Chapter 6:
Structural Analysis

SIMPLE TRUSSES

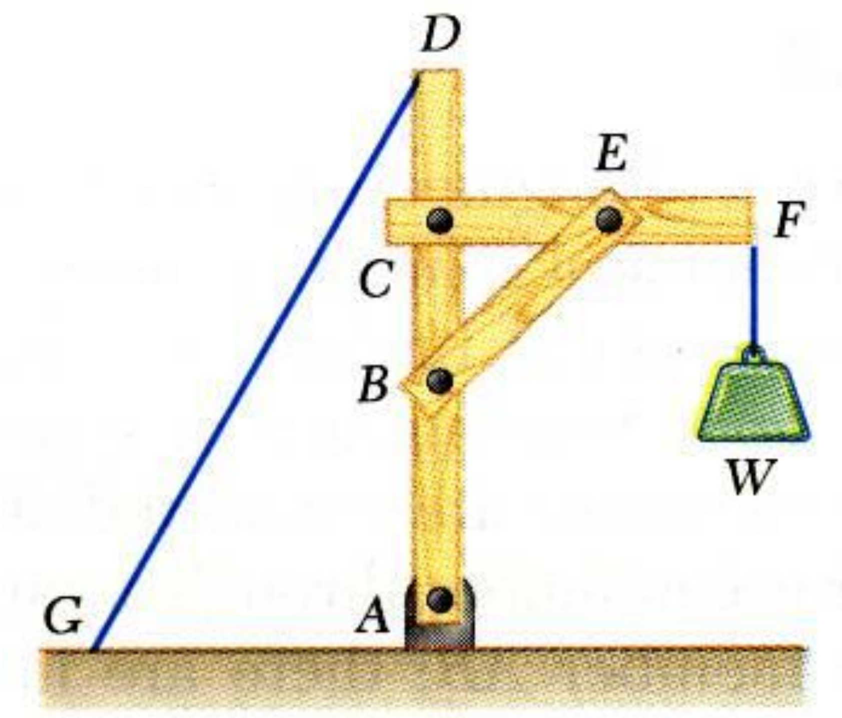
Trusses are commonly used to support a roof.

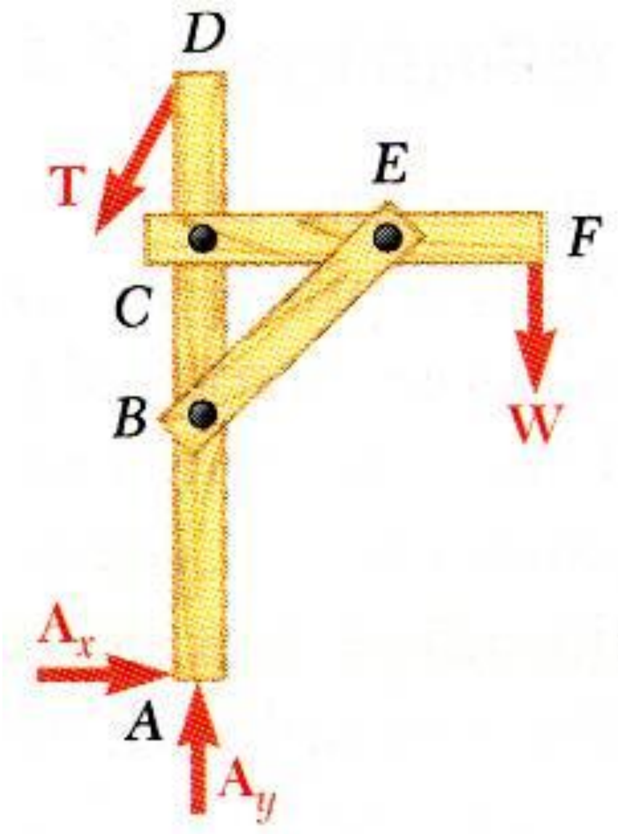
For a given truss geometry and load, how can we determine the forces in the truss members and select their sizes?

A more challenging question is that for a given load, how can we design the trusses’ geometry to minimize cost?

❑ Introduction

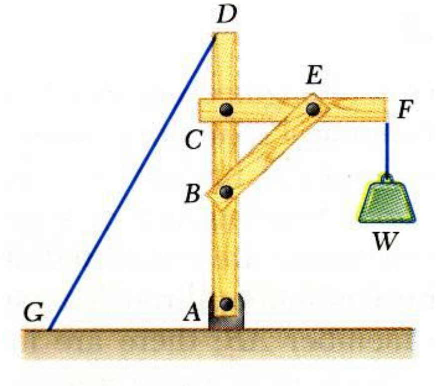
- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.

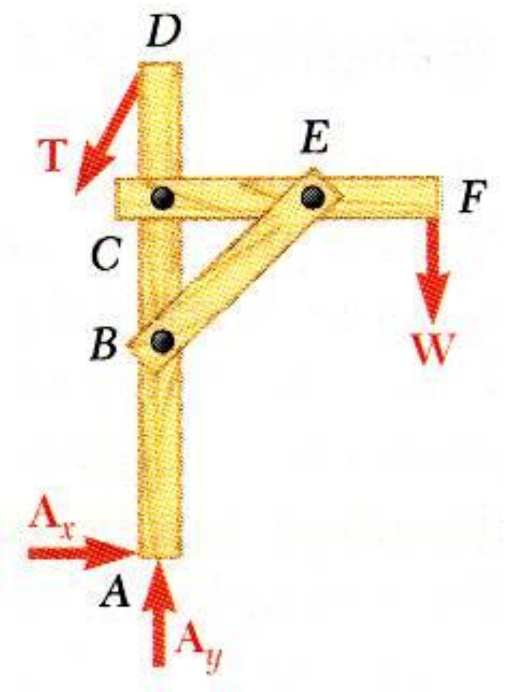


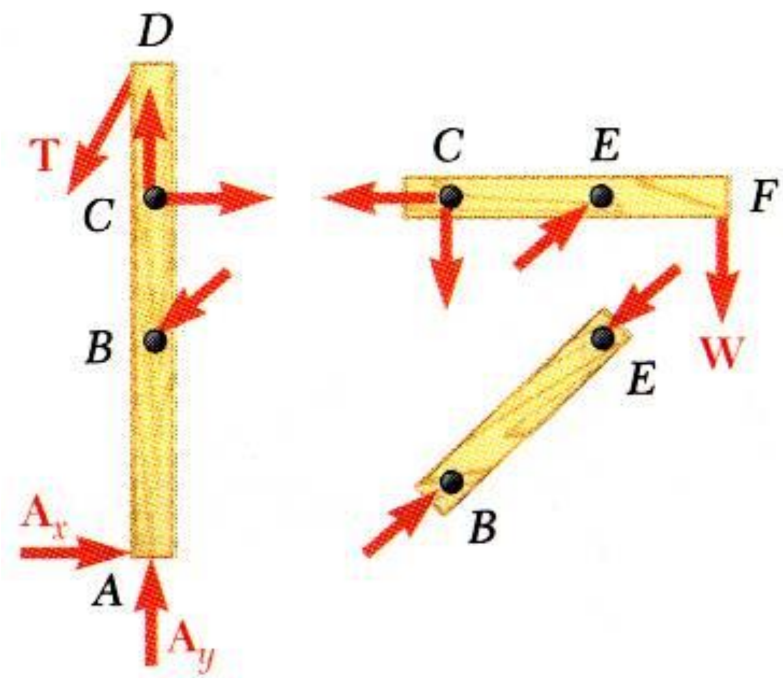


Introduction

- In the interaction between connected parts, Newton's 3rd Law states that ***the forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense.***

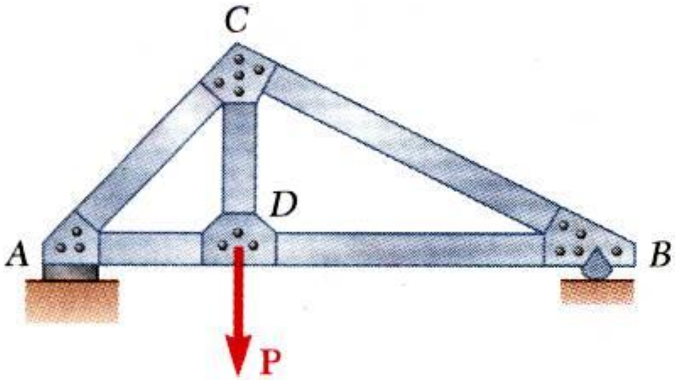




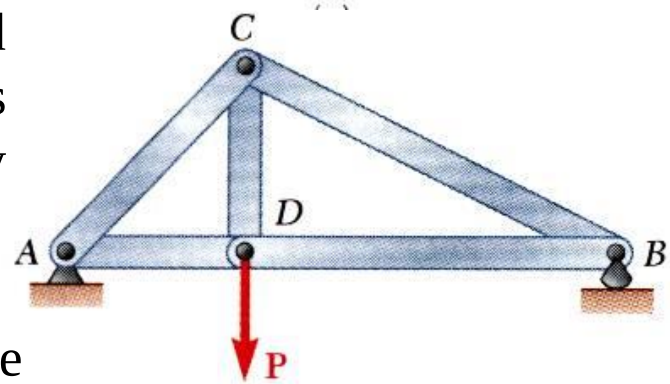


❑ Definition of a Truss

- A truss consists of straight members connected at joints. No member is continuous through a joint.

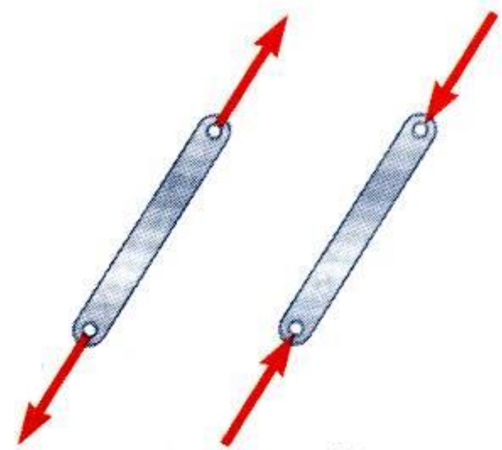


- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.



- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.

- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.



❑ Definition of a Truss

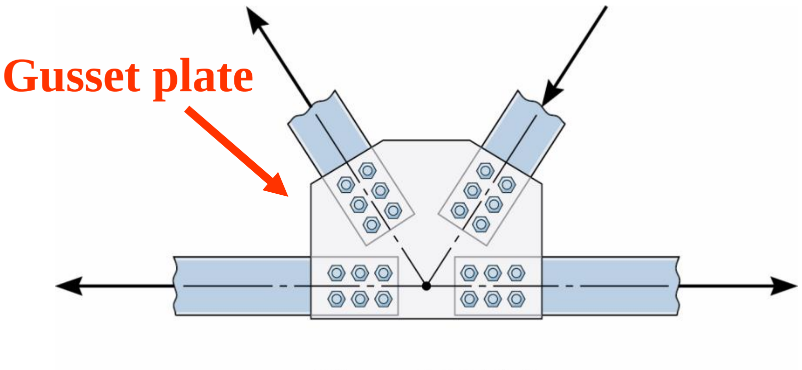
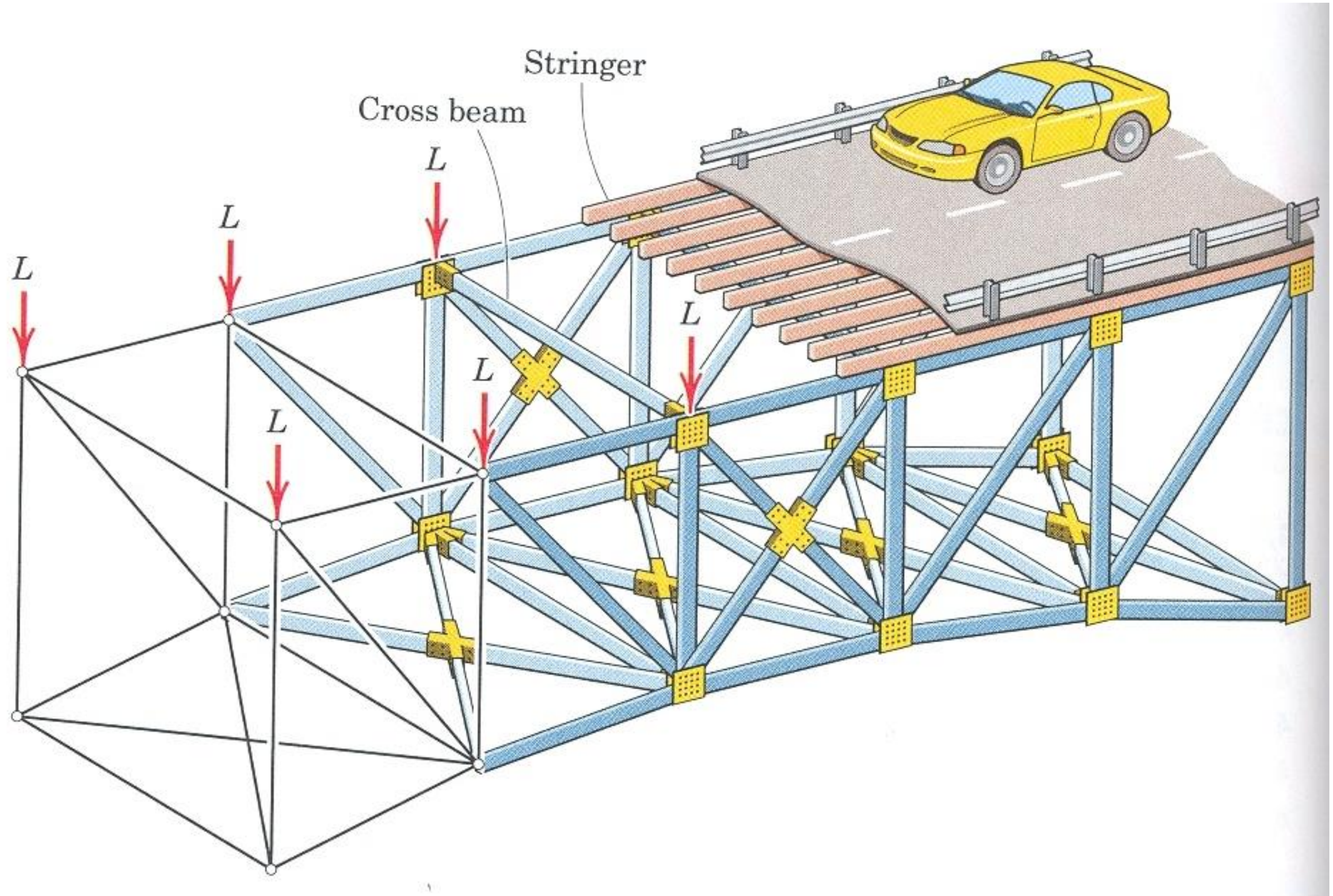


Figure 06.01(a)

Joints are often bolted, riveted, or welded. Gusset plates are also often included to tie the members together. However, the members are designed to support axial loads so assuming that the joints act as if they are pinned is a good approximation.

