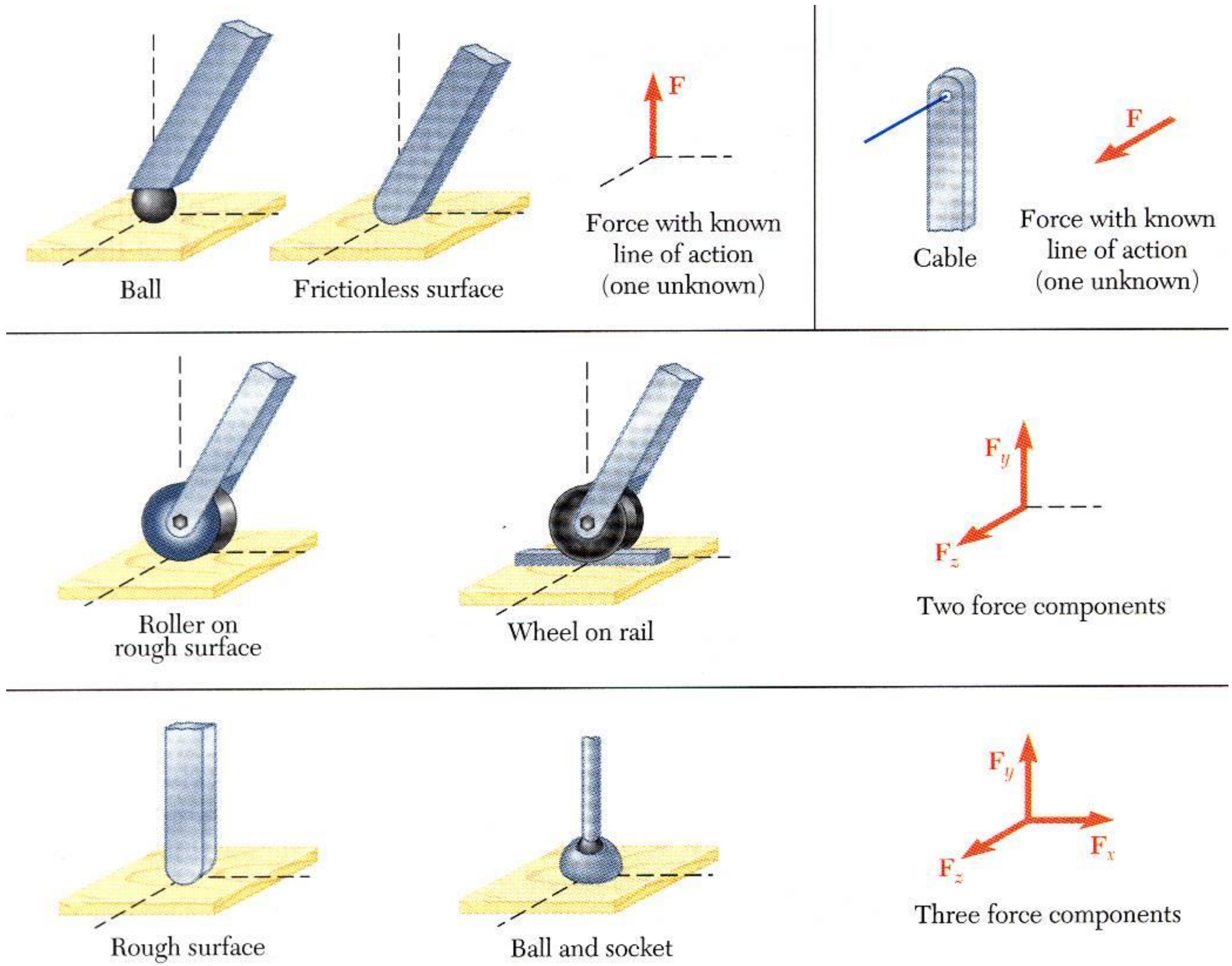


The weights of the fuselage and fuel act through A, B, and C. How will we determine the reactions at the wheels D, E and F ?

