



University of Kurdistan
Department of Electrical Engineering

Linear Control Systems

Homework 4: Routh-Hurwitz Criterion, Stability in State Space

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1. Routh-Hurwitz Criterion

1.1 Using the Routh-Hurwitz criterion, determine the stability of the following polynomials.

a) $s^4 + 2s^3 + 4s^2 + 5s + 2$

b) $s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$

c) $s^6 + s^5 + 8s^4 + 6s^3 + 20s^2 + 8s + 16$

1.2 A feedback control system has the following characteristic equation, discuss the location of the system poles in the s -plane.

$$s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$$

1.3 For the unity feedback system shown in Figure 1.1, find the following:

- The range of K that keeps the system stable
- The value of K that makes the system oscillate
- The frequency of oscillation when K is set to the value that makes the system oscillate.

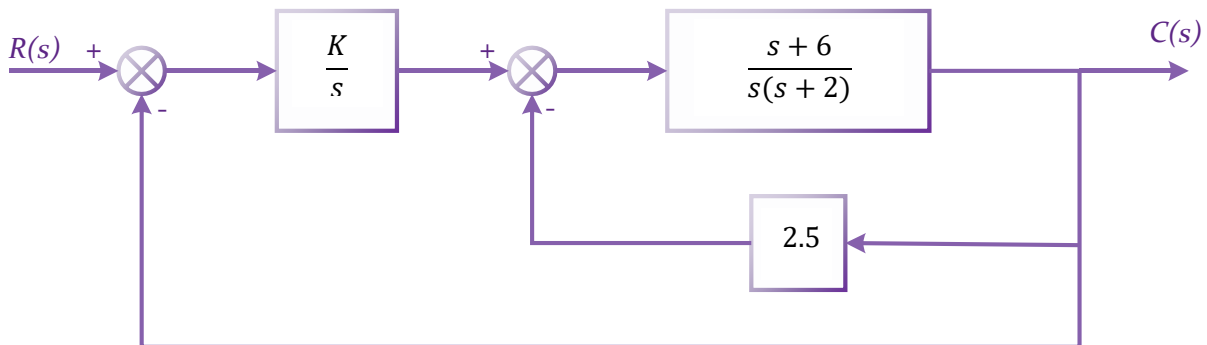


Figure 1. 1

1.4 Consider the control system of Figure 1.2. Find the range of k that will locate only two poles of the closed-loop system in the interval $-2 < \text{Re}(s) < 0$.

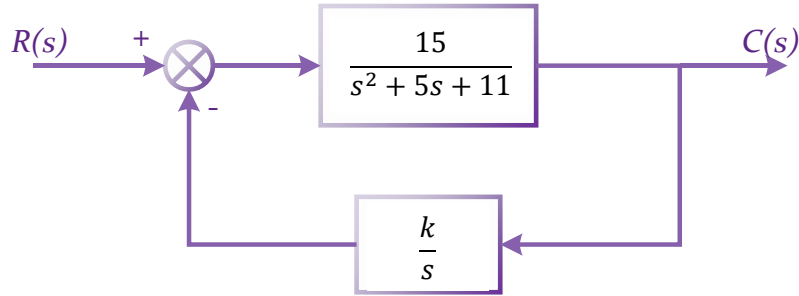


Figure 1. 2

1.5 Consider the system

$$\dot{x} = Ax$$

where matrix A is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -b_3 & 0 & 1 \\ 0 & -b_2 & -b_1 \end{bmatrix}$$

(A is called Schwarz matrix.) Show that the first column of the Routh's array of the characteristic equation $|sI - A| = 0$ consists of 1, b_1 , b_2 , and $b_1 b_3$.

1.6 A control system is shown in Figure 1.3. we want the system to be stable and the steady-state error for a unit step input to be less than or equal to 0.05 (5%).

- Determine the range of α that satisfies the error requirement.
- Determine the range of α that satisfies the stability requirement.
- Select an α that meets both requirements.

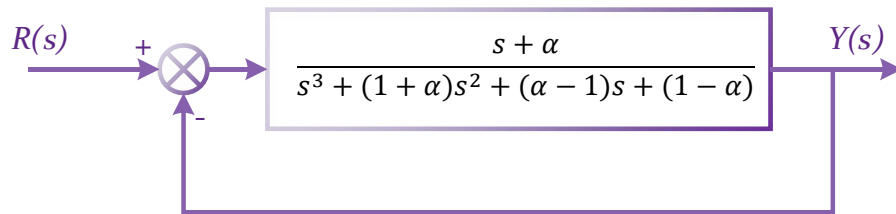


Figure 1. 3

2. Stability in State Space

2.1 A system is represented in state space as

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -14 & -10 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} u$$

$$y = [1 \quad 1 \quad 0] \mathbf{x}$$

Determine how many eigenvalues are in the right half-plane, in the left half-plane, and on $j\omega$ -axis.

- 2.2 The forward path transfer function of a negative unity feedback system is represented in state variable form as follows. Use Routh's stability criterion to determine if the closed-loop system is stable.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -8 & -12 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = [0 \quad 1 \quad 1] \mathbf{x}$$

3. MATLAB Simulation

- 3.1 Use MATLAB and the symbolic math toolbox to generate a Routh table and tell how many poles of the following function are in the rhp, in the lhp, and on $j\omega$ -axis.

$$T(s) = \frac{1}{s^5 + 2s^4 + 2s^3 + 4s^2 + s + 2}$$

- 3.2 Repeat problem 2.2 using MATLAB.
- 3.3 Develop an m-file to check the stability of an LTI system. (hint: program gets the characteristic equation of the system's transfer function and uses the Routh's stability criterion to check the stability and prints "system is stable" for stable systems and "system is not stable" for instable systems.)

"Great things can be done by great sacrifices only."

•Swami Vivekananda•