Definition of a Truss

Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.



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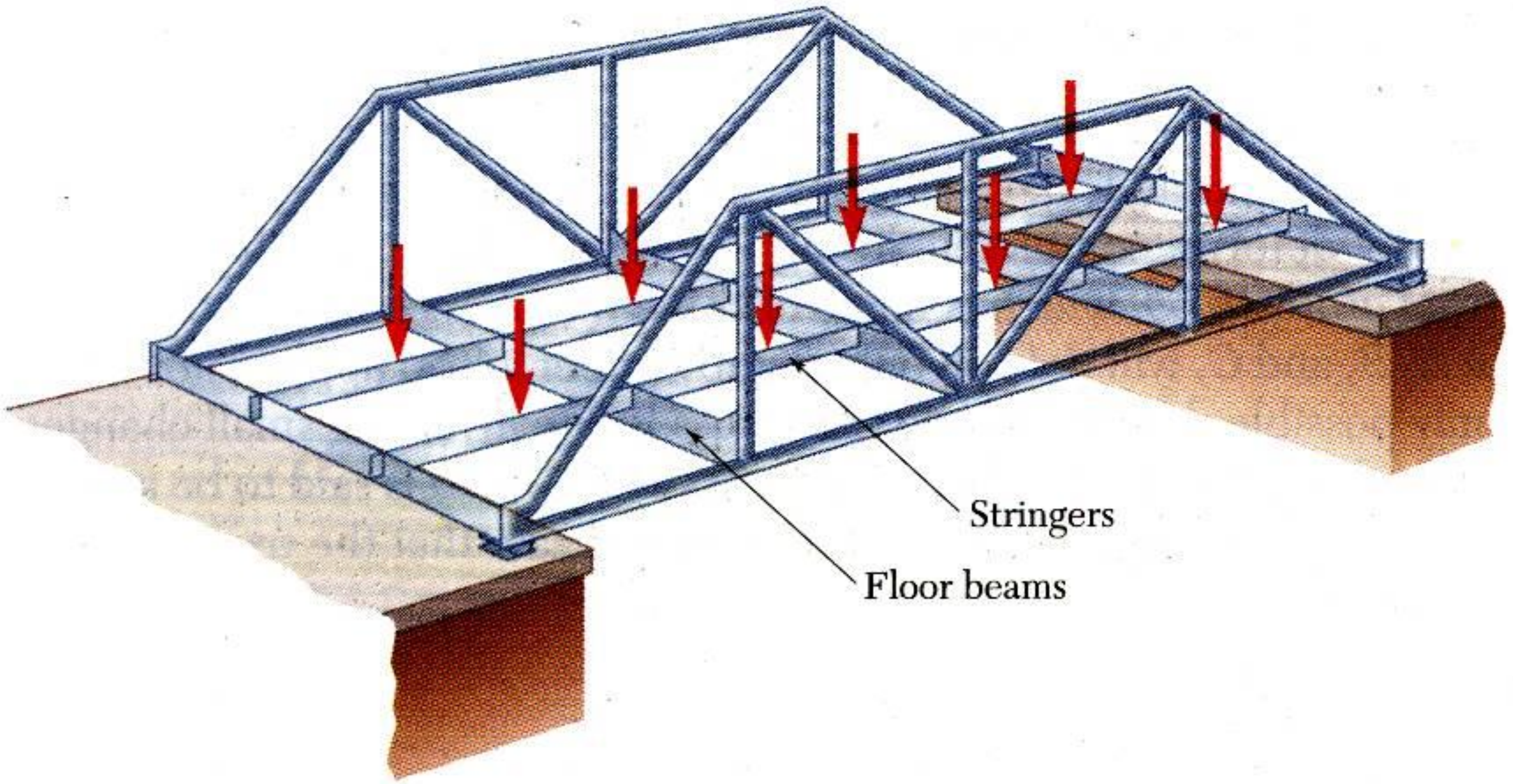
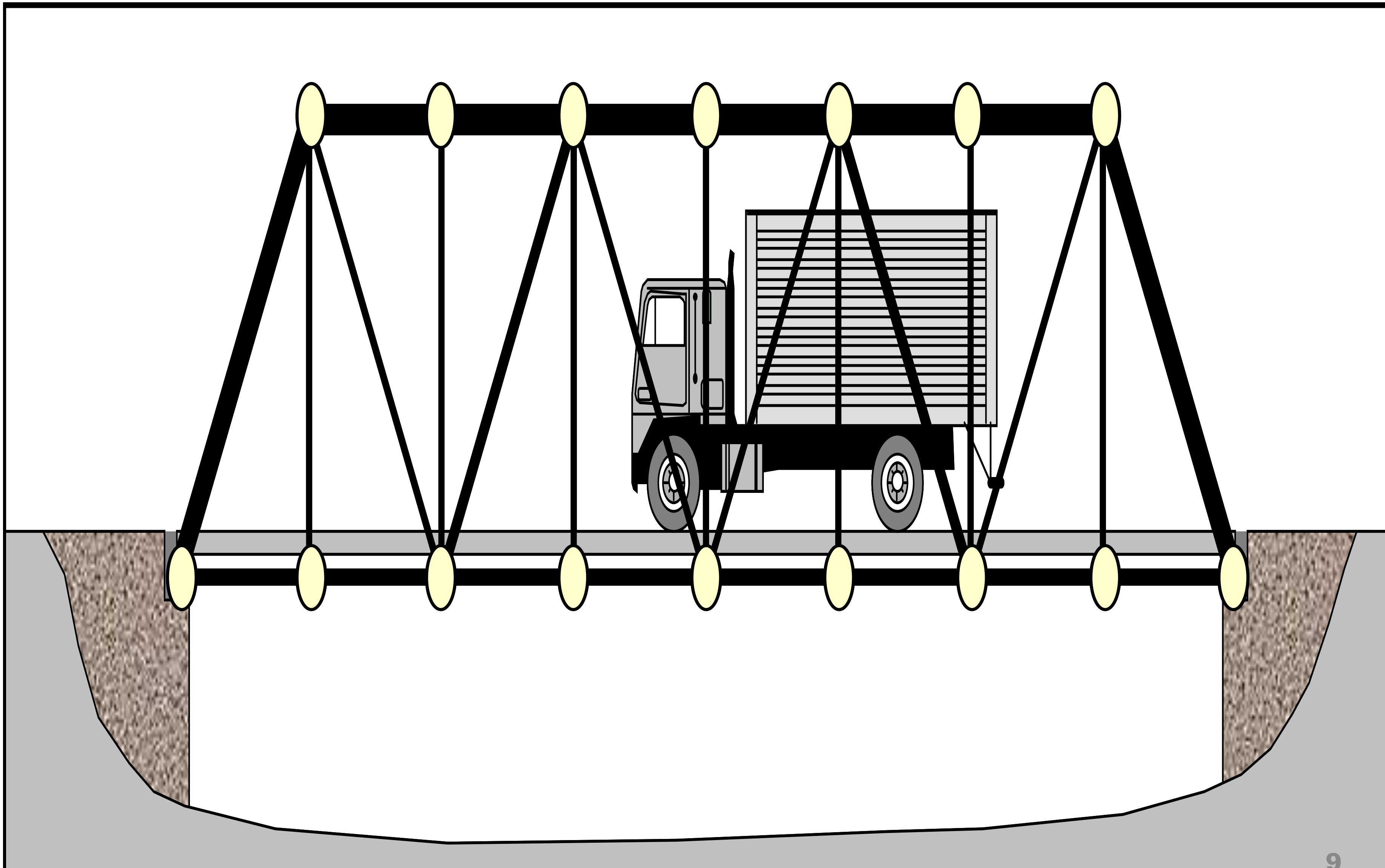