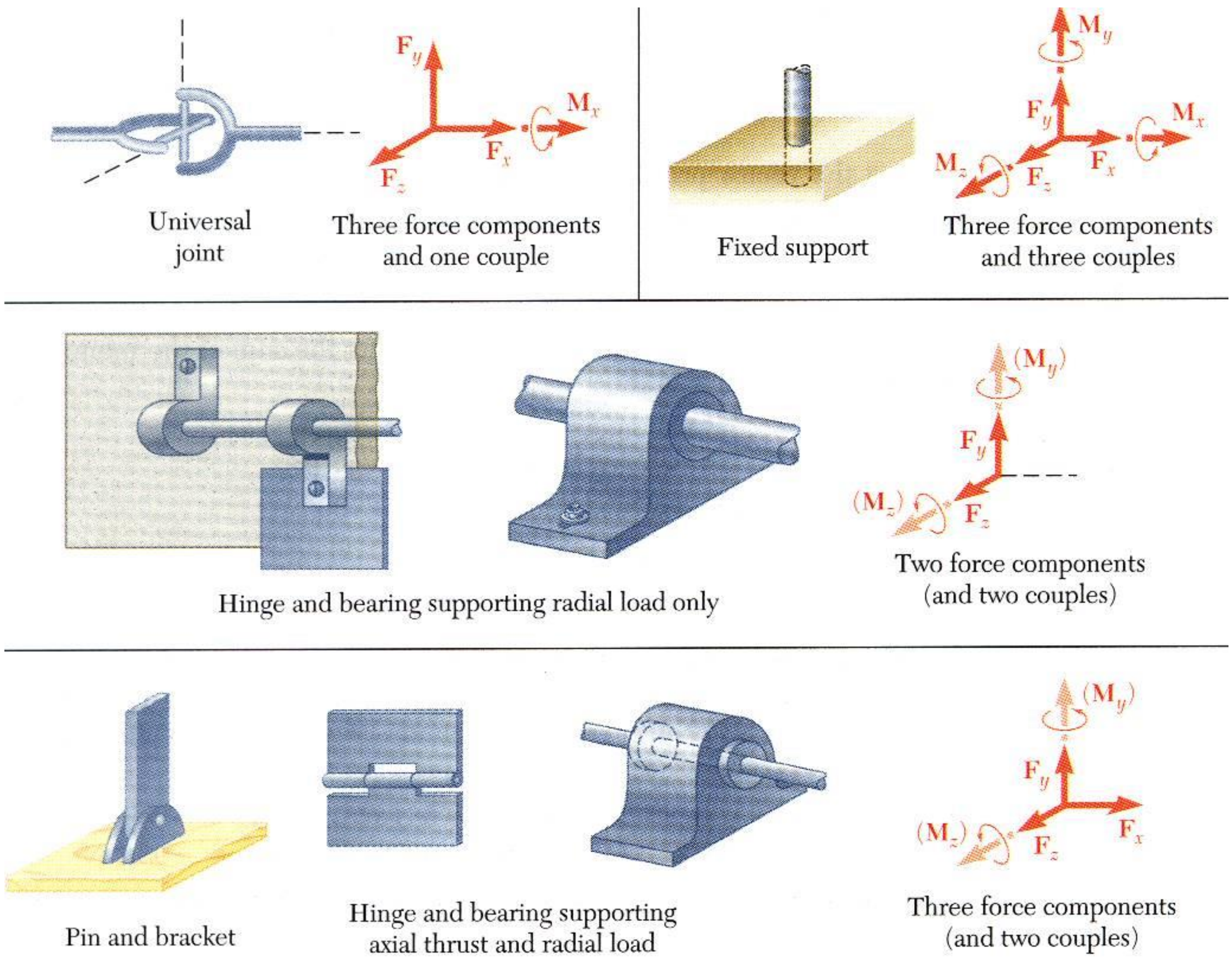


A 50 lb sign is kept in equilibrium using two cables and a smooth collar. How can we determine the reactions at these supports?

❑ Reactions at Supports and Connections for a Three-Dimensional Structure



❑ Reactions at Supports and Connections for a Three-Dimensional Structure



EQUATIONS OF EQUILIBRIUM
(Section 5.6)

As stated earlier, when a body is in equilibrium, the net force and the net moment equal zero, i.e., $\sum \boldsymbol{F} = 0$ and $\sum \boldsymbol{M_O} = 0$.

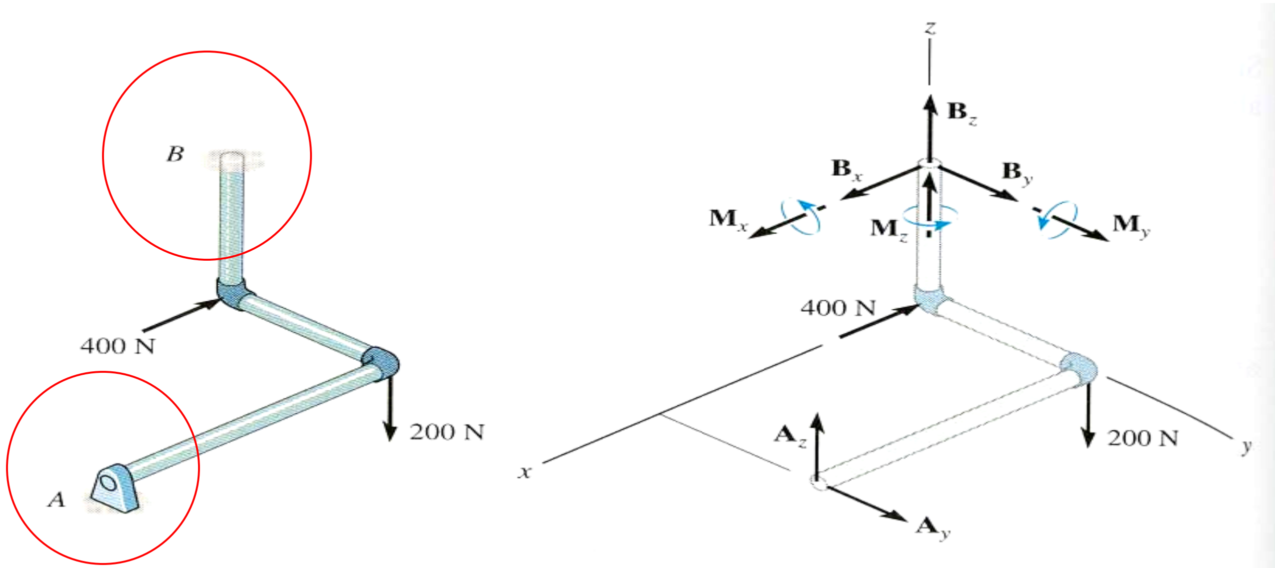
These two vector equations can be written as six scalar equations of equilibrium (EofE). These are

$$\sum F_X = \sum F_Y = \sum F_Z = 0$$

$$\sum M_X = \sum M_Y = \sum M_Z = 0$$

The moment equations can be determined about any point. Usually, choosing the point where the maximum number of unknown forces are present simplifies the solution. Those forces do not appear in the moment equation since they pass through the point. Thus, they do not appear in the equation.

CONSTRAINTS FOR A RIGID BODY



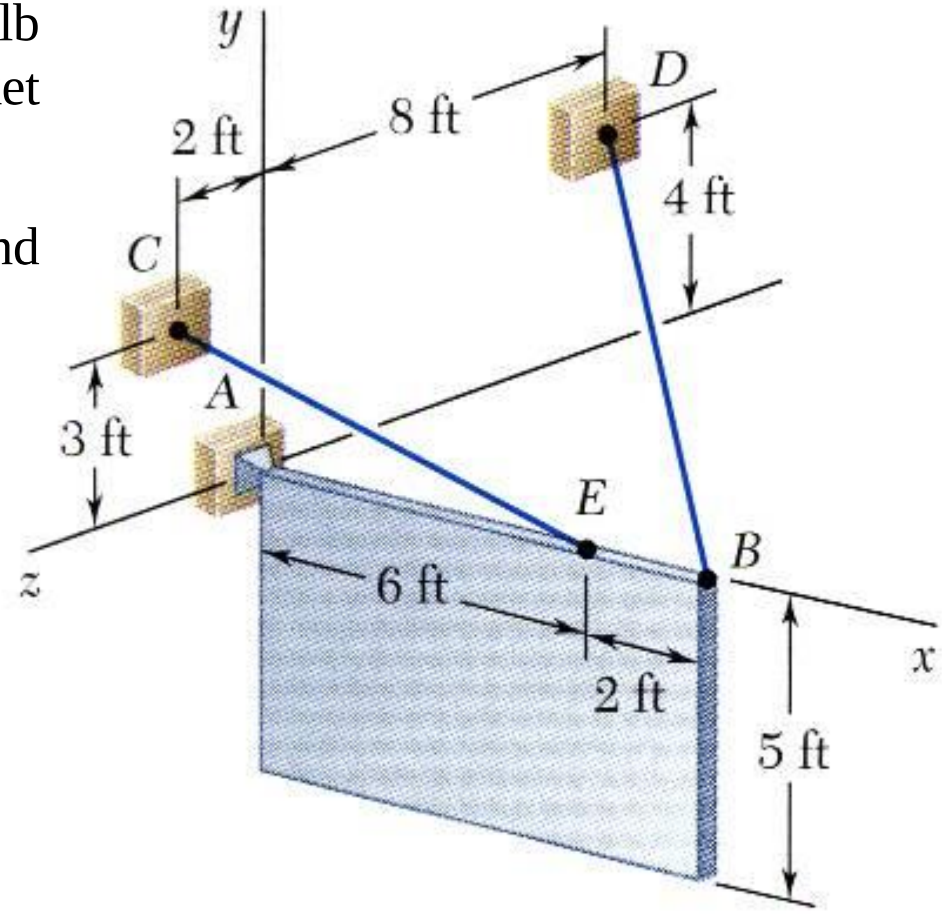
Redundant Constraints: When a body has more supports than necessary to hold it in equilibrium, it becomes statically indeterminate.