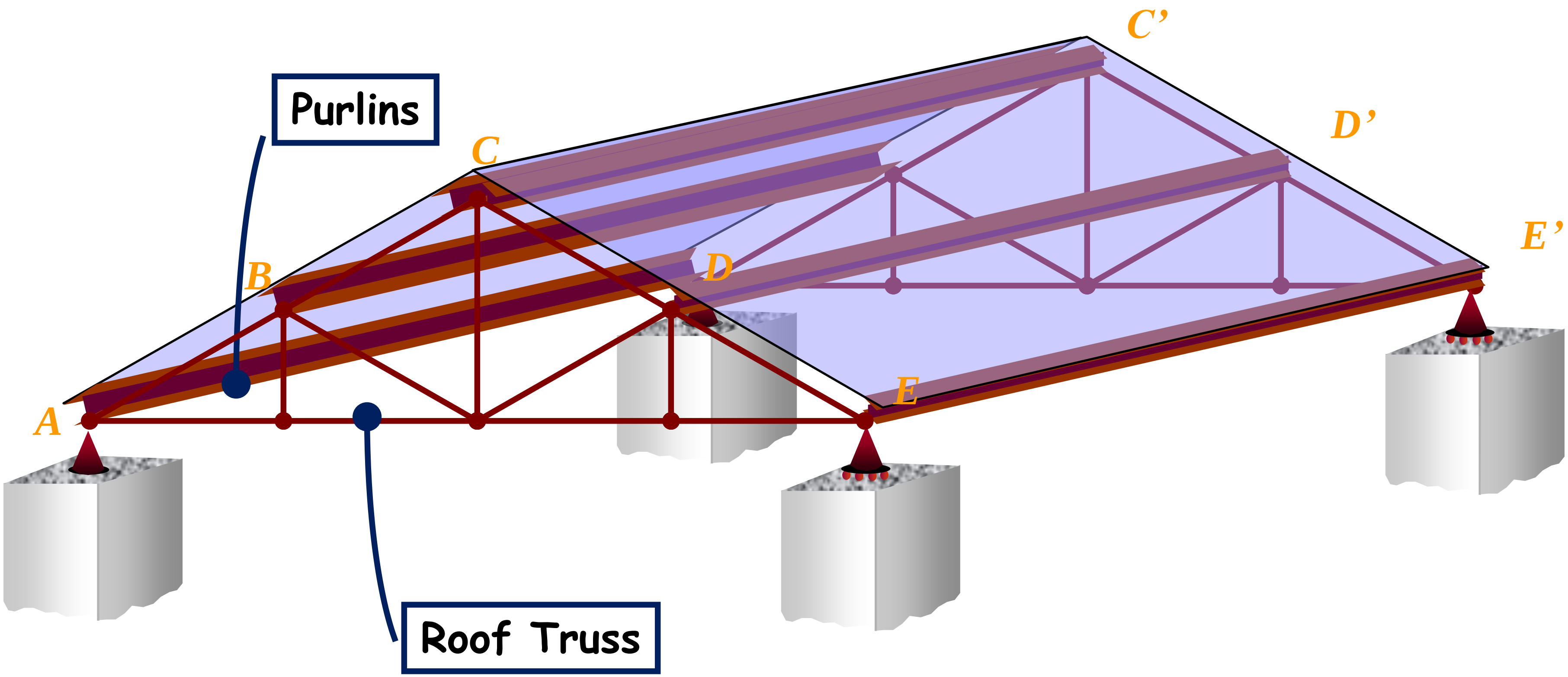
Definition of a Truss



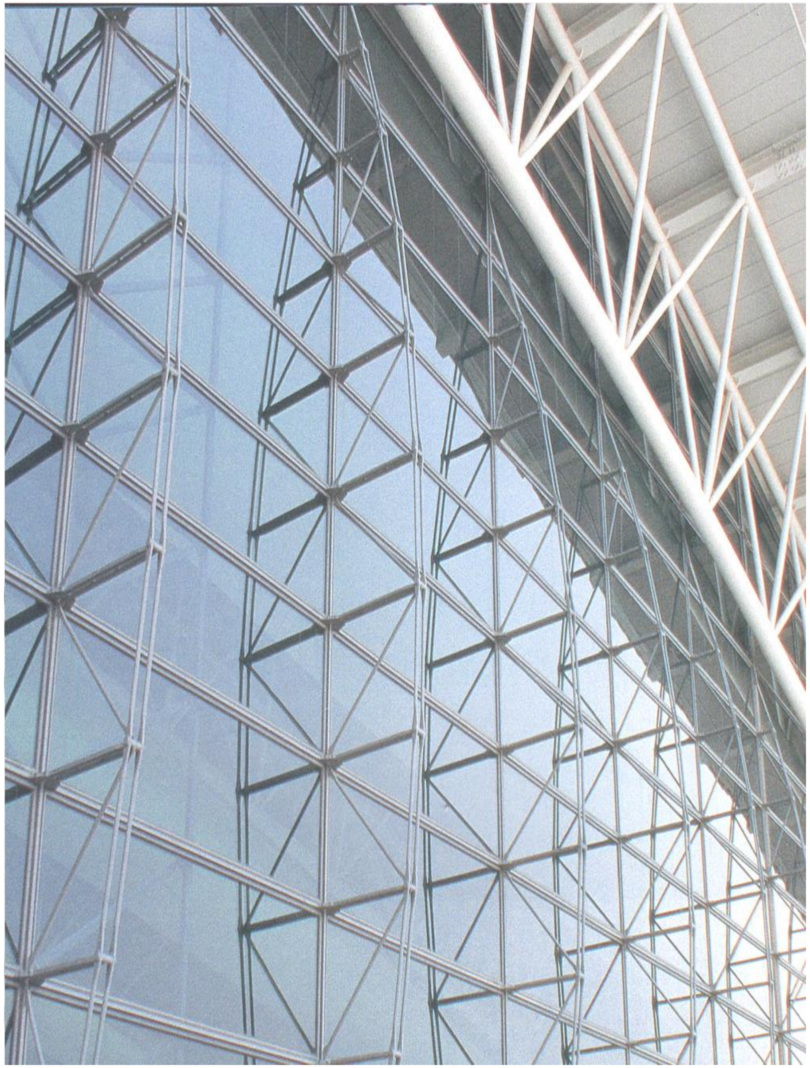
Definition of a Truss

Roof trusses

❑ Definition of a Truss



❑ Definition of a Truss

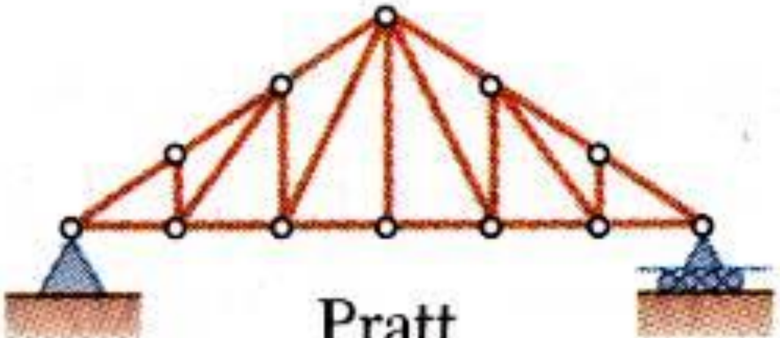


Definition of a Truss

❑ Definition of a Truss

❑ Definition of a Truss

<u>Typical Roof Trusses</u>



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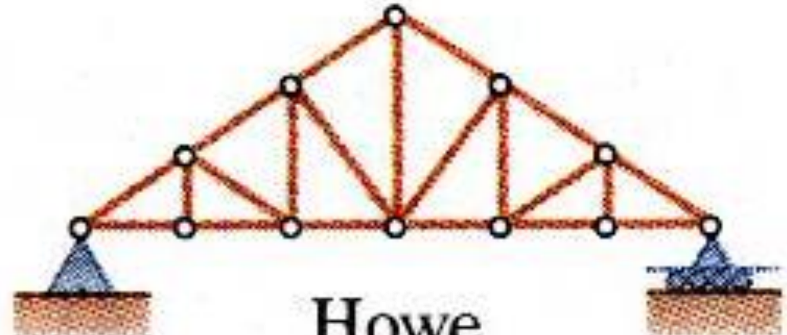

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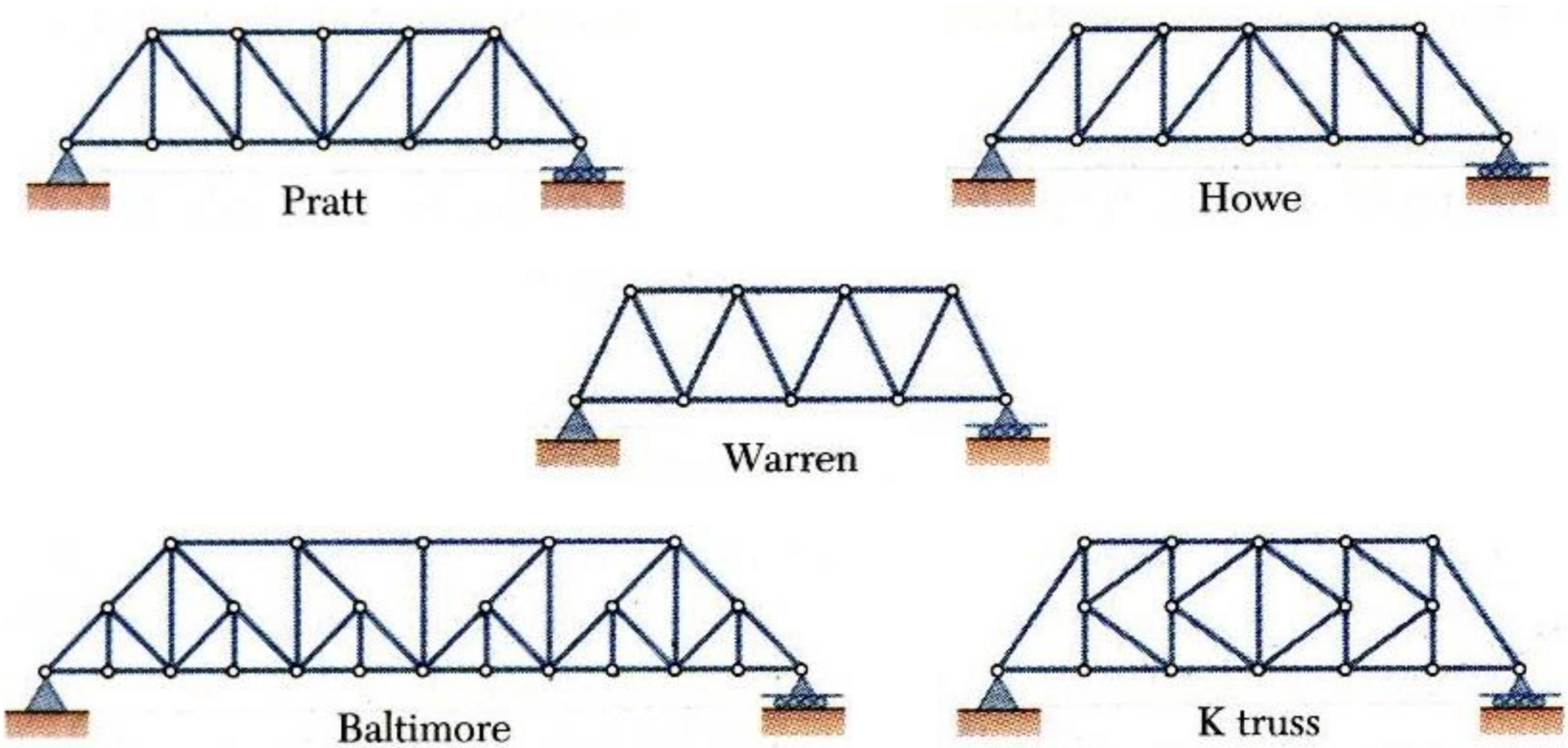
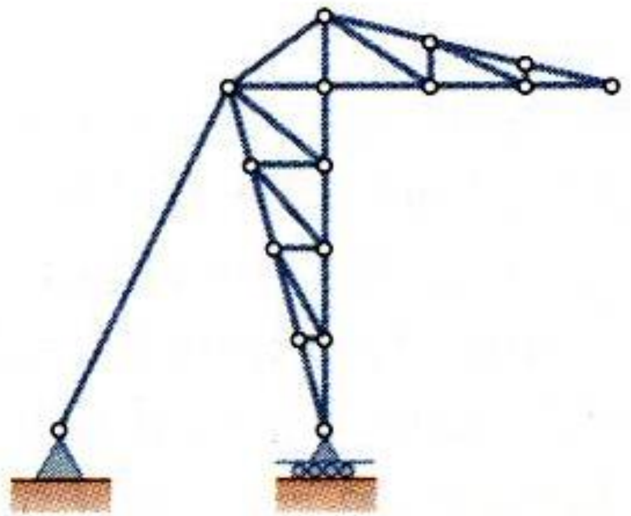
❑ Definition of a Truss

<u>Typical Bridge Trusses</u>

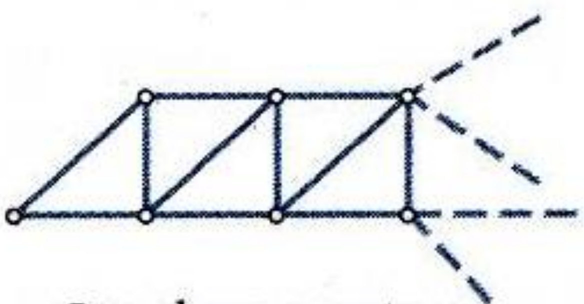


❑ Definition of a Truss

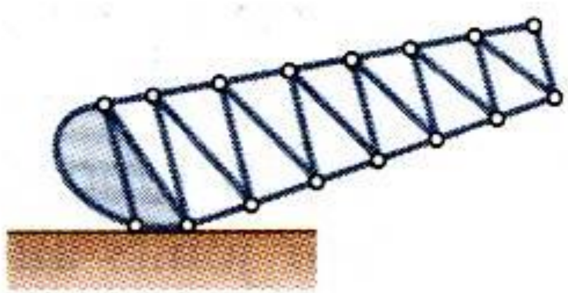
Other Type of Trusses



Stadium



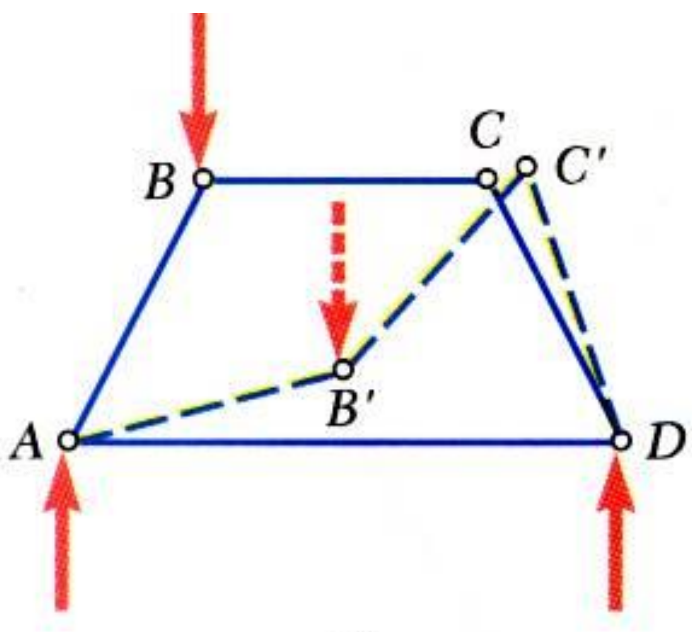
Cantilever portion of a truss

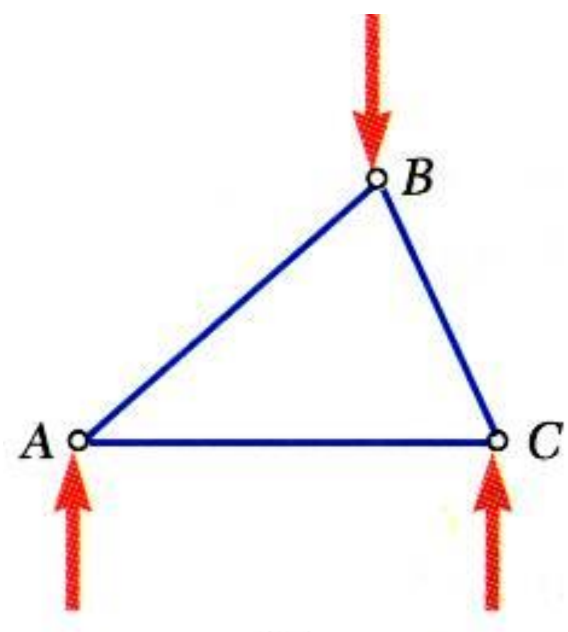


Bascule

❑ Simple Trusses

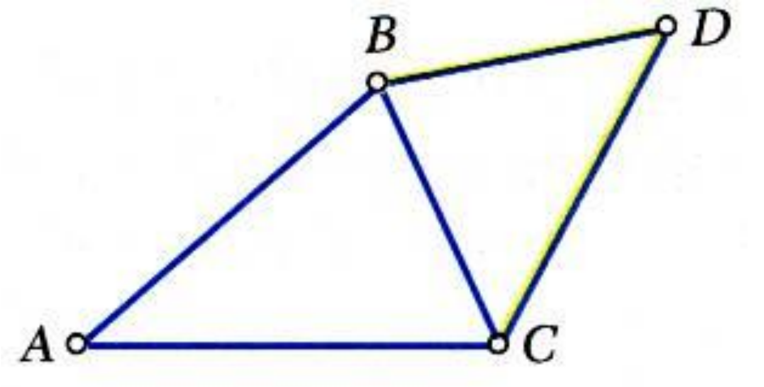
- A *rigid truss* will not collapse under the application of a load.





Simple Trusses

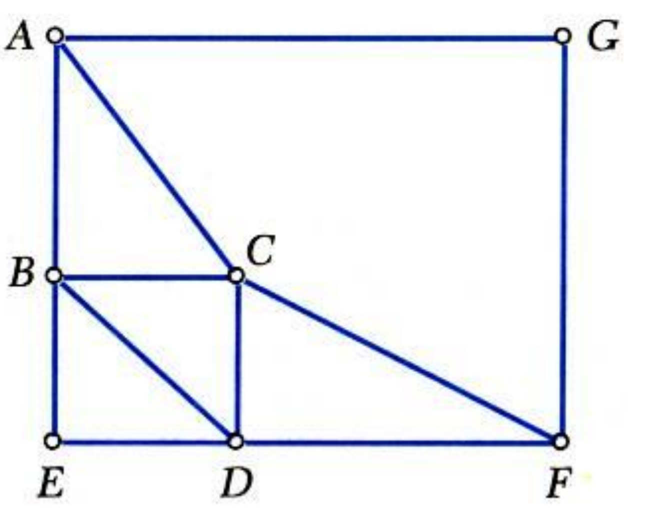
- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.