A problem that is statically indeterminate has more unknowns than equations of equilibrium.

❑ Sample Problem

A sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at *A* and by two cables.

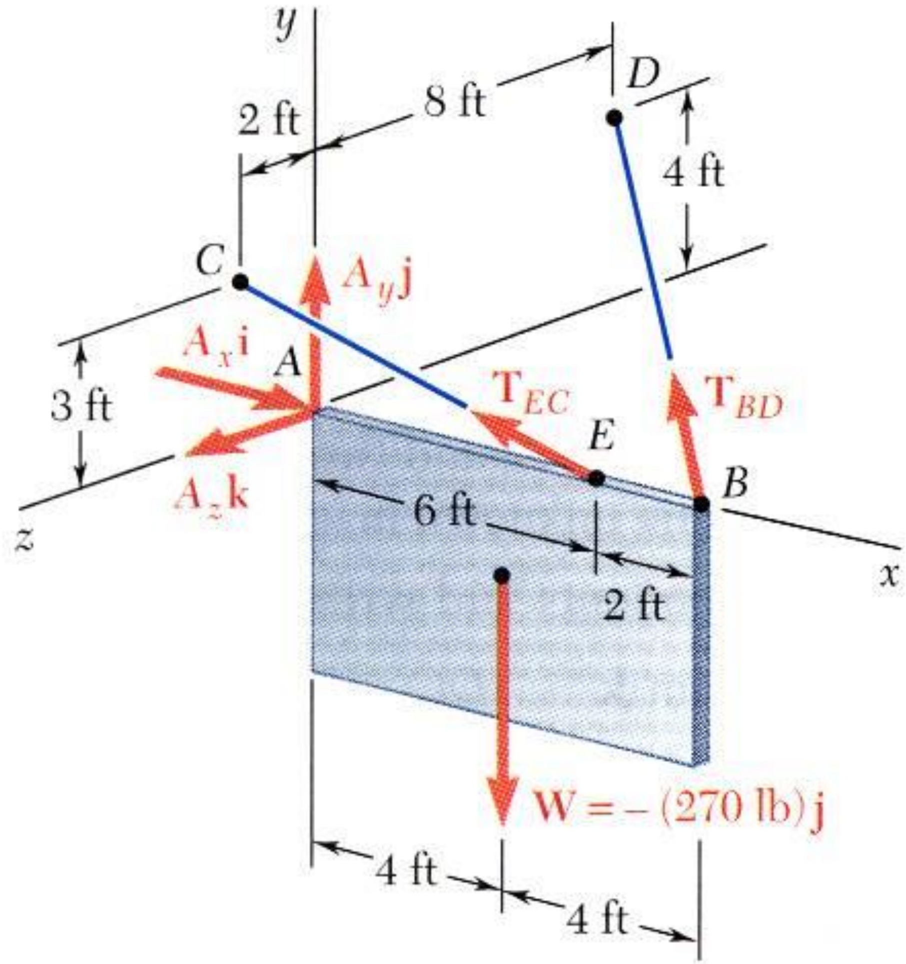
Determine the tension in each cable and the reaction at *A*.



❑ Sample Problem

SOLUTION:

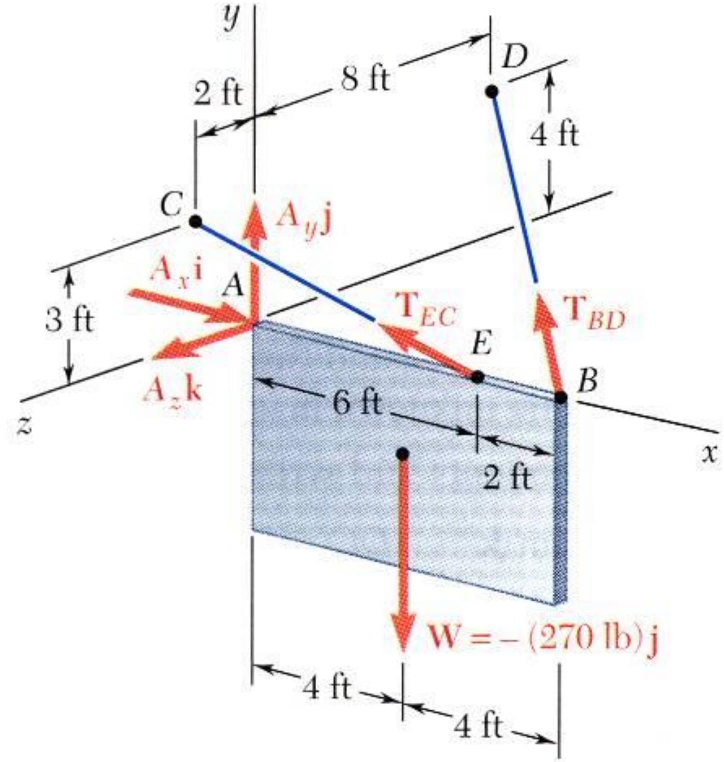
- Create a free-body diagram for the sign.



Sample Problem

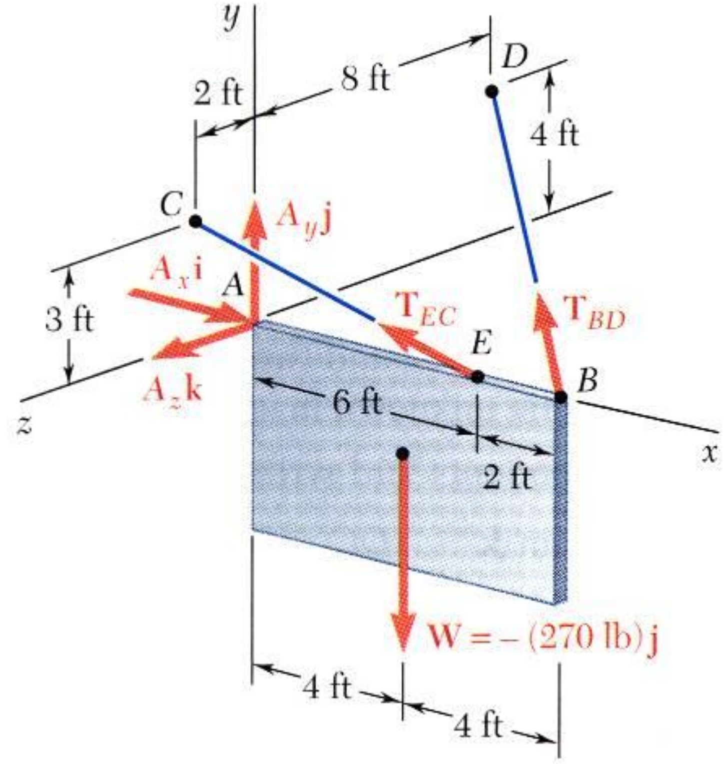
SOLUTION:

- Apply the conditions for static equilibrium to develop equations for the unknown reactions.