- In a simple truss,

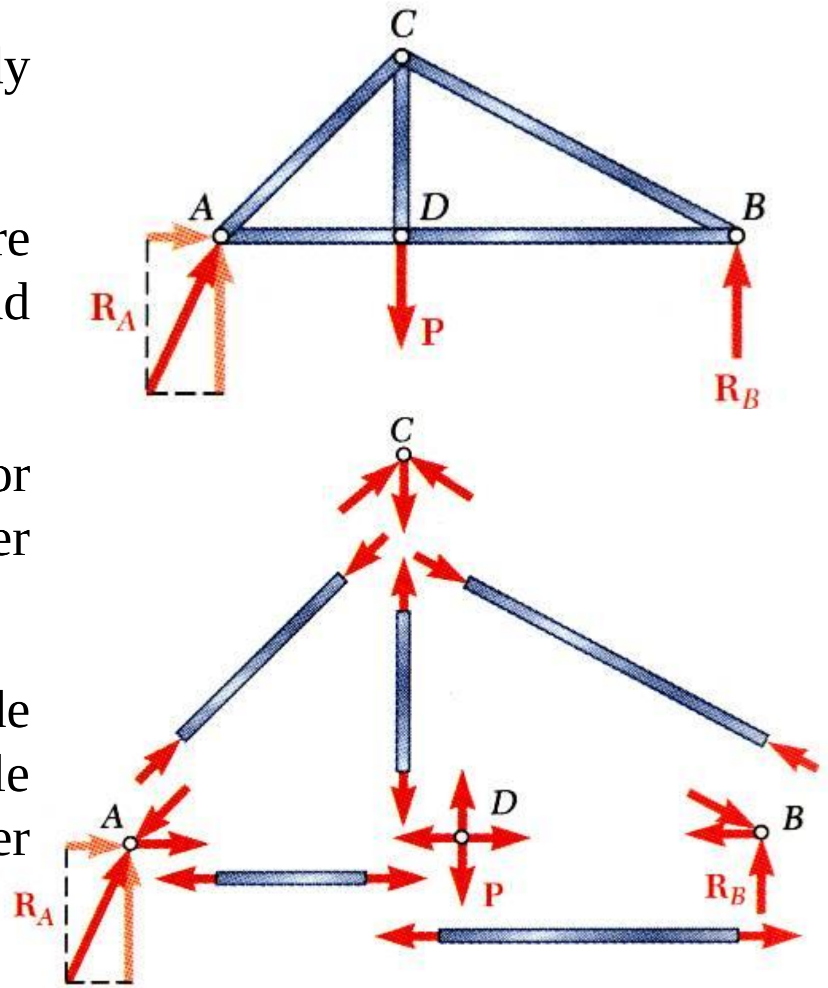
$$m = 2n - 3$$

where m is the total number of members and n is the number of joints.



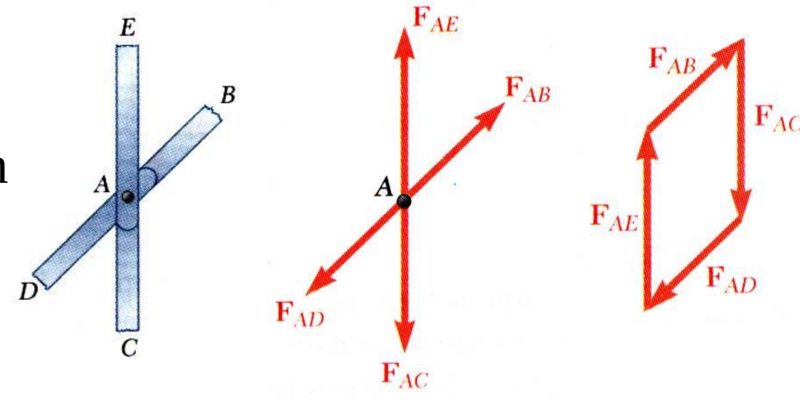
Analysis of Trusses by the Method of Joints

- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

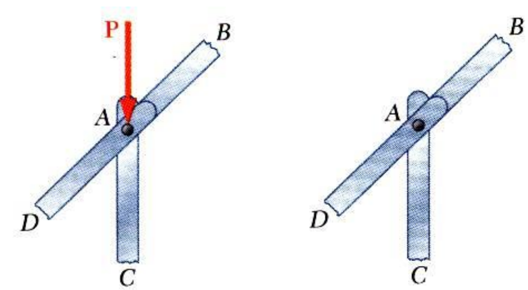


❑ Joints Under Special Loading Conditions

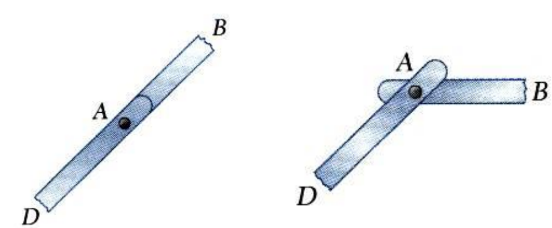
- Forces in opposite members intersecting in two straight lines at a joint are equal.



- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).

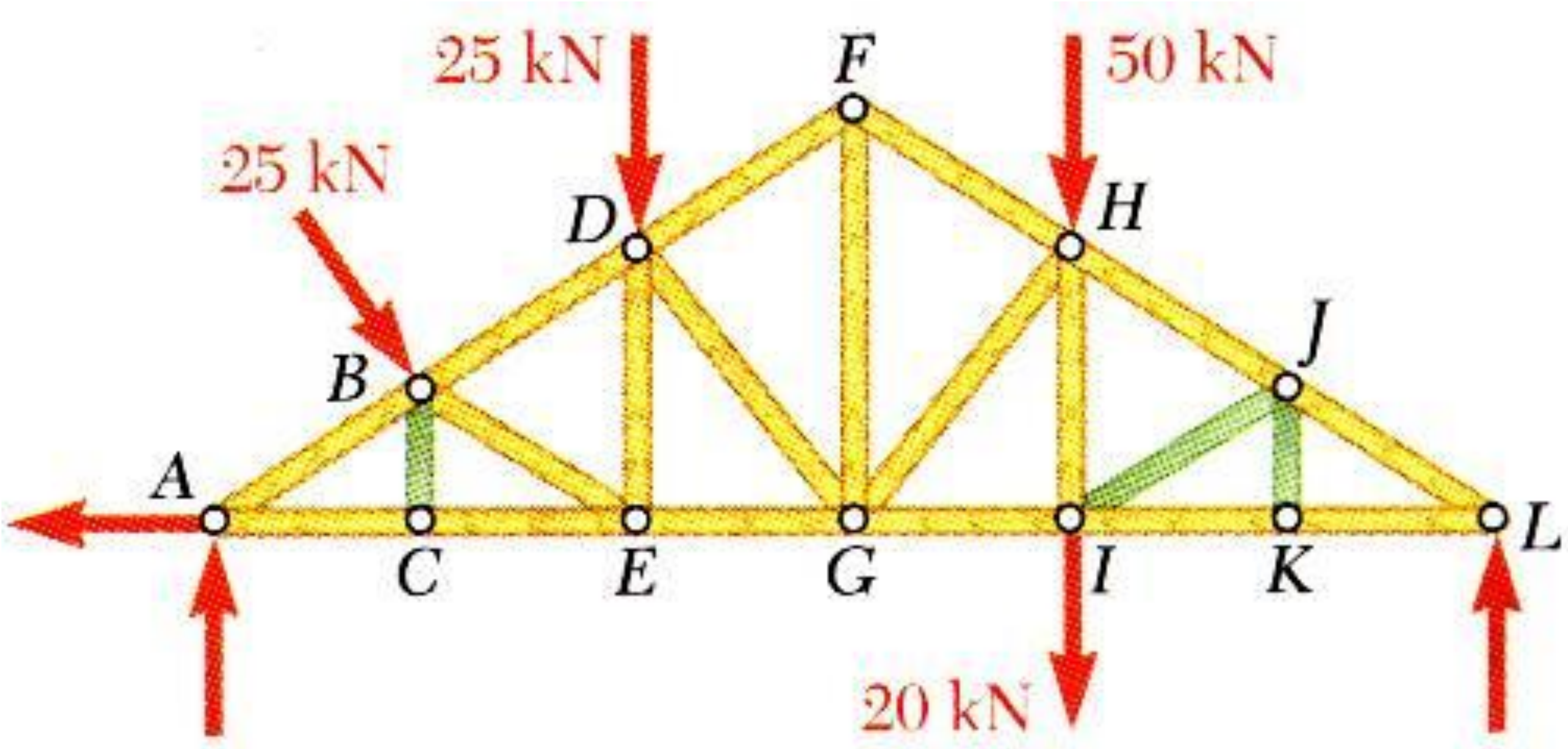


- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.



❑ Joints Under Special Loading Conditions

- Recognition of joints under special loading conditions simplifies a truss analysis.

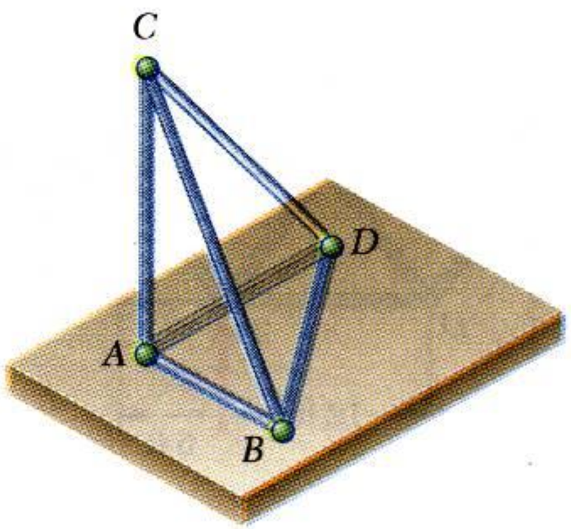


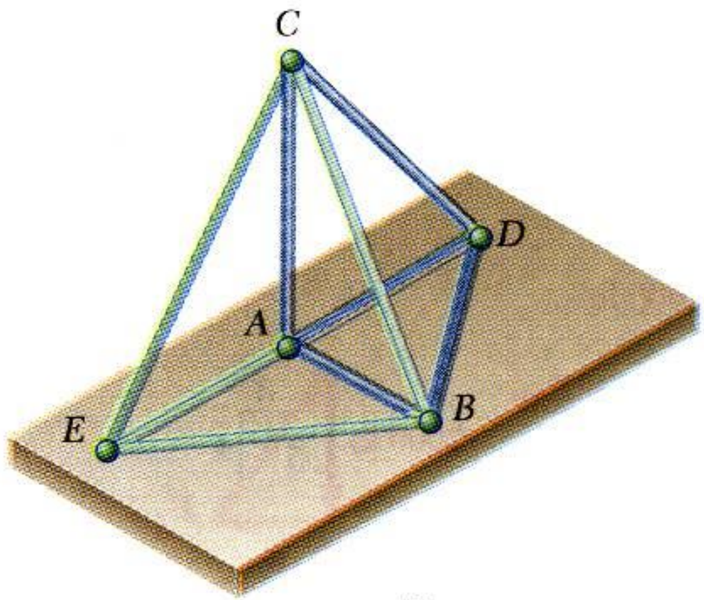
Space Trusses

- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss,

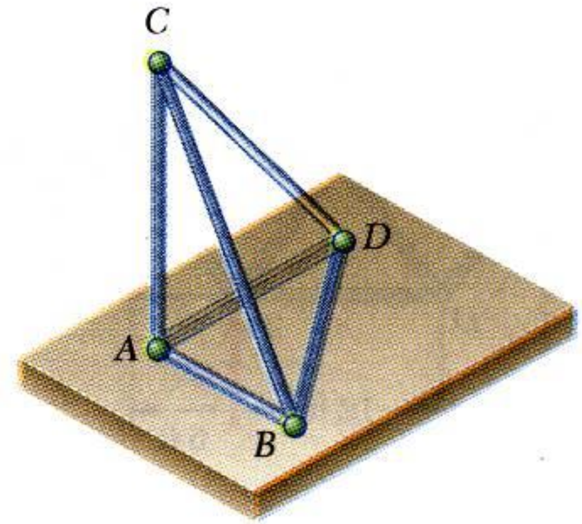
$$m = 3n - 6$$

where m is the number of members and n is the number of joints.

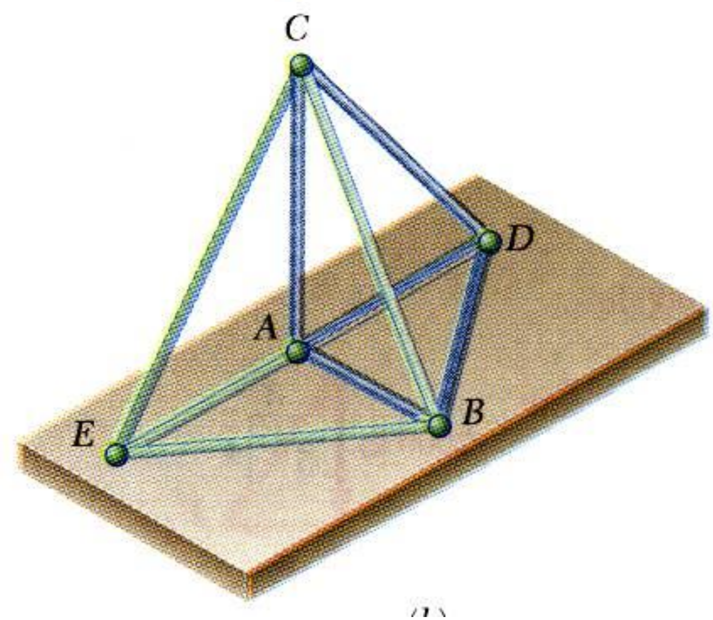




❑ Space Trusses

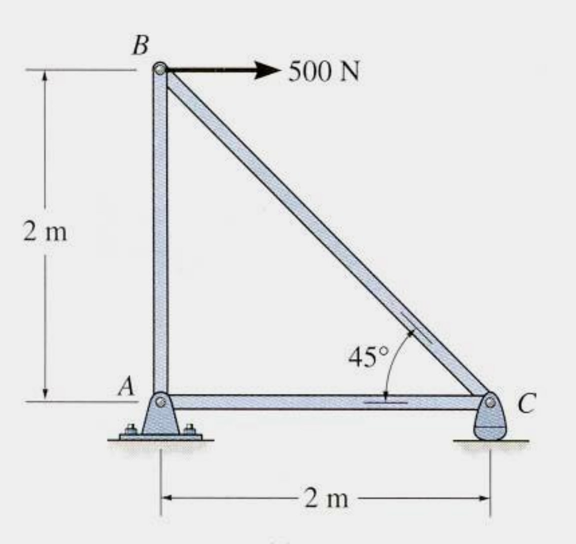


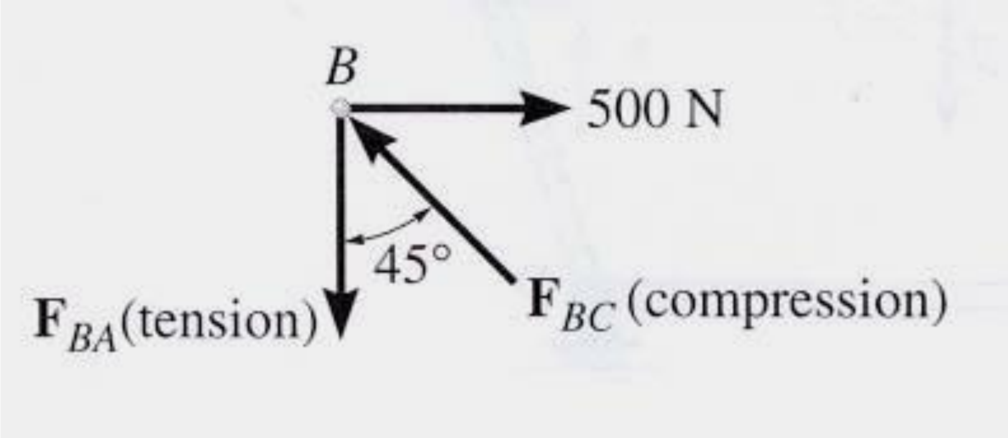
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.



- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

THE METHOD OF JOINTS





In this method of solving for the forces in truss members, the equilibrium of a joint (pin) is considered. All forces acting at the joint are shown in a FBD. This includes all external forces (including support reactions) as well as the forces acting in the members. Equations of equilibrium ($\sum F_X = 0$ and $\sum F_Y = 0$) are used to solve for the unknown forces acting at the joints.

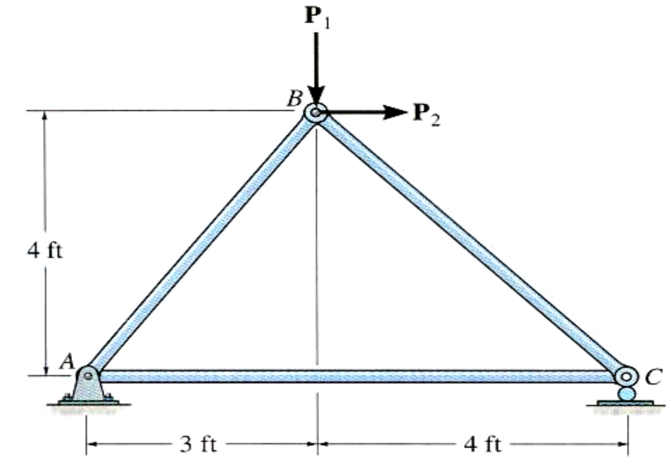
STEPS FOR ANALYSIS

1. If the support reactions are not given, draw a FBD of the entire truss and determine all the support reactions using the equations of equilibrium.

2. Draw the free-body diagram of a joint with one or two unknowns. Assume that all unknown member forces act in tension (pulling the pin) unless you can determine by inspection that the forces are compression loads.

3. Apply the scalar equations of equilibrium, $\sum F_X = 0$ and $\sum F_Y = 0$, to determine the unknown(s). If the answer is positive, then the assumed direction (tension) is correct, otherwise it is in the opposite direction (compression).

4. Repeat steps 2 and 3 at each joint in succession until all the required forces are determined.



EXAMPLE

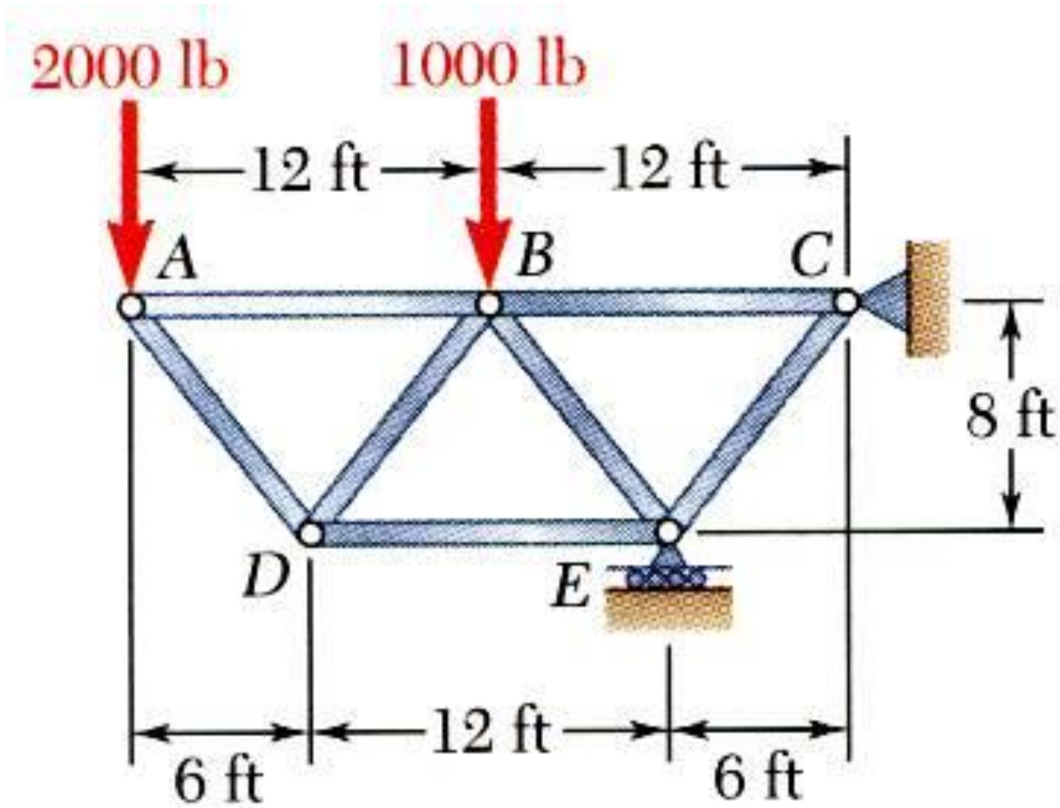
Given: $P_1 = 200$ lb, $P_2 = 500$ lb

Find: The forces in each member of the truss.

Plan: First analyze pin B and then pin C

❑ Sample Problem

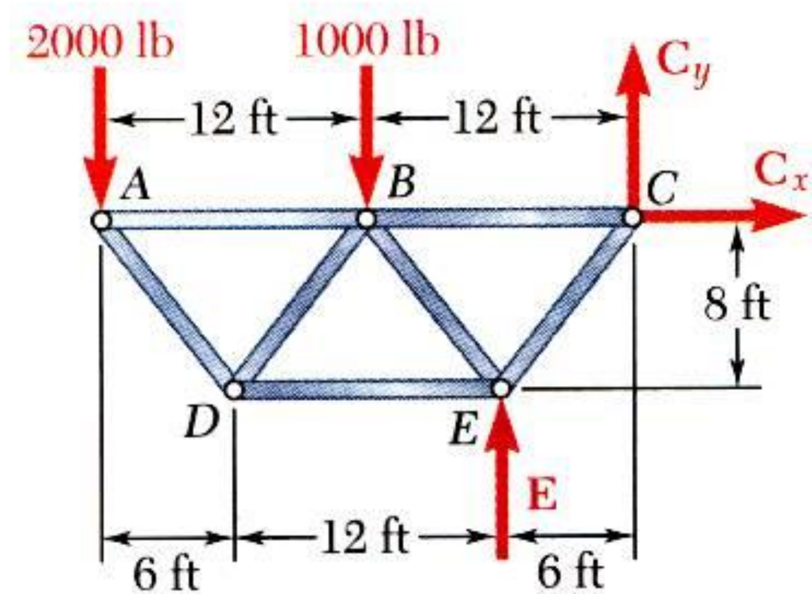
Using the method of joints, determine the force in each member of the truss.



❑ Sample Problem

SOLUTION:

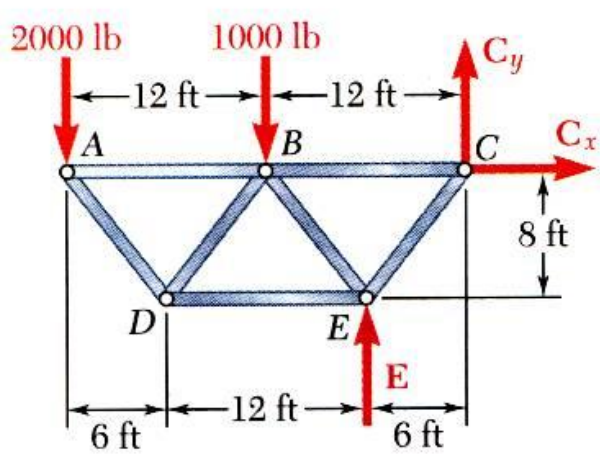
- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.



❑ Sample Problem

SOLUTION:

- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.



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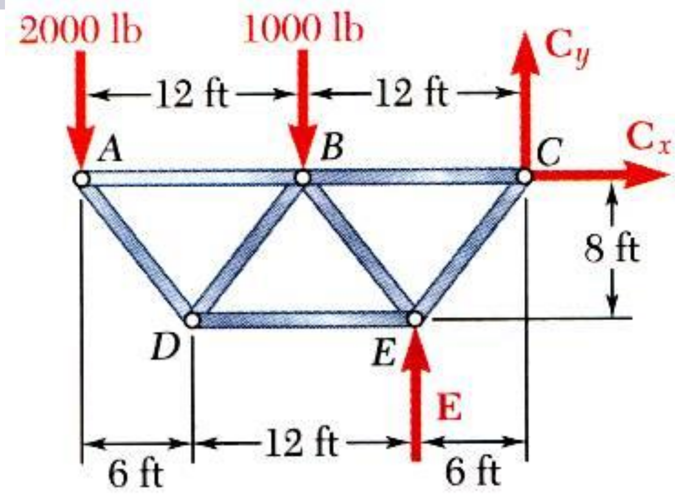
❑ Sample Problem

SOLUTION:

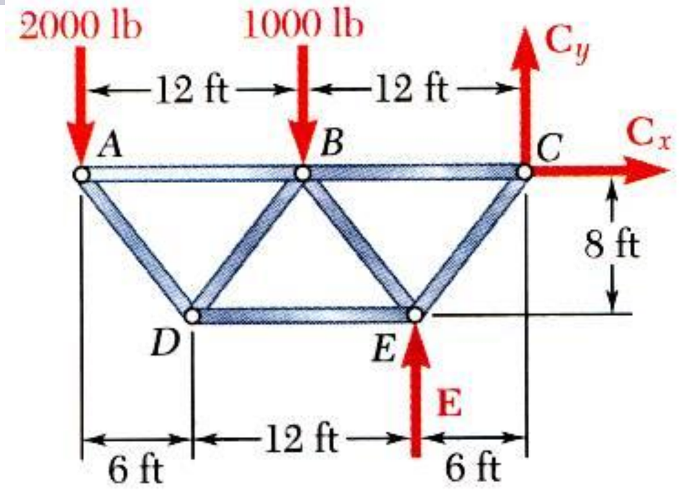
- There are now only two unknown member forces at joint D.



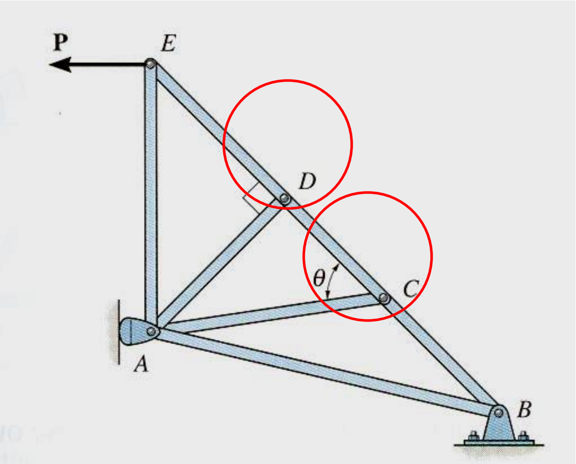
❑ Sample Problem

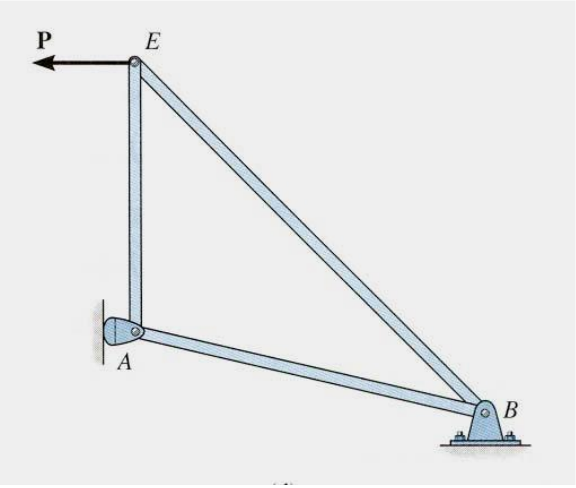
SOLUTION:

- There are now only two unknown member forces at joint B. Assume both are in tension.



ZERO – FORCE MEMBERS

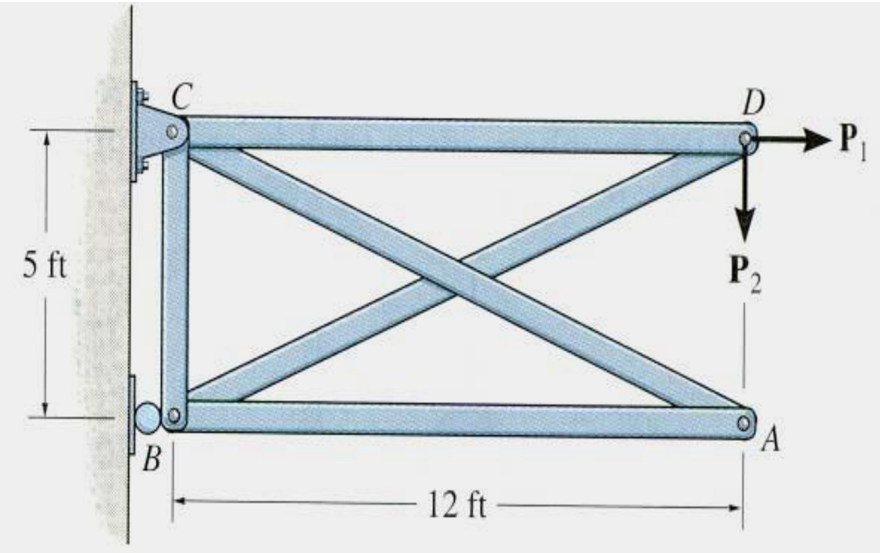




If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member.