❑ Sample Problem

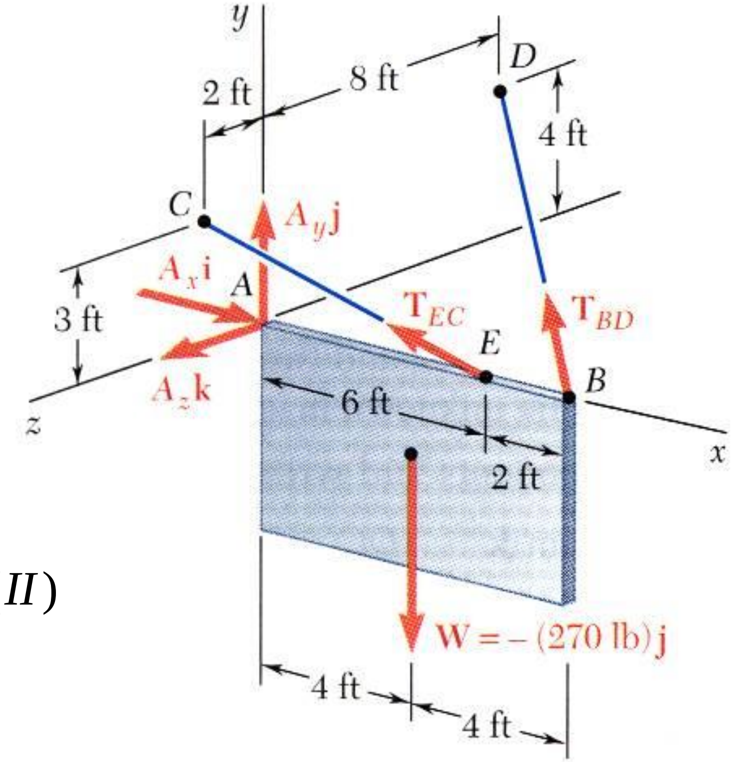
SOLUTION:



❑ Sample Problem

SOLUTION:

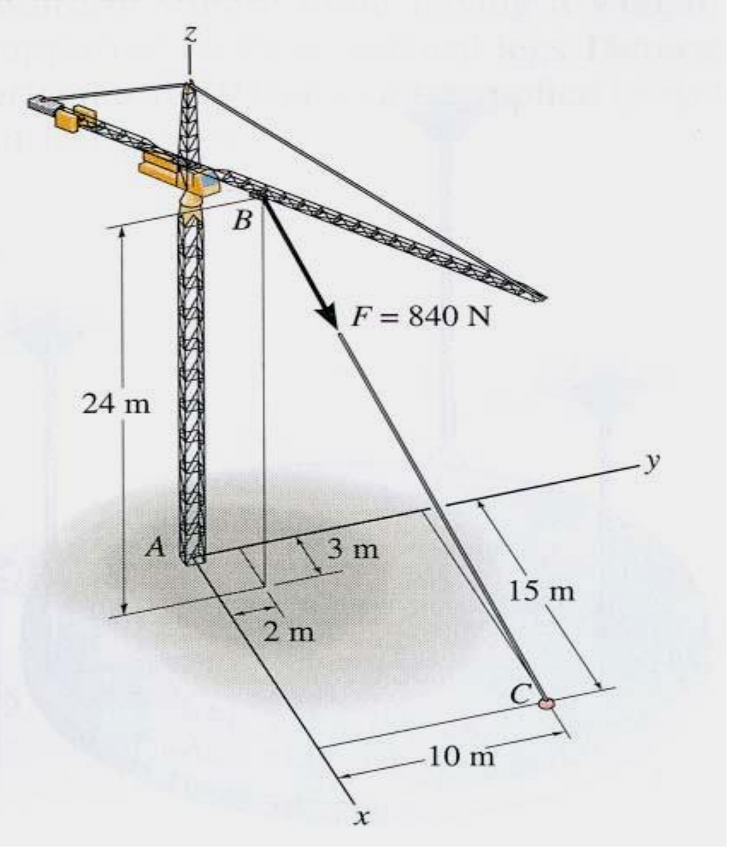
(II)



$(I)\,\&\,(II) \quad \Rightarrow$ Solve the 5 equations for the 5 unknowns,

Given: The cable of the tower crane is subjected to 840 N force. A fixed base at A supports the crane.

Find: Reactions at the fixed base A.



Plan:

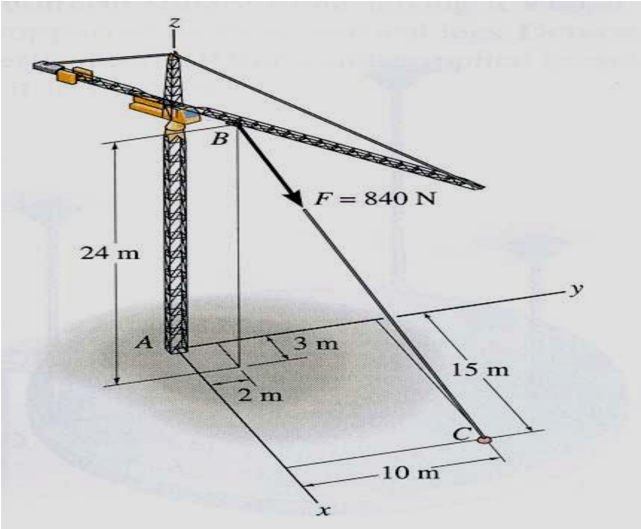
a) Establish the x, y and z axes.