Again, this can easily be proven. One can also remove the zero-force member, as shown, on the left, for analyzing the truss further.

Please note that zero-force members are used to increase stability and rigidity of the truss, and to provide support for various different loading conditions.



Given: P_1 = 240 lb and

P_2 = 100 lb

Find: Determine the force in all the truss members (do not forget to mention whether they are in T or C).

Plan:

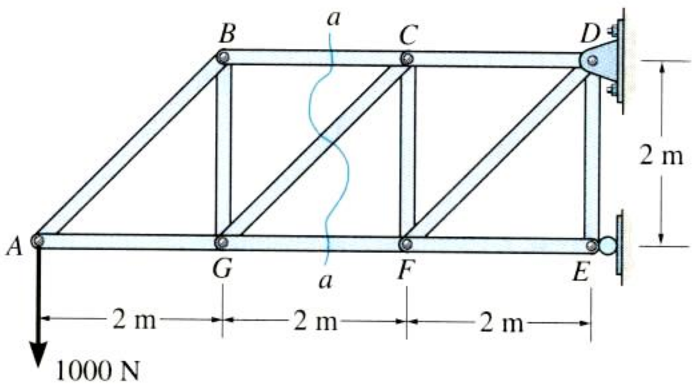
a) Check if there are any zero-force members.

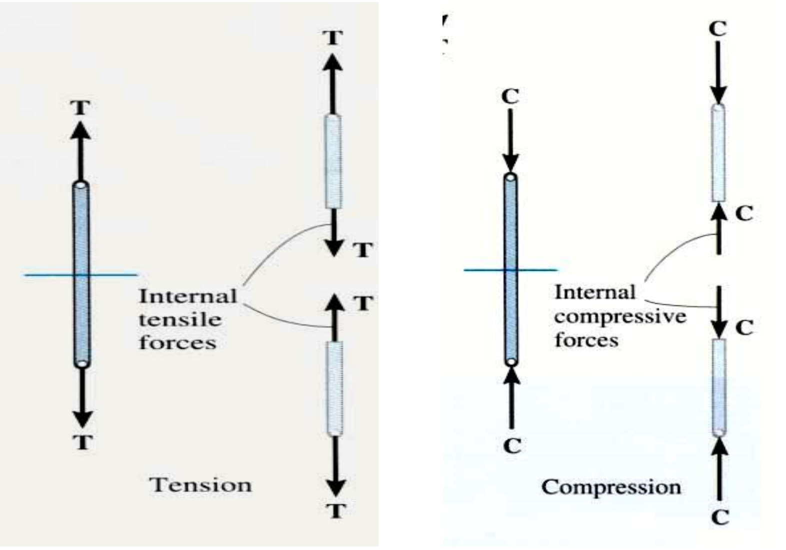
b) Draw FBDs of pins D and B, and then apply EE at those pins to solve for the unknowns.

Solution:

Members AB and AC are zero-force members.

THE METHOD OF SECTIONS

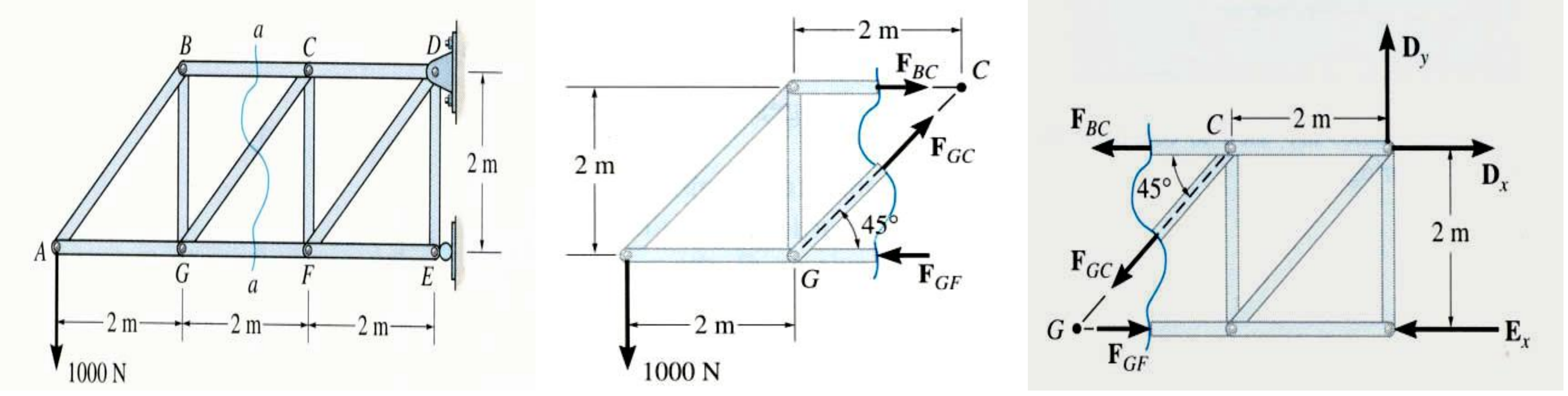




In the method of sections, a truss is divided into two parts by taking an imaginary "cut" (shown here as a-a) through the truss.

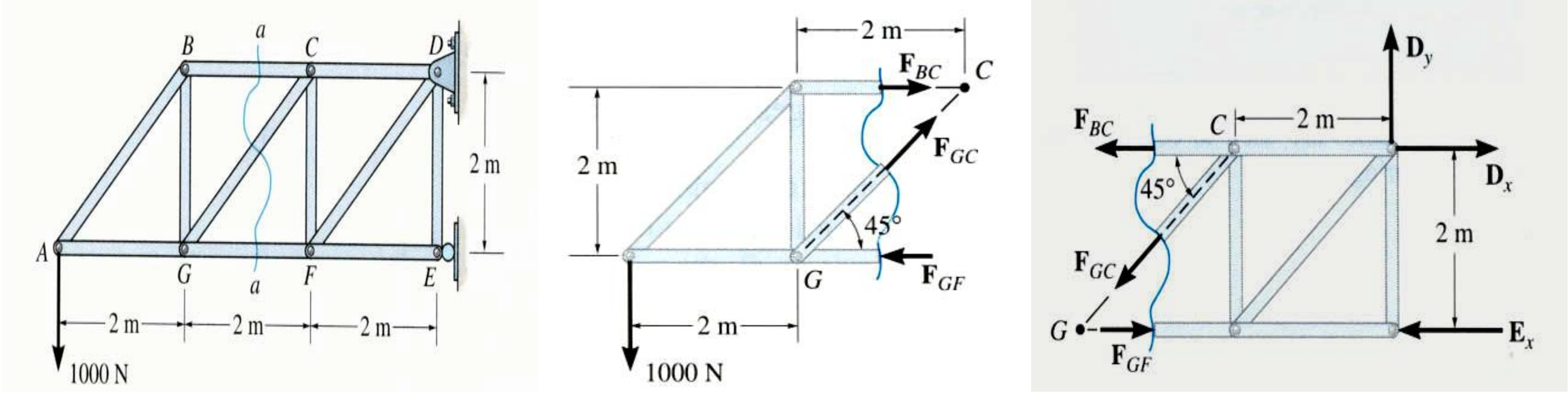
Since truss members are subjected to only tensile or compressive forces along their length, the internal forces at the cut member will also be either tensile or compressive with the same magnitude. This result is based on the equilibrium principle and Newton's third law.

STEPS FOR ANALYSIS



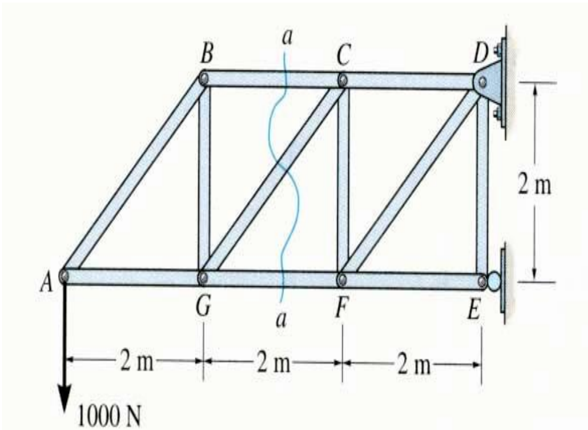
1. Decide how you need to “cut” the truss. This is based on: a) where you need to determine forces, and, b) where the total number of unknowns does not exceed three (in general).
2. Decide which side of the cut truss will be easier to work with (minimize the number of reactions you have to find).
3. <u>If required</u>, determine the necessary support reactions by drawing the FBD of the entire truss and applying the E-of-E.

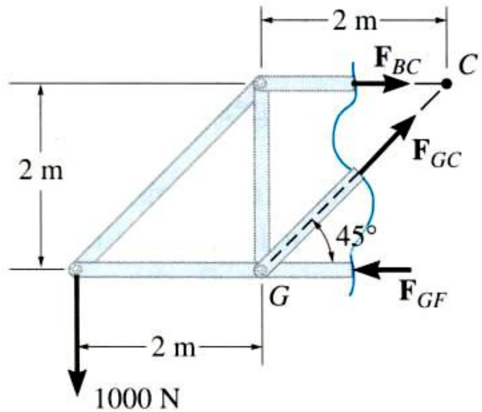
STEPS FOR ANALYSIS (continued)

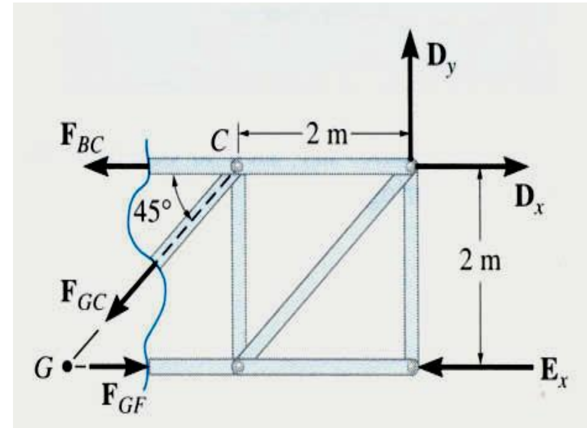


4. Draw the FBD of the selected part of the cut truss. We need to indicate the unknown forces at the cut members. Initially we may assume all the members are in tension, as we did when using the method of joints. Upon solving, if the answer is positive, the member is in tension as per our assumption. If the answer is negative, the member must be in compression. (Please note that you can also assume forces to be either tension or compression by inspection as was done in the figures above.)

STEPS FOR ANALYSIS (continued)

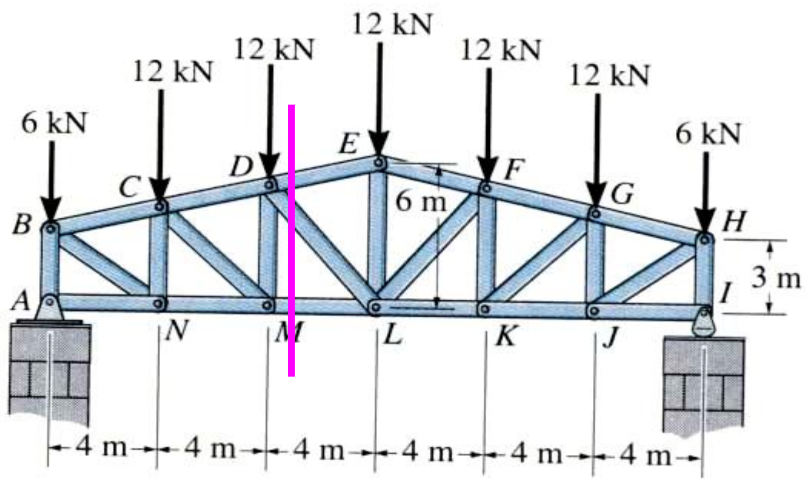






5. Apply the equations of equilibrium (E-of-E) to the selected cut section of the truss to solve for the unknown member forces. <u>Please note</u> that in most cases it is possible to write one equation to solve for one unknown directly.

EXAMPLE



Given: Loads as shown on the roof truss.

Find: The force in members DE, DL, and ML.

Plan:

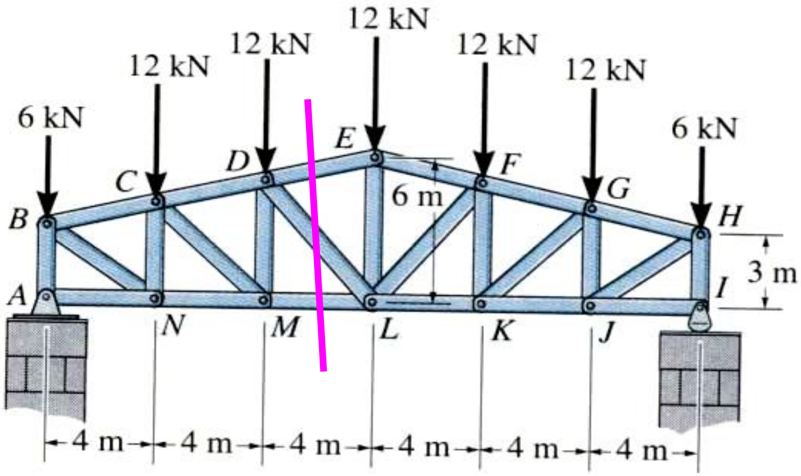
a) Take a cut through the members DE, DL, and ML.

b) Work with the left part of the cut section. Why?

c) Determine the support reaction at A. What are they?

d) Apply the EofE to find the forces in DE, DL, and ML.

d)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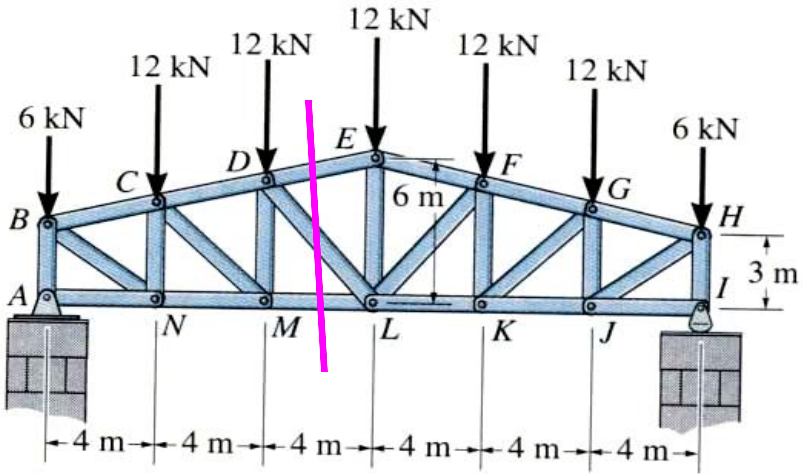
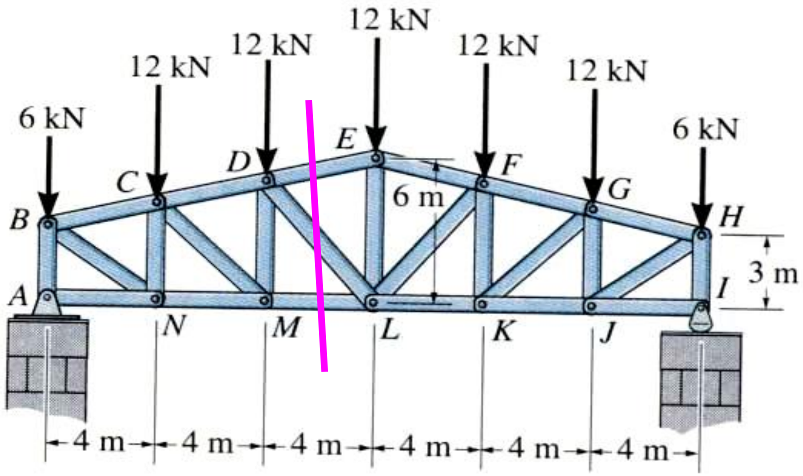
EXAMPLE (continued)

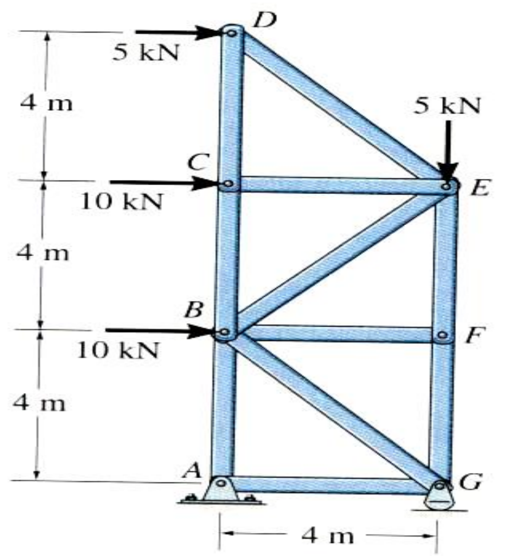


EXAMPLE (continued)



Given: Loading on the truss as shown.

Find: The force in members BC, BE, and EF.



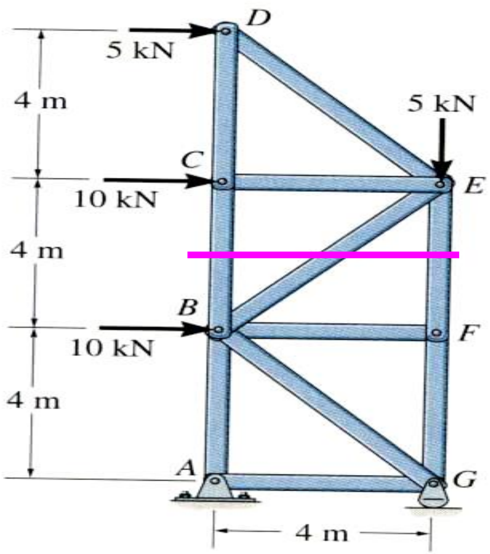
Plan:

a) Take a cut through the members BC, BE, and EF.

b) Analyze the top section (no support reactions!).

c) Draw the FBD of the top section.

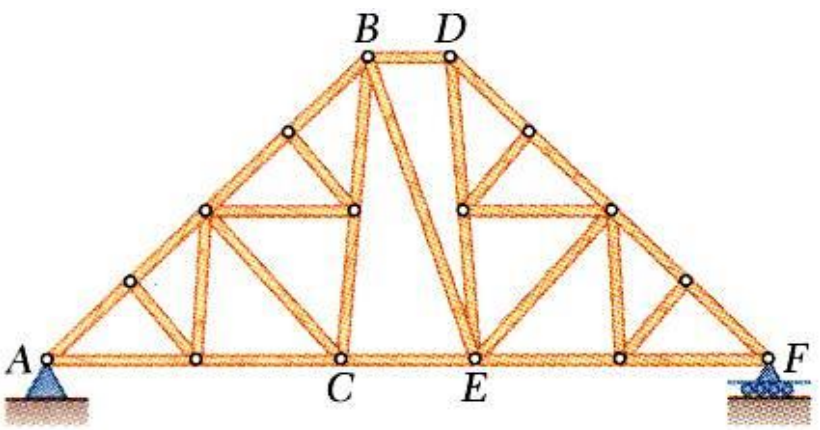
d) Apply the equations of equilibrium such that every equation yields answer to one unknown.

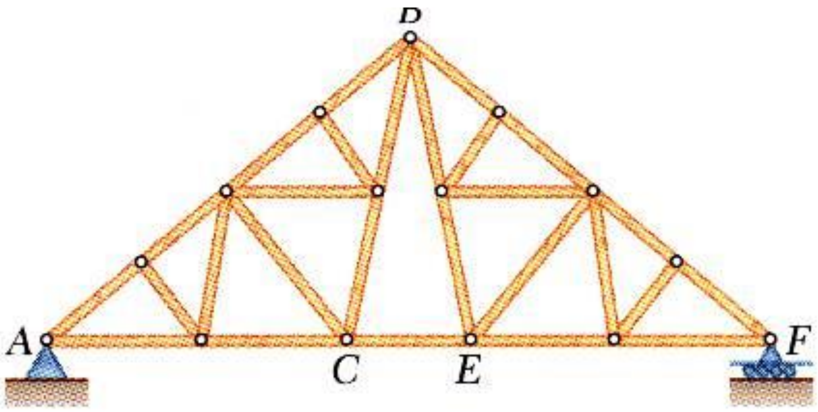


D
5 kN
4 m
5 kN
C
E
10 kN
4 m
B
F
10 kN
4 m
A
G
4 m

❑ Trusses Made of Several Simple Trusses

- *Compound trusses* are statically ***determinant***, ***rigid***, and ***completely constrained***.





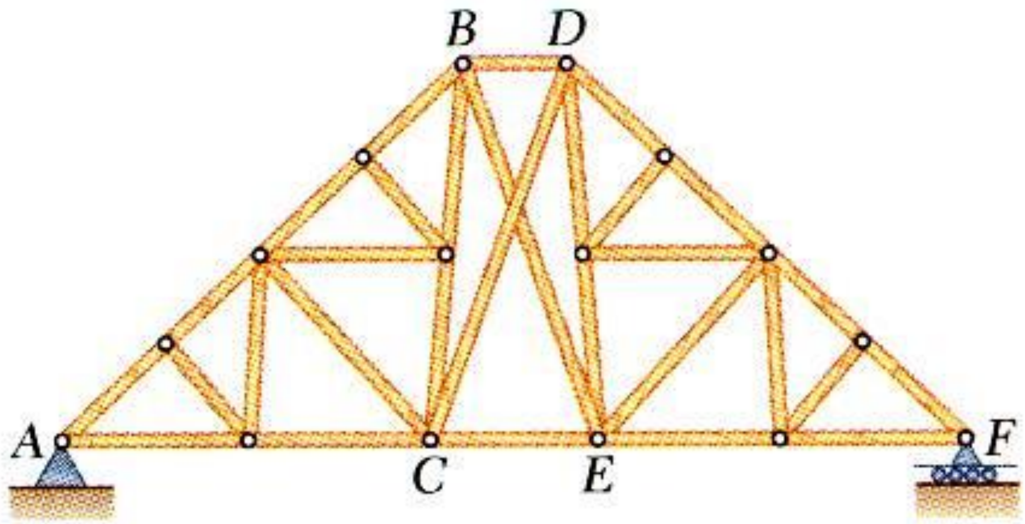
Unknown: $m+r$

Equations: $2n$

$$\Rightarrow \boxed{m+r=2n}$$

Trusses Made of Several Simple Trusses

- Truss contains a ***redundant member*** and is ***statically indeterminate***.



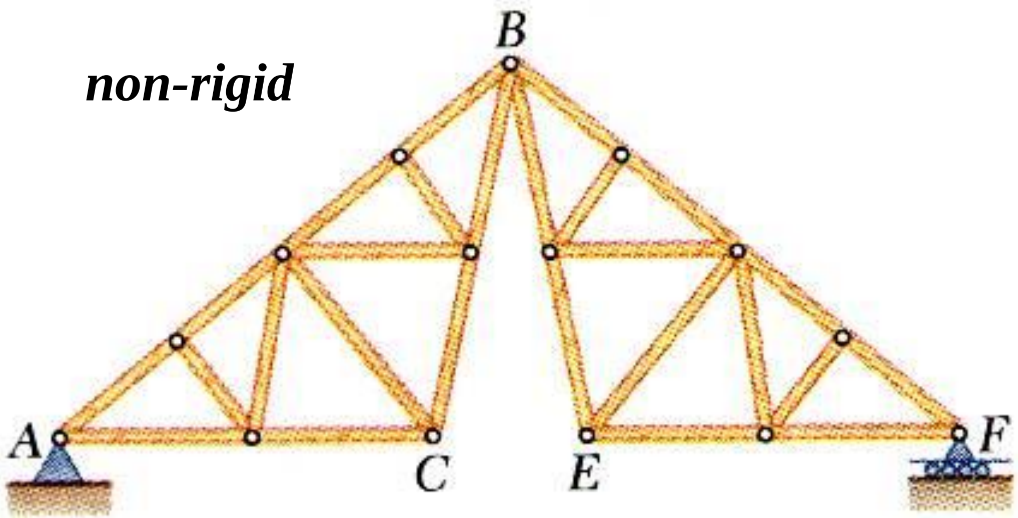
Unknown: $m+r$

Equations: $2n$

$\Rightarrow$ $m+r > 2n$

Trusses Made of Several Simple Trusses

- Additional reaction forces may be necessary for a rigid truss.

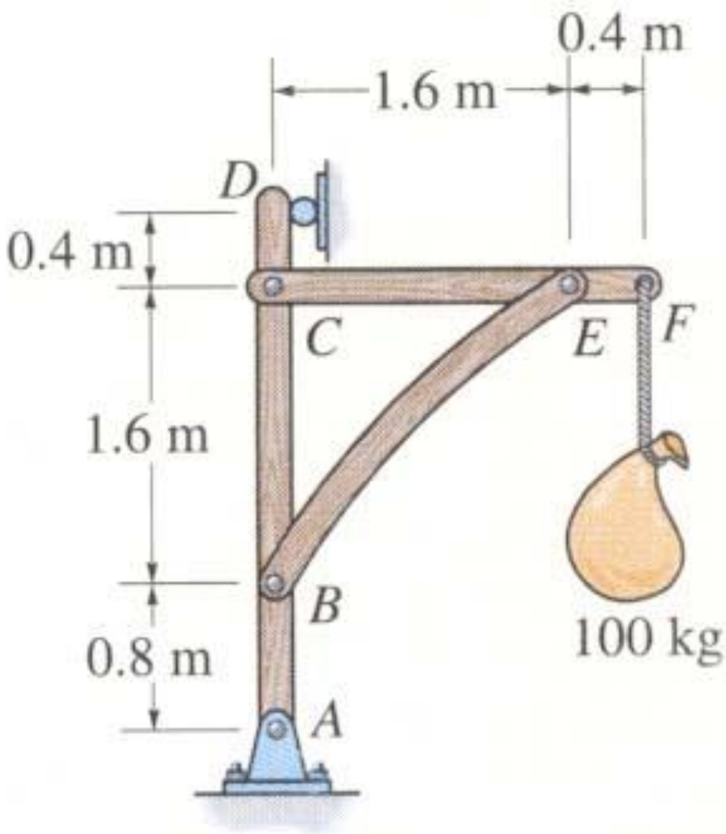


Unknown: $m+r$

Equations: $2n$

$\Rightarrow$ $m+r<2n$

FRAMES AND MACHINES

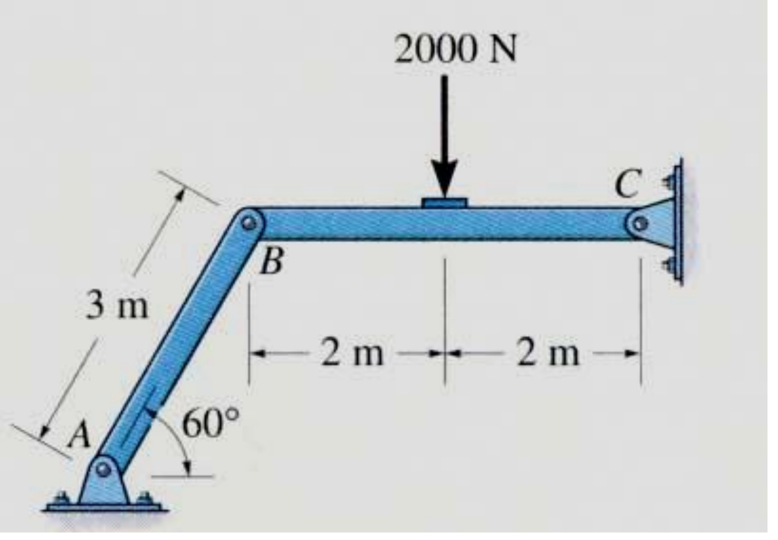


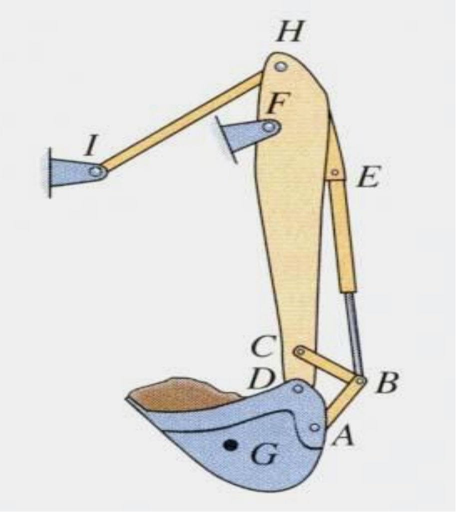
Frames are commonly used to support various external loads.