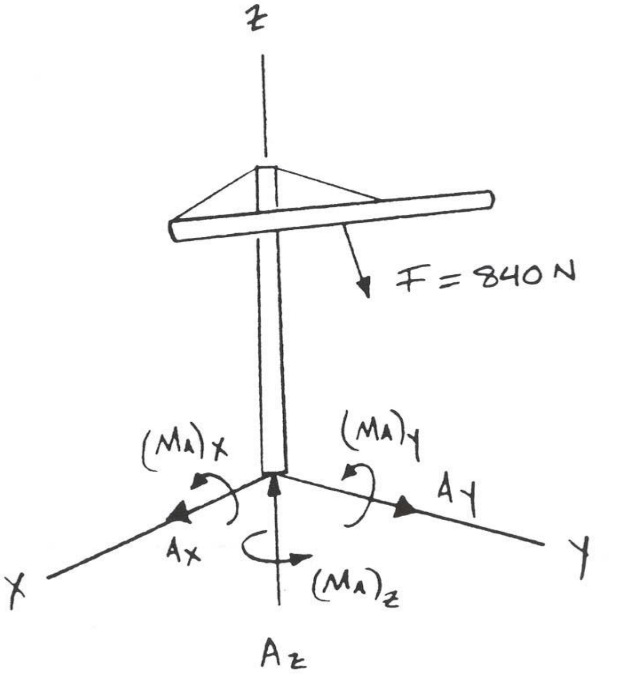
b) Draw a FBD of the crane.

c) Write the forces using Cartesian vector notation.

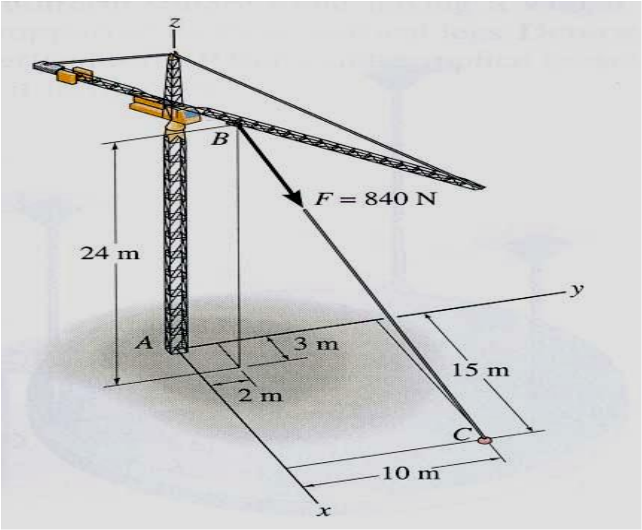
d) Apply the equations of equilibrium (vector version) to solve for the unknown forces.

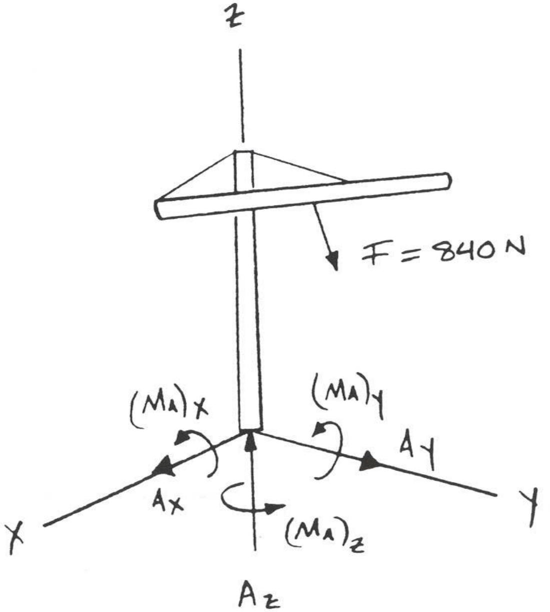
EXAMPLE
(continued)





EXAMPLE
(continued)





EXAMPLE
(continued)