How is a frame different than a truss?

How can you determine the forces at the joints and supports of a frame?

FRAMES AND MACHINES: DEFINITIONS



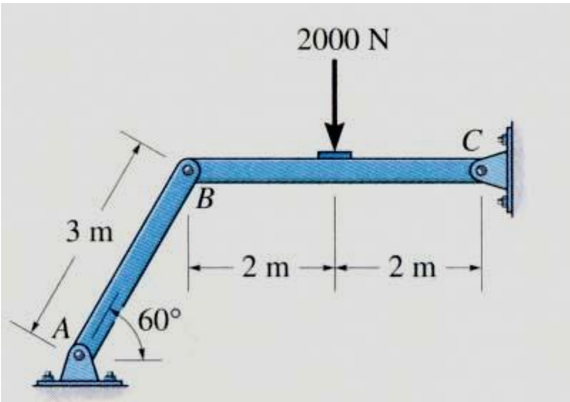


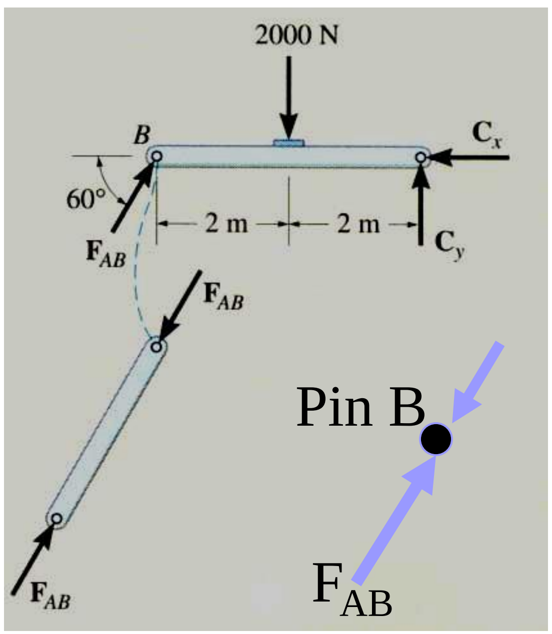
Frames and machines are two common types of structures that have <u>at least one multi-force member</u>. (Recall that trusses have nothing but two-force members).

<u>Frames</u> are generally <u>stationary</u> and support external loads.

<u>Machines</u> contain <u>moving parts</u> and are designed to alter the effect of forces.

STEPS FOR ANALYZING A FRAME OR MACHINE





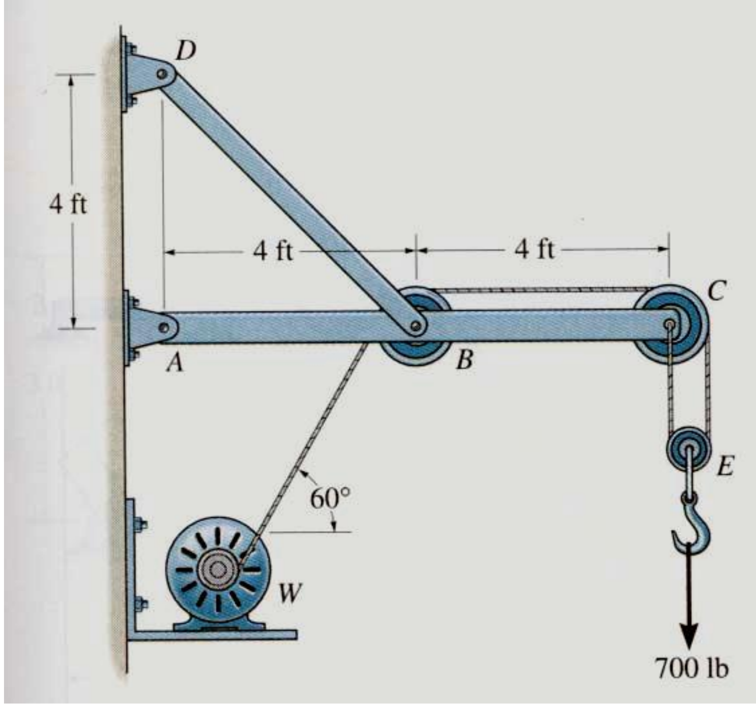
1. Draw the FBD of the frame or machine and its members, as necessary.

Hints:

a) Identify any two-force members, b) Forces on contacting surfaces (usually between a pin and a member) are equal and opposite, and,
c) For a joint with more than two members or an external force, draw a FBD of the pin.

2. Develop a strategy to apply the equations of equilibrium to solve for the unknowns.

Problems are going to be challenging since there are usually several unknowns. A lot of practice is needed to develop good strategies.



Given: The wall crane supports an external load of 700 lb.

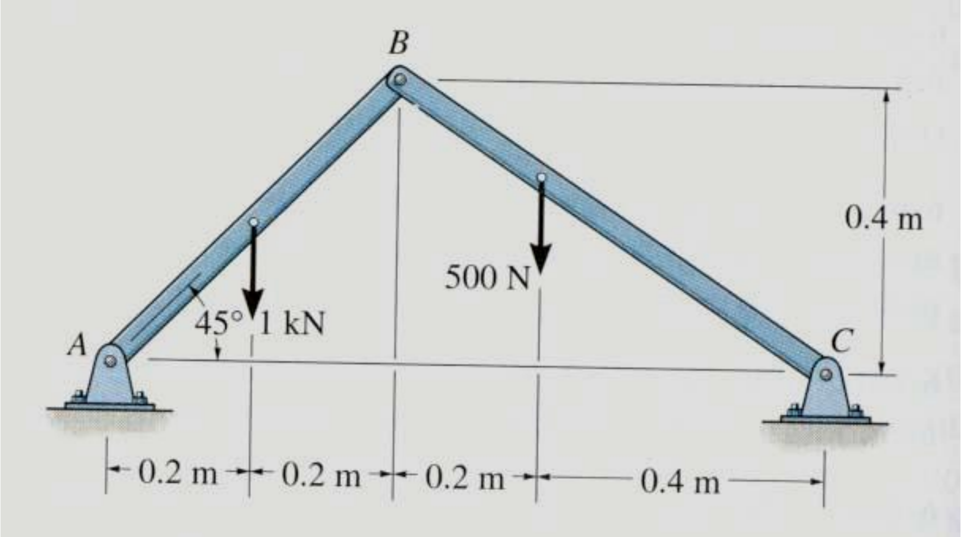
Find: The force in the cable at the winch motor W and the horizontal and vertical components of the pin reactions at A, B, C, and D.

Plan:

a) Draw FBDs of the frame's members and pulleys.

b) Apply the equations of equilibrium and solve for the unknowns.

EXAMPLE (continued)



Given: A frame and loads as shown.

Find: The reactions that the pins exert on the frame at A, B and C.

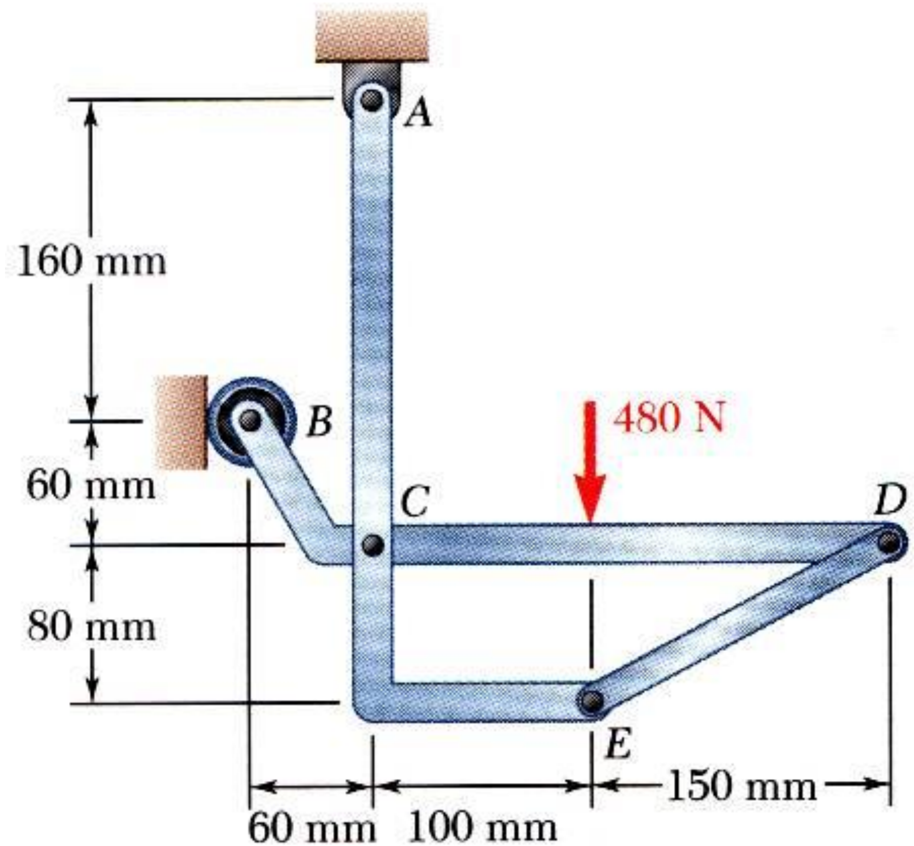
Plan:

a) Draw a FBD of members AB and BC.

b) Apply the equations of equilibrium to each FBD to solve for the six unknowns. Think about a strategy to easily solve for the unknowns.

❑ Sample Problem

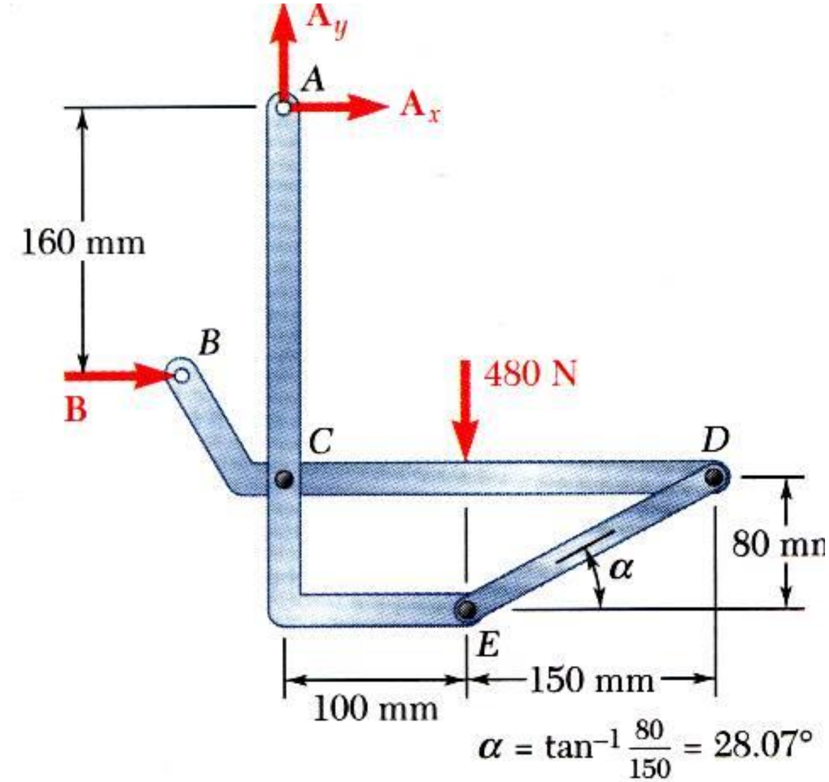
Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.



❑ Sample Problem

SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.



Determine the greatest force **P** that can be applied to the frame if the largest force resultant acting at A can have a magnitude F_{max}.

Units Used:

$$kN = 10^3\ N$$

Given:

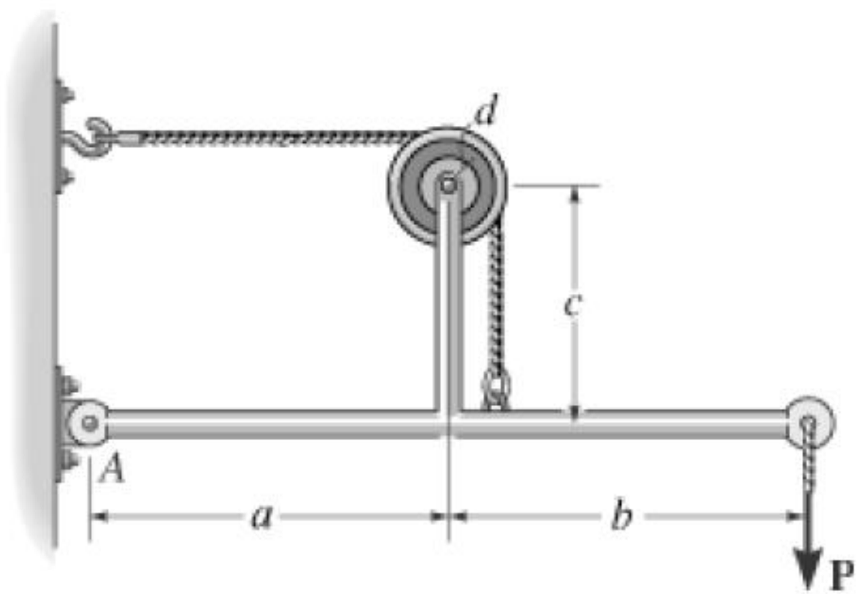
$$F_{max} = 2\ kN$$

$$a = 0.75\ m$$

$$b = 0.75\ m$$

$$c = 0.5\ m$$

$$d = 0.1\ m$$



Determine the reactions at supports A and B.

Units Used:

$kip = 10^3\ lb$

Given:

$w_1 = 500\ \frac{lb}{ft}$

$w_2 = 700\ \frac{lb}{ft}$

$a = 6\ ft$

$b = 8\ ft$

$c = 9\ ft$

$d = 6\ ft$